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Perturbative and nonperturbative QCD to the Bjorken sum rule up to four loop

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We present results of applying the common QCD perturbation theory and the singularity-free analytic perturbation theory to the polarized Bjorken sum rule, $\Gamma_1^{p-n}(Q^2)$, at low momentum transfers. We use the four-loop expression for the coefficient function $C_{\rm Bj}(\alpha_{\rm s})$ available now and the most precise experimental data from the Jefferson Lab on $\Gamma_1^{p-n}(Q^2)$ in the range $0.05 < Q^2 < 3.0$ GeV^2. We observe that the ordinary perturbative series for the function $C_{\rm Bj}$ gives a hint to its asymptotic nature manifesting itself in the region $Q^2 < 1$ GeV^2. Besides, the related values of the higher twists coefficients turn out to be highly unstable with respect to the order approximations. On the contrary, the usage of the analytic perturbation theory allows to describe the Jefferson Lab data down to $Q \sim \Lambda_{\rm QCD}$ and gives a possibility for reliable extraction of the higher twist coefficients.

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1. Introduction

Among the moments of the proton and neutron structure functions, the Bjorken sum rule [2] is one of the convenient tests of the perturbative QCD (pQCD). Since this sum rule relates the difference of the proton and neutron first moments, only flavor non-singlet quark operators appear in the operator product expansion. The Q^2-evolution of the Bjorken sum rule is given by a double series in powers of $1/Q^2$ (nonperturbative power corrections) and in powers of the QCD running coupling $\alpha_s(Q^2)$ (pQCD radiative corrections). Until very recently, the pQCD contribution to the Bjorken sum rule has been known up to a third order in perturbative α_s expansion [3]. So far, the corresponding expression have been used in many studies aimed, in particular, to extraction of the α_s values at low momentum scales [4, 5].

The four-loop expression for the pQCD contribution to the Bjorken sum rule, which became recently available [6], gives us a reasonable motivation for a new extended QCD analysis of the precise low energy combined data on $\Gamma_1^{p-n}(Q^2)$ [7, 8, 9] accounting for up to α_s^4-order in both the standard perturbation theory (PT) and the ghost-free analytic perturbation theory (APT) [10]. The APT approach takes into account basic principles of local quantum field theory which in the simplest cases is reflected in the form of Q^2-analyticity of the Källén–Lehmann type (see, e.g., [11] for a review). As has been demonstrated in [12] (see also [13, 14]), the moments of the structure functions are analytic functions in the complex Q^2-plane with a cut along the negative real axis. The usage of the APT approach gives the possibility of combining the renormalization group resummation with correct analytic properties of the pQCD correction [15, 16].

In this report, we concentrate on the features of the four-loop PT and APT expansions in the analysis of the Bjorken sum rule and on the interplay between the nonperturbative power corrections (higher twists) and higher order pQCD corrections at low momentum scales.

2. The perturbative part

Away from the large Q^2 limit, the Bjorken sum rule can be written as the perturbative QCD part and the higher twist contribution

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}\left[1-\Delta_{\rm Bj}(Q^2)\right] + \sum_{i=2}^{\infty}\frac{\mu_{2i}}{Q^{2i-2}}\,, \qquad (2.1)$$

where g_A is the nucleon axial charge defined from the neutron β-decay data, $g_A = 1.267 \pm 0.004$ [17], the higher twist (HT) coefficients $\mu_4, \mu_6, \ldots$ contain the information on quark-gluon correlations in nucleons. The perturbative correction, $\Delta_{\rm Bj}(Q^2)$, is defined by the coefficient function $C_{\rm Bj}$: $\Delta_{\rm Bj}(Q^2) \equiv 1 - C_{\rm Bj}(\alpha_s)$. In the standard PT case, the approximation for $\Delta_{\rm Bj}(Q^2)$ has a form of the power series in the PT running coupling. At the up-to-date four-loop (N^{3}LO) level in the massless case

$$\Delta_{\rm Bj}^{\rm PT}(Q^2) = c_1\,\alpha_s + c_2\,\alpha_s^2 + c_3\,\alpha_s^3 + c_4\,\alpha_s^4\,, \qquad (2.2)$$

where the expansion coefficients c_i in the modified minimal subtraction scheme, $\overline{\rm MS}$, for three active flavors are $c_1 = 1/\pi = 0.31831$, $c_2 = 0.36307$ [18], $c_3 = 0.65197$ [3] and $c_4 = 1.8042$ [6].

