

Wall-shock waves with charge in AdS_5

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We study the formation of a quasi trapped surface in the collision of two charged wall-shock waves in $(A)dS$ space-time. The advantage of this model is the essential simplicity of calculations. The quasi trapped surface has an infinite extent in transversal directions. and also has a sharp bend in the location of the domain wall. We consider a regularization that deforms the quasi trapped surface to make it a smooth compact surface. Taking into account that the entropy density of the TS admits the remove of the regularization one can restrict ourself to quasi TS calculations. We study the TS equation, more precisely, the quasi TS equation, for a charged wall-wall collision and reduce it to an algebraic equation that we study numerically.

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1. Introduction

Black holes are expected to form in collisions of ultrarelativistic particles with energies above the Planck scale [1, 2, 3, 4]. The Planck energy could be few TeV in the framework of TeV-gravity, where our space is a 3-brane situated in a large extra dimensional space and elementary particles are confined on the brane [5]. In this scenario the black hole production in collisions of particles with the center-mass energy of a few TeV and their experimental signatures [6] at the LHC became the subject of intensive analytical and numerical investigations [7, 8, 9, 10, 11, 12, 13]. We also note a discussion of the possible production of wormholes and others more exotic objects at the LHC [14, 15, 16].

Gubser, Yarom and Pufu [17] have proposed the gravitational shock wave in AdS_5 as a possible holographic dual for the heavy ion and have related the area of the trapped surface formed in a collision of such waves to the entropy of matter formed after collision of heavy ions. See [18] and refs therein about applications of AdS/CFT duality for others aspects of heavy ion physics. The main result of [17, 19] is that in the limit of a very large collision energy E the multiplicity (the entropy S) grows as $S \sim E^{2/3}$. Alvarez-Gaume, Gomez, Sabeo Vera, Tavanfar, and Vazquez-Mozoand [20] have considered central collision of shock waves sourced by a nontrivial matter distribution in the transverse space and they have found critical phenomenon occurring as the shock wave reaches some diluteness limit. This criticality may be related to criticality found in [21]. The numerical results of [21] show the existence of a simple scaling relation between the critical impact parameter and the energy of colliding waves.

Formation of marginally trapped surfaces in the head-on collision of two ultrarelativistic charges in $(A)dS$ space-time has been studied in [22]. Formation of trapped surfaces is only possible when charge is below the critical value. The situation result takes place for the collision of two ultrarelativistic charges in Minkowski space-time[23]. In [24] the critical value is identified via AdS/CFT duality with a point on the boundary separated the quark gluon plasma and the hadronic matter. It would be interesting to perform calculations similar to [20] for the charged case.

The model of infinite homogenous wall has been proposed and analyzed by Shuryak and Lin [21, 25]. The advantage of this model is the essential simplicity of calculations. However, the legitimacy of these calculations requires some additional examinations (see our discussion in [24]). The point is that by definition the trapped surface (TS) is a smooth compact surface. But the surface considered in [21] has an infinite extent in transversal directions. It has also a sharp bend in the location of the domain wall. By this reason we first reconsider calculations performed in [21] and regularize the geometry of wall-on-wall collisions. This give a regularized version of trapped surface calculations for wall-on-wall collisions. After these preliminary considerations we discuss the charged wall collision. We study the corresponding TS equation (more precisely, the quasi TS) and reduce it to an algebraic equation that we study numerically.

2. Remarks about the regularization of TS calculations in the case of wall-on-wall collisions

In [21] has been proposed a simple dual description of the colliding nuclei that uses a wall-on-wall collision in the bulk. The Einstein equation for the profile of the wall shock wave [21] has the

form:

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi(z) = J_{uu}^{WP}, \quad J_{uu}^{WP} = -16\pi G_5 \frac{E}{L^2} \frac{z_0^3}{L^3} \delta(z - z_0) \quad (2.1)$$

To find a TS that can be formed in the collision of two wall shock waves one needs to find a solution to the Einstein eq.(2.1) that satisfies two conditions. It is convenient to write these conditions in terms of function $\psi(z)$ related to ϕ via

$$\phi(z) = \frac{z}{L} \psi. \quad (2.2)$$

They have the form

$$\psi(z_a) = \psi(z_b) = 0, \quad (2.3)$$

$$\psi'(z_a) \frac{z_a}{L} = 2, \quad \psi'(z_b) \frac{z_b}{L} = -2 \quad (2.4)$$

where z_a, z_b are supposed to be the boundaries of TS [21]. But as we will see in the moment, strictly speaking, one cannot call the solution to the equation (2.1) with boundary conditions (2.3) and (2.4) as TS, since by definition this surface supposed to be smooth and compact meanwhile the solution [21] is *non-smooth* and *noncompact*.

