PROCEEDINGS OF SCIENCE

Schwinger mechanism in QCD

Joannis Papavassiliou*
Department of Theoretical Physics and IFIC,
University of Valencia-CSIC,
E-46100, Valencia, Spain.
E-mail: Joannis.Papavassiliou@uv.es

The generation of a momentum-dependent gluon mass proceeds through a sophisticated implementation, at the level of the Schwinger-Dyson equation for the gluon propagator, of the Schwinger mechanism, whose central dynamical ingredient is the nonperturbative formation of longitudinally coupled massless bound-state excitations. In addition to triggering the aforementioned mechanism, these excitations introduce poles in the various off-shell Green's functions of the theory, in such a way as to maintain the Slavnov-Taylor identities intact in the presence of massive gluon propagators, acting effectively as composite Nambu-Goldstone bosons. In this work we focus on the dynamics leading to the actual formation of such bound states. Specifically, we derive and solve numerically an approximate version of the homogeneous Bethe-Salpeter equation governing the wave function of this special bound state. It is found that this integral equation admits physically meaningful non-trivial solutions, indicating that the QCD dynamics produce one of the crucial ingredients required for the gauge-invariant generation of a gluon mass.

International Workshop on QCD Green's Functions, Confinement and Phenomenology,
September 05-09, 2011
Trento Italy

*Speaker.

PoS(QCD-TNT-II)034

1. Introduction

It is by now a well-established fact that large-volume lattice simulations in the Landau gauge yield a gluon propagator that reaches a finite non-vanishing value in the deep infrared [1, 2, 3, 4, 5, 6, 7]. Without a doubt, the most physical way of explaining this observed finiteness is to invoke the mechanism of dynamical gluon mass generation, first introduced in the seminal work of Cornwall [8], and subsequently studied in a series of articles [9, 10, 11]. In this picture the fundamental Lagrangian of the Yang-Mills theory (or that of QCD) remains unaltered, and the generation of the gluon mass takes place dynamically, through the well-known Schwinger mechanism [12, 13, 14, 15, 16, 17, 18], without violating any of the underlying symmetries (for related contributions and alternative approaches, see, e.g., [19, 20, 21, 22, 23, 24, 25, 26]).

The way how the Schwinger mechanism generates a mass for the gauge boson (gluon) can be seen most directly at the level of its inverse propagator, $\Delta^{-1}(q^2) = q^2[1 + i\mathbf{\Pi}(q^2)]$, where $\mathbf{\Pi}(q)$ is the dimensionless vacuum polarization. According to Schwinger's fundamental observation, if $\mathbf{\Pi}(q^2)$ develops a pole at zero momentum transfer ($q^2 = 0$), then the vector meson acquires a mass, even if the gauge symmetry forbids a mass term at the level of the fundamental Lagrangian. Indeed, if $\mathbf{\Pi}(q^2) = m^2/q^2$, then (in Euclidean space) $\Delta^{-1}(q^2) = q^2 + m^2$, and so the vector meson becomes massive, $\Delta^{-1}(0) = m^2$, even though it is massless in the absence of interactions ($g = 0$, $\mathbf{\Pi} = 0$) [14, 15].

The key assumption when invoking the Schwinger mechanism in Yang-Mills theories, such as QCD, is that the required poles may be produced due to purely dynamical reasons; specifically, one assumes that, for sufficiently strong binding, the mass of the appropriate bound state may be reduced to zero [14, 15, 16, 17, 18]. In addition to triggering the Schwinger mechanism, these massless composite excitations are crucial for preserving gauge invariance. Specifically, the presence of massless poles in the off-shell interaction vertices guarantees that the Ward identities (WIs) and Slavnov Taylor identities (STIs) of the theory maintain exactly the same form before and after mass generation (i.e. when the the massless propagators appearing in them are replaced by massive ones) [8, 17, 18, 11]. Thus, these excitations act like dynamical Nambu-Goldstone scalars, displaying, in fact, all their typical characteristics, such as masslessness, compositeness, and longitudinal coupling; note, however, that they differ from Nambu-Goldstone bosons as far as their origin is concerned, since they are not associated with the spontaneous breaking of any global symmetry [8]. Finally, every such Goldstone-like scalar, "absorbed" by a gluon in order to acquire a mass, is expected to actually cancel out of the S-matrix against other massless poles or due to current conservation [14, 15, 16, 17, 18].

The main purpose of this presentation is to report on recent work [27], where the central assumption of the dynamical scenario outlined above, namely the possibility of actual formation of such massless excitations, has been examined. Specifically, the entire mechanism of gluon mass generation hinges on the appearance of massless poles inside the nonperturbative three-gluon vertex, which enters in the Schwinger Dyson equation (SDE) governing the gluon propagator. These poles correspond to the propagator of the scalar massless excitation, and interact with a pair of gluons through a very characteristic proper vertex, which, of course, must be non vanishing, or else the entire construction is invalidated. The way to establish the existence of this latter vertex is by finding non-trivial solutions to the homogeneous Bethe-Salpeter equation (BSE) that it satisfies.