The PT running coupling can be obtained numerically by integration of the renormalization group equation with the four-loop β-function (see [19, 20, 21] for additional details).

As outlined above, the moments of the structure functions are analytic functions of Q^2 in the complex Q^2-plane with a cut along the negative part of the real axis. The perturbative representation (2.2) violates these analytic properties due to the unphysical singularities of the PT running coupling for $Q^2 > 0$. To avoid this problem, we apply the APT method [10, 11]. In the framework of the APT, the correct analytic properties of the perturbative expansions are preserved at any fixed order including the four-loop one. The corresponding expression for $\Delta_{\rm Bj}(Q^2)$ reads as

$$\Delta_{\rm Bj}^{\rm APT}(Q^2) = c_1 \mathscr{A}^{(1)}(Q^2) + c_2 \mathscr{A}^{(2)}(Q^2) + c_3 \mathscr{A}^{(3)}(Q^2) + c_4 \mathscr{A}^{(4)}(Q^2)\,, \tag{2.3}$$

where coefficients c_1, c_2, c_3 and c_4 are the same as in (2.2), and functions $\mathscr{A}^{(k)}(Q^2)$ can be expressed through the spectral functions $\rho_k(\sigma) \equiv {\rm Im}\left[\alpha_{\rm s}^k(-\sigma - i\varepsilon)\right]$ by the Källen-Lehman representation

$$\mathscr{A}^{(k)}(Q^2) = \frac{1}{\pi}\int_0^\infty d\sigma\, \frac{\rho_k(\sigma)}{\sigma + Q^2}\,. \tag{2.4}$$

At large momentum transfers, analytic functions $\mathscr{A}^{(k)}(Q^2)$ become proportional to k-th power of the usual perturbative coupling, $[\alpha_{\rm s}(Q^2)]^k$, and the expansion (2.3) reduces to the power series (2.2). However, at small Q^2 the properties of the non-power expansion (2.3) become considerably different from the PT power series (2.2) (see, e.g., [15] for details).

2.1 The Q^2-dependence

Let us analyze the Q^2-dependence of the Bjorken sum rule in the framework of both PT and APT approaches in different orders (NLO, $\rm N^2LO$ and $\rm N^3LO$) of the perturbative expansions (2.2) and (2.3), respectively. As a normalization point, we use the most accurate $\alpha_{\rm s}$-value at $Q^2 = M_Z^2$, $\alpha_{\rm s}(M_Z^2) = 0.1184 \pm 0.0007$ [17, 21]. In order to take into account flavor thresholds, we apply the matching conditions for the values of $\alpha_{\rm s}$ which are rather nontrivial in higher PT orders (see [19, 21, 22]). Following to analysis in [23], our matched calculation for the four-loop $\overline{\rm MS}$-coupling gives $\Lambda^{(n_f=3)} = 340 \pm 10$ MeV.

In Fig. 1, we illustrate the behavior of the perturbative part of the Bjorken sum rule in different orders in $\alpha_{\rm s}$ in both PT and APT approaches. For completeness, we also show here the combined SLAC and Jefferson Lab (JLab) data on $\Gamma_1^{p-n}(Q^2)$ which are used in our analysis. The SLAC data points [8] are denoted by squares, the JLab CLAS Hall A 2002 data – by downward pointing triangles, the JLab CLAS Hall B 2003 data – by circles [9], and the most recent JLab data [7] – by stars. The horizontal dotted line represents the limiting value $\Gamma_1^{p-n}(Q^2 \to \infty) = g_A/6$. As it follows from this figures, in the framework of the standard PT, the low energy behavior of $\Gamma_1^{p-n}(Q^2)$ is strongly dependent on the order of the initial expansion, and the lower border of satisfactory description of the JLab data shifts towards the larger Q^2 values when increasing the number of loops in the pQCD expansion (2.2). In the framework of the APT, we observe the higher-loop stability given the fact that curves corresponding to different orders in APT are very close to each other (practically coincide with each other). At the same time, the deviation of APT curve from the data shows for necessity of including HT terms.