By this reason we call the solution found in [21] a quasi-trapped surface. Let us remind the construction presented in [21].

In [25], the solution to the Einstein equation (2.1) is written in such a way that the property (2.3) is satisfied automatically. This solution has the form

$$\begin{aligned} \psi(z) &= \psi_a(z)\Theta(z_0 - z) + \psi_b(z)\Theta(z - z_0) \quad (2.5) \\ \psi_a(z) &= -\frac{4G\pi E \left(\frac{z_0^4}{z_b^4} - 1\right) z_b^4 z_a^3 \left(\frac{z^3}{z_a^3} - \frac{z_a}{z}\right)}{L^4 (z_b^4 - z_a^4)} \\ \psi_b(z) &= -\frac{4G\pi E \left(\frac{z_0^4}{z_a^4} - 1\right) z_a^4 z_b^3 \left(\frac{z^3}{z_b^3} - \frac{z_b}{z}\right)}{L^4 (z_b^4 - z_a^4)} \end{aligned}$$

Let us first note that solution (2.5) is not smooth. There is a non-smooth part of the solution (2.5)

$$\Xi = \frac{\mathcal{K}}{z} \left(-\frac{z_b}{z_a^3} \Upsilon_1 - \frac{z_a}{z_b^3} \Upsilon_2 \right), \quad \text{where } \mathcal{K} = \frac{4G\pi E}{L^4} \frac{z_a^3 z_b^3}{z_b^4 - z_a^4}, \quad (2.6)$$

$$\Upsilon_1 = z^4 \Theta(z_0 - z) + z_0^4 \Theta(z - z_0) \quad \Upsilon_2 = z_0^4 \Theta(z_0 - z) + z^4 \Theta(z - z_0) \quad (2.7)$$

Thus, in order to smooth the solution we have to smooth the function Ξ . We can do it by performing the regularization of the Heaviside step function

$$\Theta(z_0 - z) \approx \Gamma_1 = \frac{\arctan(R(z_0 - z))^3}{\pi} + \frac{1}{2} \quad (2.8)$$

$$\Theta(z - z_0) \approx \Gamma_2 = \frac{\arctan(R(z - z_0))^3}{\pi} + \frac{1}{2} : \quad (2.9)$$

and considering the regularized functions \tilde{Y}_1 and \tilde{Y}_2

$$\tilde{Y}_1 = z^4 \left(\frac{\arctan(R(z_0 - z))^3}{\pi} + \frac{1}{2} \right) + z_0^4 \left(\frac{\arctan(R(z - z_0))^3}{\pi} + \frac{1}{2} \right) \quad (2.10)$$

$$\tilde{Y}_2 = z_0^4 \left(\frac{\arctan(R(z_0 - z))^3}{\pi} + \frac{1}{2} \right) + z^4 \left(\frac{\arctan(R(z - z_0))^3}{\pi} + \frac{1}{2} \right) \quad (2.11)$$

For derivatives we have

$$\frac{d\tilde{Y}_1}{dz} \approx 4z^3 \Theta(z_0 - z), \quad \frac{d\tilde{Y}_1}{dz} \approx \frac{4z^3 (\arctan(R(z_0 - z))^3 + \pi)}{\pi}; \quad (2.12)$$

$$\frac{d\tilde{Y}_2}{dz} \approx 4z^3 \Theta(z - z_0), \quad \frac{d\tilde{Y}_2}{dz} \approx \frac{4z^3 (\arctan(R(z - z_0))^3 + \pi)}{\pi}. \quad (2.13)$$

In Fig.1 we present the derivatives of functions Y_1, Y_2 as well as derivatives of the smoothed functions \tilde{Y}_1, \tilde{Y}_2 .