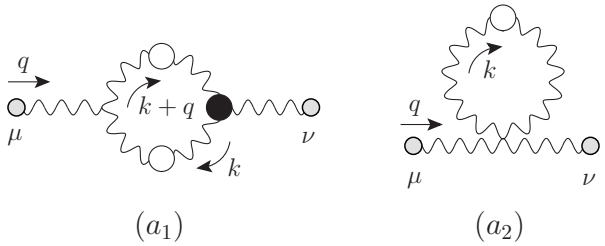

Figure 1: The "one-loop dressed" gluon contribution to the PT-BFM gluon self-energy. White (black) circles denote fully dressed propagators (vertices); a gray circle attached to the external legs indicates that they are background gluons. Within the PT-BFM framework these two diagrams constitute a transverse subset of the full gluon SDE.

2. Basic concepts

The full gluon propagator $\Delta^{ab}_{\mu\nu}(q) = \delta^{ab}\Delta_{\mu\nu}(q)$ in the Landau gauge is defined as

$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)\,, \tag{2.1}$$

where

$$P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\,, \tag{2.2}$$

is the usual transverse projector, and the scalar cofactor $\Delta(q^2)$ is related to the (all-order) gluon self-energy $\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q^2)$ through

$$\Delta^{-1}(q^2) = q^2 + i\Pi(q^2). \tag{2.3}$$

One may define the dimensionless vacuum polarization $\mathbf{\Pi}(q^2)$ by setting $\Pi(q^2) = q^2\mathbf{\Pi}(q^2)$ so that (2.3) becomes

$$\Delta^{-1}(q^2) = q^2[1 + i\mathbf{\Pi}(q^2)]\,. \tag{2.4}$$

Alternatively, one may define the gluon dressing function $J(q^2)$ as

$$\Delta^{-1}(q^2) = q^2 J(q^2)\,. \tag{2.5}$$

In the presence of a dynamically generated mass, the natural form of $\Delta^{-1}(q^2)$ is given by (Euclidean space)

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)\,, \tag{2.6}$$

where the first term corresponds to the "kinetic term", or "wave function" contribution, whereas the second is the (positive-definite) momentum-dependent mass. If one insists on maintaining the form of (2.5) by explicitly factoring out a q^2, then

$$\Delta^{-1}(q^2) = q^2\left[J(q^2) + \frac{m^2(q^2)}{q^2}\right]\,, \tag{2.7}$$

and the presence of the pole, with residue given by $m^2(0)$, becomes manifest.

The Schwinger mechanism is integrated into the SDE of the gluon propagator through the form of the three-gluon vertex. In particular, a crucial condition for the realization of the gluon mass generation scenario is the existence of a special vertex, to be denoted by $V_{\alpha\mu\nu}(q,r,p)$, which must be completely *longitudinally coupled*, i.e. must satisfy

$$P^{\alpha'\alpha}(q)P^{\mu'\mu}(r)P^{\nu'\nu}(p)V_{\alpha\mu\nu}(q,r,p)=0\,. \tag{2.8}$$

The role of the vertex $V_{\alpha\mu\nu}(q,r,p)$ is instrumental for maintaining gauge invariance, given that the massless poles that it must contain in order to trigger the Schwinger mechanism, act, at the same time, as composite, longitudinally coupled Nambu-Goldstone bosons. Specifically, in order to preserve the gauge invariance of the theory in the presence of masses, the vertex $V_{\alpha\mu\nu}(q,r,p)$ must be added to the conventional (fully-dressed) three-gluon vertex $I\!\Gamma_{\alpha\mu\nu}(q,r,p)$, giving rise to the new full vertex, $I\!\Gamma'_{\alpha\mu\nu}(q,r,p)$, defined as

$$I\!\Gamma'_{\alpha\mu\nu}(q,r,p)=I\!\Gamma_{\alpha\mu\nu}(q,r,p)+V_{\alpha\mu\nu}(q,r,p)\,. \tag{2.9}$$

Gauge invariance remains intact because $I\!\Gamma'$ satisfies the same WI (or STI) as $I\!\Gamma$ before, but now replacing the gluon propagators appearing on their rhs by massive ones; schematically, $\Delta^{-1}\to\Delta_m^{-1}$, where the former denotes the propagator given in (2.5), while the latter that of (2.6).

To see this in detail, let us employ the formalism provided by the synthesis of the pinch technique (PT) [8, 28, 29] with the background field method (BFM) [30]. In this framework, the natural quantity to consider is the vertex *BQQ*, to be denoted by $I\!\Gamma_{\alpha\mu\nu}(q,r,p)$, connecting a background gluon (B) with two quantum gluons (Q). With the Schwinger mechanism turned off, this vertex satisfies the WI

$$q^\alpha I\!\Gamma_{\alpha\mu\nu}(q,r,p)=p^2J(p^2)P_{\mu\nu}(p)-r^2J(r^2)P_{\mu\nu}(r)\,, \tag{2.10}$$

when contracted with respect to the momentum of the background gluon. Then, gauge invariance requires that

$$q^\alpha V_{\alpha\mu\nu}(q,r,p)=m^2(r^2)P_{\mu\nu}(r)-m^2(p^2)P_{\mu\nu}(p)\,, \tag{2.11}$$