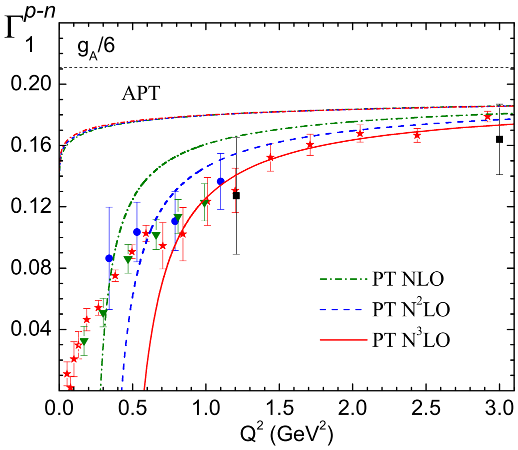

Figure 1: The Bjorken sum rule without HT terms in different orders in the PT and APT.

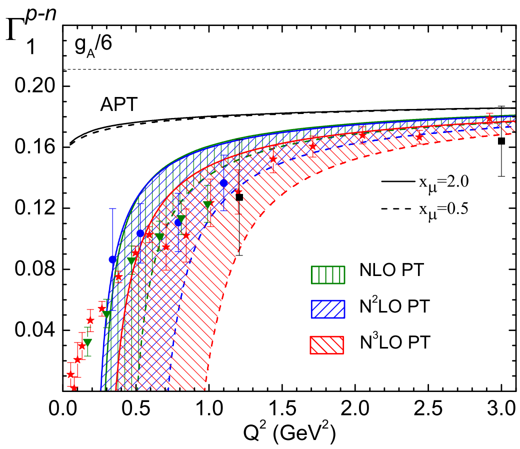

Figure 2: The μ-scale ambiguities vs. Q^2 in different orders in the PT and APT.

A truncation of a perturbative expansion leads to uncertainties in the theoretical predictions arising from the renormalization scale μ-dependence of the partial sum of the series. To estimate the ambiguity in choosing the renormalization scale μ, we use the following four-loop expression for the coefficient function $C_{\rm Bj}$ [6]

$$\begin{aligned} C_{Bj}\left(x_\mu,\alpha_{\rm s}\right) \;=\; & 1-0.31831\,\alpha_{\rm s}(\mu^2)+\left[-0.36307-0.22797\ln\left(x_\mu\right)\right]\alpha_{\rm s}^2(\mu^2) \\ & +\left[-0.65197-0.64906\ln\left(x_\mu\right)-0.16327\ln^2\left(x_\mu\right)\right]\alpha_{\rm s}^3(\mu^2) \\ & +\left[-1.8042-1.7984\ln\left(x_\mu\right)-0.78968\ln^2\left(x_\mu\right)-0.11694\ln^3\left(x_\mu\right)\right]\alpha_{\rm s}^4(\mu^2)\,, \end{aligned} \tag{2.5}$$

where the dimensionless parameter x_μ is introduced as $x_\mu=\mu^2/Q^2$ and in our analysis is changed within the interval $0.5\div 2$.

In Fig. 2 we compare the μ-scale ambiguities between the two-, three- and four-loop PT and APT series at low Q^2. This figure demonstrates that in the considered region of Q^2 there is an essential difference between μ-dependence of PT and APT expansions. The APT result is practically μ-scale independent, and the standard PT is rather sensitive to μ-scale variations indicating quite significant theoretical uncertainties of the corresponding PT expansions. As it is seen from this figure, the N^{3}LO PT approximation does not improve the μ-scale ambiguities compared to the N^{2}LO one (see the μ-scale analysis in [24] for comparison).

Let us turn now to the perturbative QCD correction $\Delta_{\rm Bj}(Q^2)$ truncated after four-loop order and demonstrate the convergence properties of the PT power series (2.2) and the APT series (2.3). For this purpose we consider the relative contributions, $\mathrm{N}_i(Q^2)$, for i-th term of $\Delta_{\rm Bj}$-series as a function of the Q^2 in the PT and APT cases.

We present our results in Figs. 3 and 4. One can see from Fig. 3 that the dominant contribution for the PT series (2.2) comes from the four-loop term in the region of small momentum transfers $Q^2<1$ GeV^2. Moreover, when decreasing Q^2 its relative contribution increases. In the region $Q^2>2$ GeV^2 the situation changes – the major contribution comes from one- and two-loop orders there. So, the fourth order PT correction to the Bjorken sum rule does not improve the convergence that is presumably due to an asymptotic character of the PT series.

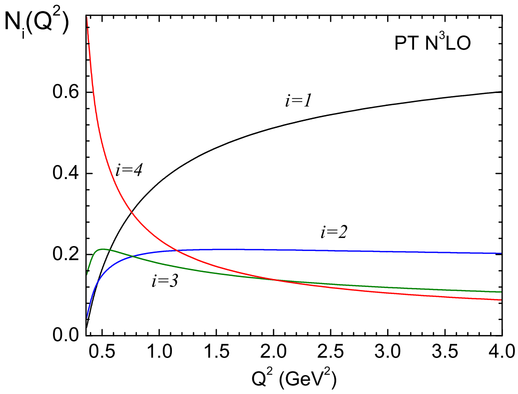

Figure 3: The Q^2-dependence of the relative contributions at the four-loop level in the PT.

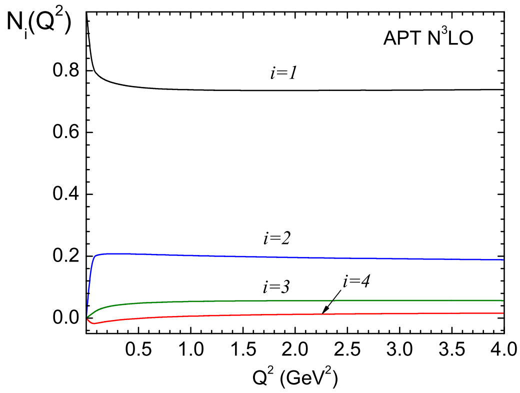

Figure 4: The Q^2-dependence of the relative contributions at the four-loop level in the APT.

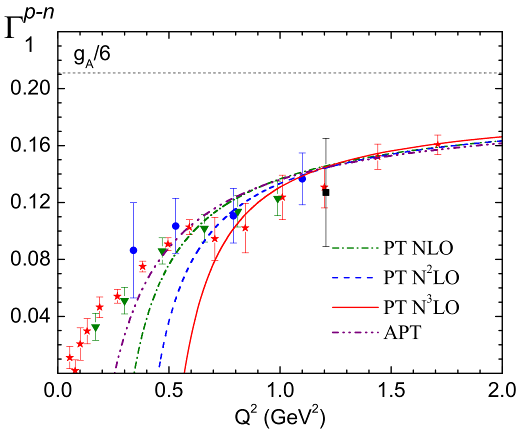

Figure 5: The μ_4-fits of the JLab data in various orders of PT and APT.

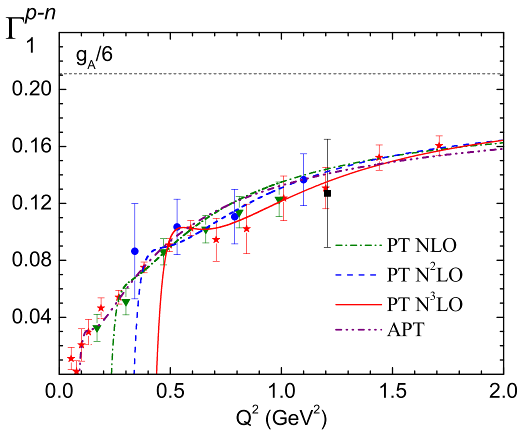

Figure 6: The $\mu_{4,6,8}$-fits of the JLab data in various orders of PT and APT.