so that, after turning the Schwinger mechanism on, the corresponding WI satisfied by $I\!\Gamma'$ would read

$$\begin{aligned} q^\alpha I\!\Gamma'_{\alpha\mu\nu}(q,r,p) &= q^\alpha\left[I\!\Gamma(q,r,p)+V(q,r,p)\right]_{\alpha\mu\nu}\\ &= [p^2J(p^2)-m^2(p^2)]P_{\mu\nu}(p)-[r^2J(r^2)-m^2(r^2)]P_{\mu\nu}(r)\\ &= \Delta_m^{-1}(p^2)P_{\mu\nu}(p)-\Delta_m^{-1}(r^2)P_{\mu\nu}(r)\,, \end{aligned} \tag{2.12}$$

which is indeed the identity in Eq. (2.10), with the aforementioned replacement $\Delta^{-1}\to\Delta_m^{-1}$ enforced. The remaining STIs, triggered when contracting $I\!\Gamma'_{\alpha\mu\nu}(q,r,p)$ with respect to the other two legs are realized in exactly the same fashion.

The next step is to insert $I\!\Gamma'_{\alpha\mu\nu}(q,r,p)$ into the SDE equation satisfied by the gluon propagator, see Fig. 1. Then, a rather elaborate analysis [11] gives rise to an integral equation for the momentum-dependent gluon mass, of the type

$$m^2(q^2)=\int_k m^2(k^2)\,K(q,k)\,, \tag{2.13}$$

where the kernel K survives the $q\to 0$ limit, i.e., $\lim_{q\to 0}K(q,k)\neq 0$, precisely because it includes the term $1/q^2$ contained inside $V_{\alpha\mu\nu}(q,r,p)$.

3. Structure of the pole vertex

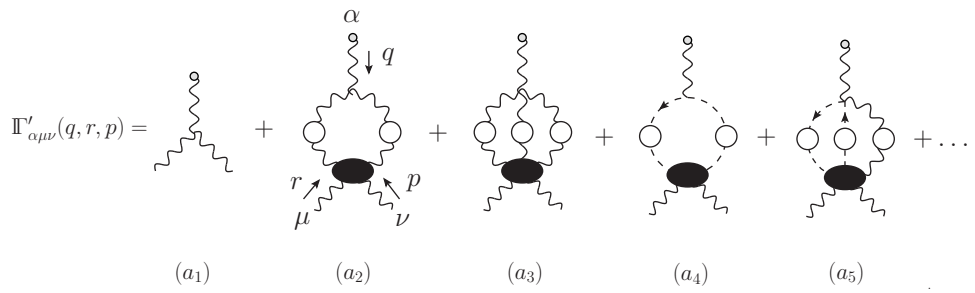

Figure 2: The SDE for the *BQQ* vertex which connects a background gluon (*B*) with two quantum gluons (*Q*).

The main characteristic of the vertex V, which sharply differentiates it from ordinary vertex contributions, is that it contains massless poles, originating from the contributions of bound-state excitations. Specifically, all terms of the vertex V are proportional to $1/q^2$, $1/r^2$, $1/p^2$, and products thereof. Such dynamically generated poles are to be clearly distinguished from poles related to ordinary massless propagators, associated with elementary fields in the original Lagrangian.

To see how such poles enter into the vertex, let us focus on the general structure of the SDE for the *BQQ* vertex. With the Schwinger mechanism turned off, the various multiparticle kernels appearing in this SDE have a complicated skeleton expansion (not shown here), but their common characteristic is that they are one-particle-irreducible with respect to cuts in the direction of the momentum q

When the Schwinger mechanism is turned on, the structure of the kernels is modified by the presence of composite massless excitation, described by a propagator of the type i/q^2, as shown in Fig. 3. The sum of such dynamical terms, coming from all multiparticle kernels, shown in Fig. 4, constitutes a characteristic part of the vertex V, to be denoted by U in Eq. (3.2), namely the part that contains at least a massless propagator i/q^2. The remaining parts, to be denoted by R, contain massless excitations in the other two channels, namely r_μ/r^2 and p_ν/p^2 (but no q_α/q^2), and are not relevant for the purposes of this presentation. Thus,

$$V_{\alpha\mu\nu}(q,r,p) = U_{\alpha\mu\nu}(q,r,p) + R_{\alpha\mu\nu}(q,r,p)\,, \tag{3.1}$$

with

$$U_{\alpha\mu\nu}(q,r,p) = q_\alpha \Big(V_1 g_{\mu\nu} + V_2 q_\mu q_\nu + V_3 p_\mu p_\nu + V_4 r_\mu q_\nu + V_5 r_\mu p_\nu \Big)\,, \tag{3.2}$$

where the V_i are form factors depending on the various momenta.

At this point we can make the nonperturbative pole manifest, and cast $U_{\alpha\mu\nu}(q,r,p)$ in the form of Fig. 4, by setting

$$U_{\alpha\mu\nu}(q,r,p) = I_\alpha(q) \left(\frac{i}{q^2}\right) B_{\mu\nu}(q,r,p)\,, \tag{3.3}$$

where the nonperturbative quantity

$$B_{\mu\nu}(q,r,p) = B_1 g_{\mu\nu} + B_2 q_\mu q_\nu + B_3 p_\mu p_\nu + B_4 r_\mu q_\nu + B_5 r_\mu p_\nu\,, \tag{3.4}$$

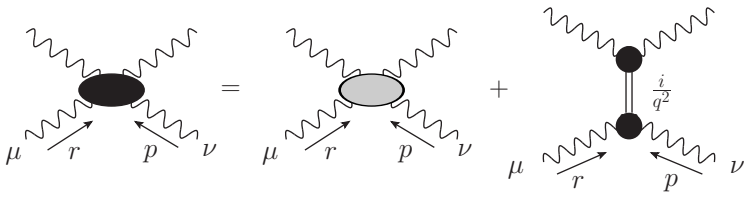

Figure 3: The gray kernel (regular part with respect to q, and the composite massless excitation in the q-channel.

is the effective vertex describing the interaction between the massless excitation and two gluons. $B_{\mu\nu}(q,r,p)$ is to be identified with the "bound-state wave function" (or "BS wave function") of the two-gluon bound-state shown in Fig. 3, which, as we will see shortly, satisfies a homogeneous BSE. In addition, i/q^2 is the propagator of the scalar massless excitation. Finally, $I_\alpha(q)$ is the (nonperturbative) transition amplitude introduced in Fig. 4, allowing the mixing between a gluon and the massless excitation; note that the imaginary factor "i" from the Feynman rule in Fig. 3 is absorbed into the definition of $I_\alpha(q)$.

Evidently, by Lorentz invariance,

$$I_\alpha(q) = q_\alpha I(q)\,, \tag{3.5}$$

and the scalar cofactor, to be referred to as the "transition function", is simply given by

$$I(q) = \frac{q^\alpha I_\alpha(q)}{q^2}\,, \tag{3.6}$$

so that

$$V_j(q,r,p) = I(q)\left(\frac{i}{q^2}\right)B_j(q,r,p)\,; \quad j = 1,\ldots,5\,. \tag{3.7}$$

Note that, due to Bose symmetry with respect to the interchange $\mu \leftrightarrow \nu$ and $p \leftrightarrow r$, we must have

$$B_{1,2}(q,r,p) = -B_{1,2}(q,p,r)\,, \tag{3.8}$$

which implies that

$$B_{1,2}(0,-p,p) = 0\,. \tag{3.9}$$

4. Gluon mass and the BS wave-function: an exact relation

The WI of Eq (2.11) furnishes an exact relation between the dynamical gluon mass, the transition amplitude at zero momentum transfer, and the form factor B_1. Specifically, contracting both sides of the WI with two transverse projectors, one obtains,

$$P^{\mu'\mu}(r)P^{\nu'\nu}(p)q^\alpha V_{\alpha\mu\nu}(q,r,p) = [m^2(r) - m^2(p)]P^{\mu'}_\sigma(r)P^{\sigma\nu'}(p)\,. \tag{4.1}$$

On the other hand, contracting the full expansion of the vertex (3.3) by these transverse projectors and then contracting the result with the momentum of the background leg, we get

$$q^\alpha P^{\mu'\mu}(r)P^{\nu'\nu}(p)V_{\alpha\mu\nu}(q,r,p) = iI(q)[B_1 g_{\mu\nu} + B_2 q_\mu q_\nu]P^{\mu'\mu}(r)P^{\nu'\nu}(p)\,, \tag{4.2}$$

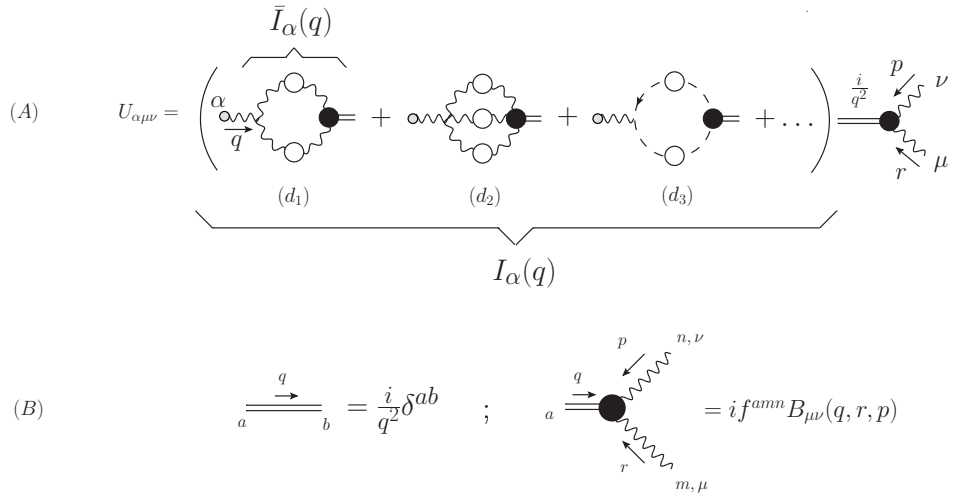

Figure 4: (A) The vertex $U_{\alpha\mu\nu}$ is composed of three main ingredients: the transition amplitude, I_α, which mixes the gluon with a massless excitation, the propagator of the massless excitation, and the (massless excitation)–(gluon)–(gluon) vertex. (**B)** The Feynman rules (with color factors included) for (i) the propagator of the massless excitation and (ii) the "proper vertex function", or, "bound-state wave function", $B_{\mu\nu}$.