The result for the APT series (2.3) is presented at Fig. 4. This figure demonstrates that there is an essential difference between PT and APT cases. The APT expansion obeys much better convergence than the PT one. In the APT case, the higher order contributions are stable at all Q^2, and one-loop contribution gives about 70 %, two-loop – 20 %, three-loop – not exceeds 5%, and four-loop – up to 1 %.

3. Higher twists terms

The QCD description of the Bjorken sum rule [see (2.1)] contains the perturbative part discussed above and the HT terms. One of the actual theoretical subjects is the interplay between these contributions. At low Q^2 these interplay may become very large. Previously, a detailed higher-twist analysis for the Bjorken sum rule at the two- and three-loop level in the framework of both PT and APT was performed in [16]. It was shown that the infrared behavior of the strong coupling is crucial for the extraction of the nonperturbative information from the low-energy data. Here, we extend the analysis started in [16] to an order $\sim \alpha_s^4$. Using the expression (2.1) being fitted to above mentioned experimental data [7, 9], we extract coefficients μ_{2i} of the HT contribution (see, for detail, [1]). In Figs. 5 and 6 we present the results of 1- and 3-parametric fits in various orders of PT and APT. The corresponding fit results for values of HT coefficients, extracted by using the different orders of PT and APT, are given in Table 1 (all numerical results are normalized to the corresponding powers of the nucleon mass M).

Method	Q^2_{min}	μ_4/M^2	μ_6/M^4	μ_8/M^6	$\chi^2_{d.f.}$
The best μ_4-fit results					
PT NLO	0.5	−0.028(3)	–	–	0.80
PT N^{2}LO	0.66	−0.014(5)	–	–	0.59
PT N^{3}LO	0.71	0.006(7)	–	–	0.51
APT	0.47	−0.050(2)	–	–	0.82
The best $\mu_{4,6,8}$-fit results					
PT NLO	0.27	−0.026(9)	−0.01(1)	0.008(4)	0.69
PT N^{2}LO	0.34	0.01(2)	−0.06(4)	0.04(2)	0.67
PT N^{3}LO	0.47	0.05(3)	−0.17(9)	0.12(6)	0.46
APT	0.08	−0.061(2)	0.009(1)	−0.0004(1)	0.91

Table 1: Results of HT extraction from the JLab data on the Bjorken sum rule in various orders of PT and all orders of APT with left border Q^2_{min} [GeV^2] of fitting domain.

One can see that the difference between the PT and APT results are significant. Since the APT approach exhibits the higher loop stability, the values of HT coefficients extracted in the NLO, N^{2}LO and N^{3}LO APT coincide within the data fits uncertainty. Besides, coefficients extracted from the data in the standard PT approach in different PT orders are different. The value of μ_4 coefficient, for example, at two- and three-loop levels is negative, whereas at the four-loop level it becomes positive. From these figures and Table follows that APT allows one to move further down to $Q^2 \sim 0.08$ GeV^2.

4. Summary

We have considered the Bjorken sum rule by using the standard PT and APT approaches up to four-loop level. At high Q^2 scales, the PT and APT results close to each other, both including the HT terms and without them, whereas at low Q^2 scales, the difference between the PT and APT results becomes very significant. We have observed that the ordinary PT series for the pQCD correction to the Bjorken sum rule gives a hint to its asymptotic nature manifesting itself in the region $Q^2 < 1$ GeV^2. It relates to the observation that the accuracy of both the three- and four-loop PT predictions happens to be at the same 10% level. Besides, the related values of the higher twists coefficients turn out to be highly unstable with respect to the order PT approximations. On the contrary, the usage of the analytic perturbation theory allows to describe the Jefferson Lab data down to $Q \sim \Lambda_{\rm QCD}$ and gives a possibility for reliable extraction of the higher twist coefficients.

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