where the relation of Eq (3.7) has been used. Thus, equating both results, one arrives at

$$iI(q)B_1(q,r,p)=m^2(r)-m^2(p)\,,\quad B_2(q,r,p)=0. \tag{4.3}$$

The above relations, together with those of Eq. (3.7), determine exactly the form factors V_1 and V_2 of the vertex $V_{\alpha\mu\nu}$, namely

$$V_1(q,r,p)=\frac{m^2(r)-m^2(p)}{q^2}\,,\quad V_2(q,r,p)=0. \tag{4.4}$$

We will now carry out the Taylor expansion of both sides of Eq (4.3) in the limit $q\to 0$. To that end, let consider the Taylor expansion of a function $f(q,r,p)$ around $q=0$ (and $r=-p$). In general we have

$$f(q,-p-q,p)=f(-p,p)+[2(q\cdot p)+q^2]f'(-p,p)+2(q\cdot p)^2f''(-p,p)+\mathscr{O}(q^3)\,, \tag{4.5}$$

where the prime denotes differentiation with respect to $(p+q)^2$ and subsequently taking the limit $q\to 0$, i.e.

$$f'(-p,p)\equiv\lim_{q\to 0}\left\{\frac{\partial f(q,-p-q,p)}{\partial\,(p+q)^2}\right\}. \tag{4.6}$$

Now, if the function is antisymmetric under $p\leftrightarrow r$, as happens with the form factors $B_{1,2}$, then $f(-p,p)=0$; thus, for the case of the form factors in question, the Taylor expansion is ($i=1,2$)

$$B_i(q,-p-q,p)=[2(q\cdot p)+q^2]B_i'(-p,p)+2(q\cdot p)^2B_i''(-p,p)+\mathscr{O}(q^3)\,. \tag{4.7}$$

Using Eq (4.7), and the corresponding expansion for the rhs,

$$m^2(r)-m^2(p)=m^2(q+p)-m^2(p)=2(q\cdot p)[m^2(p)]'+\mathscr{O}(q^2)\quad, \tag{4.8}$$

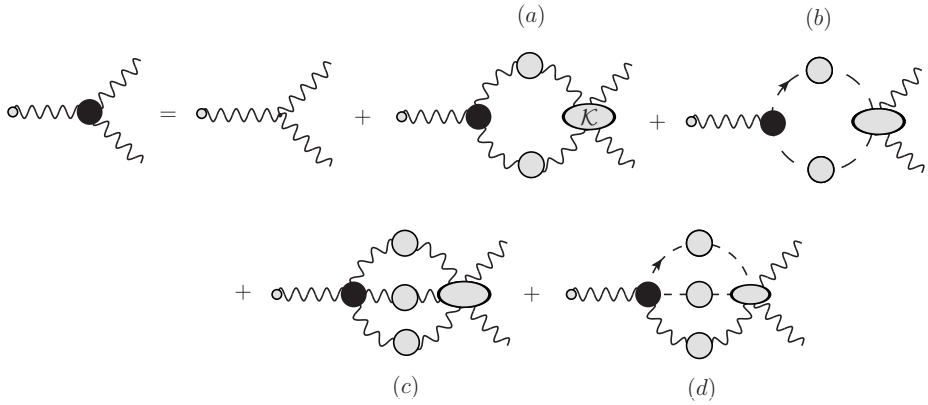

Figure 5: The complete BSE for the full three gluon vertex $I\!\Gamma'_{\alpha\mu\nu}(q,r,p)$.

assuming that the $I(0)$ is finite, and equating the coefficients in front of $(q\cdot p)$, we arrive at (Minkowski space)

$$[m^2(p)]' = iI(0)B_1'(p)\,. \tag{4.9}$$

Note that this is an exact relation, whose derivation relies only on the WI and Bose-symmetry that $V_{\alpha\mu\nu}(q,r,p)$ satisfies, as captured by Eq. (2.11) and Eq. (3.9), respectively.

5. The Bethe-Salpeter equation

As has become clear in the previous section, the existence of B_1' is of paramount importance for the mass generation mechanism envisaged here; essentially, the question boils down to whether or not the dynamical formation of a massless bound-state excitation of the type postulated above is possible. As is well-known, in order to establish the existence of such a bound state one must **(i)** derive the appropriate BSE for the corresponding bound-state wave function, $B_{\mu\nu}$, (or, in this case, its derivative), and **(ii)** find non-trivial solutions for this integral equation.

The starting point is the BSE for the vertex $I\!\Gamma'_{\alpha\mu\nu}(q,r,p)$, shown in Fig. 5. Note that, unlike the corresponding SDE of Fig. 2, the vertices where the background gluon is entering (carrying momentum q) are now fully dressed. As a consequence, the corresponding multiparticle kernels appearing in Fig. 5 are different from those of the SDE.

The general methodology of how to isolate from the BSE shown in Fig. 5 the corresponding dynamical equation for the quantity $B_{\mu\nu}$ has been explained in [15, 18]. Specifically, one separates on both sides of the BSE equation each vertex (black circle) into two parts, a "regular" part and another containing a pole $1/q^2$; this separation is shown schematically in Fig. 6. Then, omitting all other vertices, and the possible poles they too may have, the BSE for $B_{\mu\nu}(q,r,p)$ is obtained simply by equating the pole parts on both sides; specifically, [see Fig. 5]

$$B^{amn}_{\mu\nu} = \int_k B^{abc}_{\alpha\beta}\Delta^{\alpha\rho}_{br}(k+q)\Delta^{\beta\sigma}_{cs}(k)\mathscr{K}^{snmr}_{\sigma\nu\mu\rho}\,. \tag{5.1}$$

We will next approximate the four-gluon BS kernel $\mathscr{K}$ by the lowest-order set of diagrams shown in Fig. 7, where the vertices are bare, while the internal gluon propagators are fully dressed.

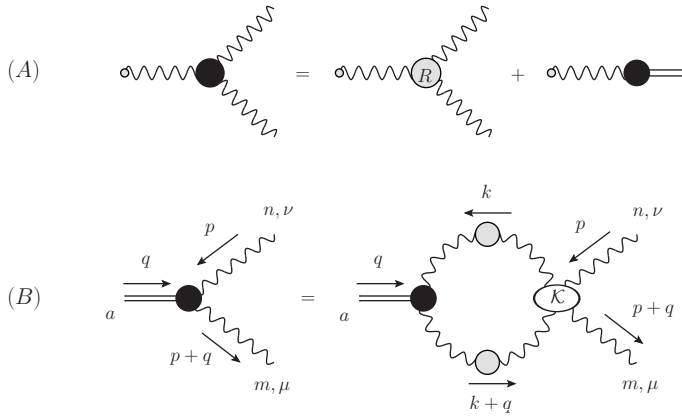

Figure 6: (A) The separation of the vertex in regular and pole parts. **(B)** The BSE for the bound-state wave function $B_{\mu\nu}$.

Going to Euclidean space, we define $x \equiv p^2$, $y \equiv k^2$, and $z \equiv (p+k)^2$; then, after appropriate Taylor expansion, and use of the fact that $B_2 = 0$ [see Eq. (4.3)], the BSE becomes

$$B_1'(x) = -\frac{\alpha_s C_A}{12\pi^2}\int_0^\infty dy y B_1'(y)\Delta^2(y) \sqrt{\frac{y}{x}}\int_0^\pi d\theta \sin^4\theta\cos\theta\left[z+10(x+y)+\frac{1}{z}(x^2+y^2+10xy)\right]\Delta(z)\,. \tag{5.2}$$

As a further simplification, we approximate the gluon propagator $\Delta(z)$ appearing in the BSE of (5.2) [but not the $\Delta^2(y)$] by its tree level value, that is, $\Delta(z) = 1/z$. Then, the angular integration may be carried out exactly, yielding

$$B_1'(x) = \frac{\alpha_s C_A}{24\pi}\left\{\int_0^x dy B_1'(y)\Delta^2(y)\frac{y^2}{x}\left(3+\frac{25}{4}\frac{y}{x}-\frac{3}{4}\frac{y^2}{x^2}\right)+\int_x^\infty dy B_1'(y)\Delta^2(y) y\left(3+\frac{25}{4}\frac{x}{y}-\frac{3}{4}\frac{x^2}{y^2}\right)\right\}. \tag{5.3}$$

6. Numerical analysis

Next we discuss the numerical solutions for Eq. (5.3) for arbitrary values of x. Evidently, the main ingredient entering into its kernel is the nonperturbative gluon propagator, $\Delta(q)$. In order to explore the sensitivity of the solutions on the details of $\Delta(q)$, we will employ three infrared-finite forms, to be denoted by $\Delta_1(q)$, $\Delta_2(q)$, and $\Delta_3(q)$, focusing on their differences in the intermediate and asymptotic regions of momenta.

(i) Let us start with the simplest such propagator, namely a tree-level massive propagator of the form

$$\Delta_1^{-1}(q^2) = q^2 + m_0^2\,, \tag{6.1}$$

where m_0^2 is a hard mass, that will be treated as a free parameter. On the left panel of Fig. 8, the (blue) dotted curve represents $\Delta_1(q^2)$ for $m_0 = 376\,\text{MeV}$.

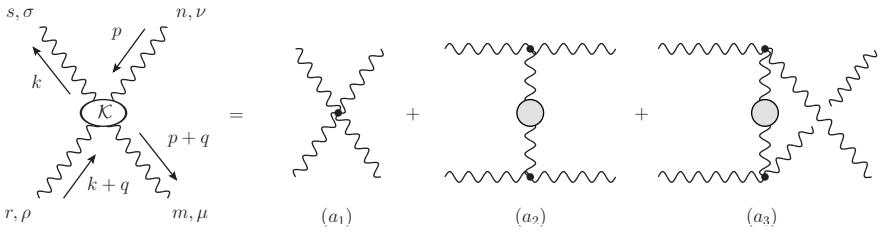

Figure 7: The Feynman diagrams considered for the BS kernel. The interaction vertices are approximated by their tree level values, while the internal gluon propagators are fully dressed.

(ii) The second model is an improved version of the first, where we introduce the renormalization-group logarithm next to the momentum q^2, more specifically

$$\Delta_2^{-1}(q^2) = q^2\left[1+\frac{13C_{\rm A}g^2}{96\pi^2}\ln\left(\frac{q^2+\rho\, m_0^2}{\mu^2}\right)\right]+m_0^2\,, \tag{6.2}$$

where ρ is an adjustable parameter varying in the range of $\rho\in[2,10]$. Notice that the hard mass m_0^2 appearing in the argument of the perturbative logarithm acts as an infrared cutoff; so, instead of the logarithm diverging at the Landau pole, it saturates at a finite value. The (black) dashed line represents the Eq. (6.2) when $\rho=16$, $m_0=376\,$MeV, and $\mu=4.3$ GeV.

(iii) The third model is simply a physically motivated fit for the gluon propagator determined by the large-volume lattice simulations of Ref. [3], and shown on the left panel of Fig. 8. The lattice data presented there correspond to a $SU(3)$ quenched lattice simulation, where $\Delta(q)$ is renormalized at $\mu=4.3$ GeV. This gluon propagator can be accurately fitted by the expression

$$\Delta_3^{-1}(q^2) = m_g^2(q^2)+q^2\left[1+\frac{13C_{\rm A}g_1^2}{96\pi^2}\ln\left(\frac{q^2+\rho_1\, m_g^2(q^2)}{\mu^2}\right)\right], \tag{6.3}$$

where $m_g^2(q^2)$ is a running mass given by

$$m_g^2(q^2) = \frac{m^4}{q^2+\rho_2 m^2}\,, \tag{6.4}$$

and the values of the fitting parameters are $m=520\,$MeV, $g_1^2=5.68$, $\rho_1=8.55$ and, $\rho_2=1.91$. On the left panel of Fig. 8, the (red) continuous line represents the fit for the lattice gluon propagator given by Eq. (6.3). Notice that, in all three cases, we have fixed the value of $\Delta^{-1}(0)=m_0^2\approx 0.14$.

Our main findings may be summarized as follows.

(a) In Fig. 8, right panel, we show the solutions of Eq. (5.3) obtained using as input the three propagators shown on the left panel. For the simple massive propagator of Eq. (6.1), a solution for $B_1'(q)$ is found for $\alpha_s=1.48$; in the case of $\Delta_2(q)$ given by Eq. (6.2), a solution is obtained when $\alpha_s=0.667$, while for the lattice propagator $\Delta_3(q)$ of Eq. (6.3) a non-trivial solution is found when $\alpha_s=0.492$.

(b) Note that, due to the fact that Eq. (5.3) is homogeneous and (effectively) linear, if $B_1'(q)$ is a solution then the function $cB_1'(q)$ is also a solution, for any real constant c. Therefore, the solutions shown on the right panel of Fig. 8 corresponds to a representative case of a family of possible solutions, where the constant c was chosen such that $B_1'(0)=1$.

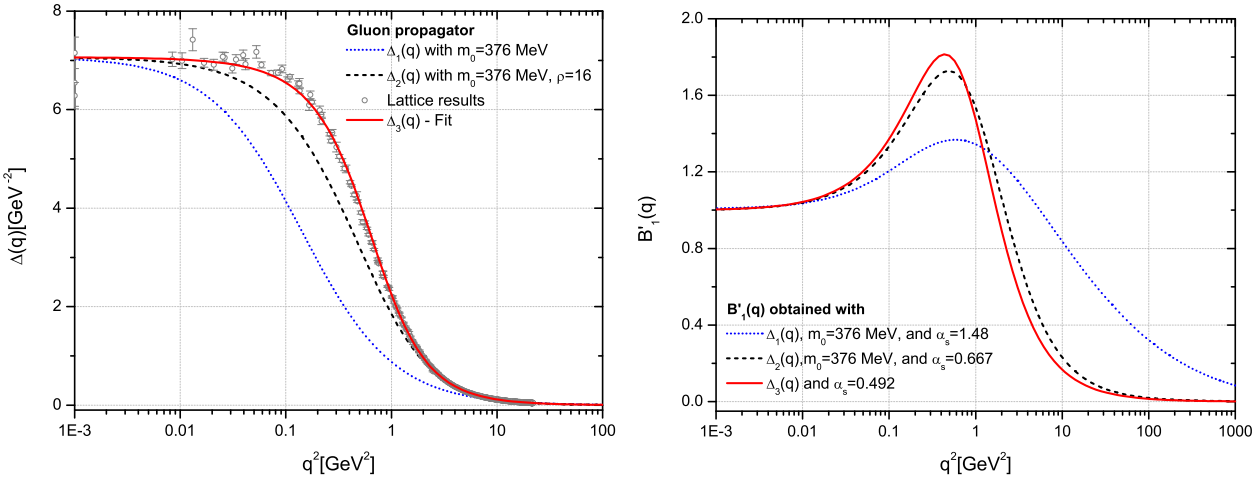

Figure 8: The three models for the gluon propagator (left) and the corresponding solutions of the BS equation for $B'(x)$ (right)

(c) Another interesting feature of the solutions of Eq. (5.3) is the dependence of the observed peak on the support of the gluon propagator in the intermediate region of momenta. Specifically, an increase of the support of the gluon propagator in the approximate range (0.3-1) GeV results in a more pronounced peak in $B_1'(q)$.

(d) In addition, observe that due to the presence of the perturbative logarithm in the expression for $\Delta_2(q)$ and $\Delta_3(q)$, the corresponding solutions $B_1'(q)$ fall off in the ultraviolet region much faster than those obtained using the simple $\Delta_1(q)$ of Eq. (6.1).

7. Conclusions

In this presentation we have reported recent progress [27] on the study of the Schwinger mechanism in QCD, which is the only self-consistent way to endow gluons with a dynamical mass. This mechanism relies on the existence of massless bound-state excitations, whose dynamical formation is controlled by a homogeneous BSE. As we have seen, under certain simplifying assumptions, this equation admits non-trivial solutions, thus furnishing additional support in favor of the specific mass generation mechanism described in a series of earlier works [9, 10, 11].

In the future it would be particularly important to consider the effects of bound-state poles in the SD kernels of not only the three-gluon vertex, as we did here, but of all other fundamental vertices of the theory. Such an investigation would eventually give rise to a coupled system of various homogeneous integral equations. Especially interesting in this context is the information that one might be able to obtain on the corresponding wave-function of the ghost-ghost channel. Specifically, according to the recent lattice findings [1, 2, 3, 4, 5], in the deep infrared the ghost dressing function F is finite, but the full ghost propagator diverges, a fact that strongly suggests that there is no dynamical mass associated with the ghost field (note that the finiteness of F can be easily accounted for by the presence of a gluon mass, saturating the perturbative logarithm of F [10]). One would expect, therefore, that the solution of the corresponding system should give rise to a non-vanishing B_1', as before, but to a vanishing ghost-ghost wave function.

Acknowledgments:

I would like to thank the ECT* for making the QCD-TNT II workshop possible. This research was supported by the European FEDER and Spanish MICINN under grant FPA2008-02878.

References

[1] A. Cucchieri and T. Mendes, PoS **LAT2007**, 297 (2007).

[2] A. Cucchieri and T. Mendes, Phys. Rev. D **81**, 016005 (2010).

[3] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, PoS LATTICE, 290 (2007).

[4] P. O. Bowman *et al.*, Phys. Rev. D **76**, 094505 (2007).

[5] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, Phys. Lett. B **676**, 69 (2009).

[6] O. Oliveira and P. J. Silva, PoS **LAT2009**, 226 (2009).

[7] H. Suganuma, T. Iritani, A. Yamamoto and H. Iida, PoSQCD -**TNT09**, 044 (2009); PoSLATTICE **2010**, 289 (2010).

[8] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982).

[9] A. C. Aguilar and J. Papavassiliou, JHEP **0612**, 012 (2006).

[10] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008).

[11] A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **84**, 085026 (2011)

[12] J. S. Schwinger, Phys. Rev. **125**, 397 (1962).

[13] J. S. Schwinger, Phys. Rev. **128**, 2425 (1962).

[14] R. Jackiw and K. Johnson, Phys. Rev. D **8**, 2386 (1973).

[15] R. Jackiw, In *Erice 1973, Proceedings, Laws Of Hadronic Matter*, New York 1975, 225-251 and M I T Cambridge - COO-3069-190 (73,REC.AUG 74) 23p.

[16] J. M. Cornwall and R. E. Norton, Phys. Rev. D **8** 3338 (1973).

[17] E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254 (1974).

[18] E. C. Poggio, E. Tomboulis and S. H. Tye, Phys. Rev. D **11**, 2839 (1975)).

[19] C. S. Fischer, A. Maas, J. M. Pawlowski, Annals Phys. **324**, 2408-2437 (2009).

[20] K. -I. Kondo, Prog. Theor. Phys. **122**, 1455-1475 (2010); [arXiv:0909.4866 [hep-th]].

[21] J. Rodriguez-Quintero, PoS **LC2010**, 023 (2010); AIP Conf. Proc. **1354**, 118 (2011).

[22] P. Watson and H. Reinhardt, Phys. Rev. D **82**, 125010 (2010).

[23] A. P. Szczepaniak and H. H. Matevosyan, Phys. Rev. D **81**, 094007 (2010).

[24] O. Oliveira and P. Bicudo, J. Phys. G **38**, 045003 (2011).

[25] D. Binosi and A. Quadri, Phys. Rev. D **84**, 065017 (2011)

[26] M. R. Pennington, D. J. Wilson, [arXiv:1109.2117 [hep-ph]].

[27] A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, arXiv:1110.2633 [hep-ph].

[28] J. M. Cornwall and J. Papavassiliou, Phys. Rev. D **40**, 3474 (1989).

[29] D. Binosi and J. Papavassiliou, Phys. Rept. **479**, 1-152 (2009).

[30] L. F. Abbott, Nucl. Phys. B **185**, 189 (1981), and references therein.