For $R = 10^4$ (see below) the differences between derivatives $\frac{d\tilde{Y}_i}{dz}$ and their approximations given by (2.12) and (2.13)

$$\Delta_1(z) = \frac{d\tilde{Y}_1}{dz} - \left(\frac{d\tilde{Y}_1}{dz} \right)_{appr}, \quad \Delta_2(z) = \frac{d\tilde{Y}_2}{dz} - \left(\frac{d\tilde{Y}_2}{dz} \right)_{appr} \quad (2.14)$$

$$\Delta_1(z) = -\Delta_2(z) = -3 \frac{z^4 R^3 (z_0 - z)^2}{(1 + R^6 (z_0 - z)^6) \pi} + 3 \frac{z_0^4 R^3 (z - z_0)^2}{(1 + R^6 (z - z_0)^6) \pi} \quad (2.15)$$

are of order $\gtrsim 10^{-3} \text{ fm}^3$ only in the interval $z \in [z'_0, z''_0]$, $z'_0 = 4.293 \text{ fm}$, $z''_0 = 4.307 \text{ fm}$.

Indeed, in our consideration (spread case) the largest value of z_a is 4.260706906 fm and the smallest value of z_b is 4.340400579 fm . At the points $z'_0 = 4.260706906 \text{ fm}$, $z''_0 = 4.340400579 \text{ fm}$ the quantity Δ_1 is less then $\leq 5 \cdot 10^{-6} \text{ fm}^3$.

At the points $z'_0 = 0.6948439783 \text{ fm}$, $z''_0 = 1018.393720 \text{ fm}$ the quantity Δ_1 is less then $\leq 2 \cdot 10^{-12} \text{ fm}^3$. The schematic picture of locations of roots and a region there $|\Delta_i(z)| \gtrsim 10^{-3}$ are presented in Fig.2. We see that the difference Δ_i is not essential in location of the roots and we can use the approximations (2.12) and (2.13).

The regularized version of the the function ψ is

$$\psi_{reg} = \psi_a(z)\Gamma_1 + \psi_b(z)\Gamma_2. \quad (2.16)$$

Now one has to put conditions (2.4) on the regularized functions

$$\frac{z_a}{2L} \frac{d}{dz} \psi_{reg} \Big|_{z=\tilde{z}_a} = 1 \quad (2.17)$$

$$\frac{z_b}{2L} \frac{d}{dz} \psi_{reg} \Big|_{z=\tilde{z}_b} = -1 \quad (2.18)$$

and find \tilde{z}_a and \tilde{z}_b from these conditions. However it is difficult to perform these calculations. Instead of finding \tilde{z}_a from condition (2.17) we propose to use such regularization that does not

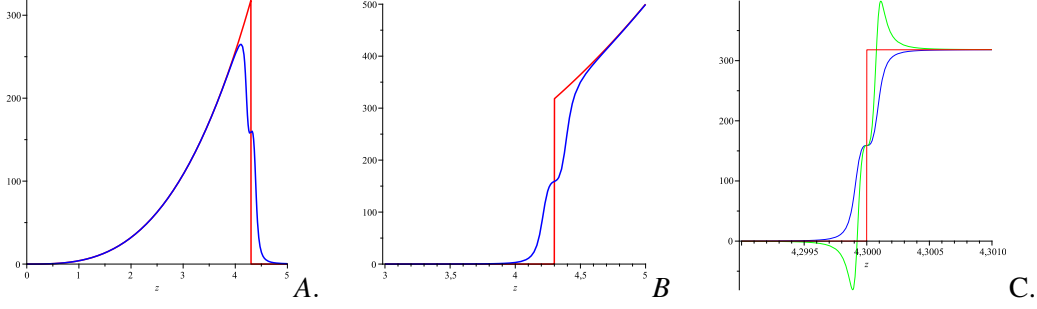


Figure 1: A. The functions $\frac{dY_1}{dz}$ (red line), $\frac{d\tilde{Y}_1}{dz} \Big|_{appr}$ (blue line). B. The functions $\frac{dY_2}{dz}$ (red line), $\frac{d\tilde{Y}_2}{dz} \Big|_{appr}$ (blue line). The regularization parameter $R = 10$ at A and B cases. C. Functions $\frac{dY_2}{dz}$ (red line), $\frac{d\tilde{Y}_2}{dz} \Big|_{appr}$ (blue line) and $\frac{d\tilde{Y}_2}{dz}$ (green line) at the regularization parameter $R = 10^4$.

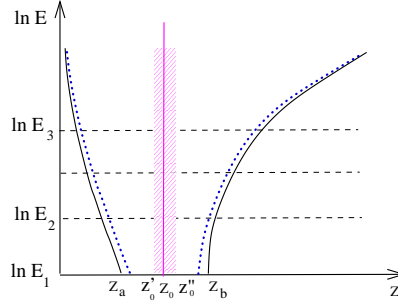


Figure 2: (color on-line) The schematic plots of locations of roots (solid black lines) dependent on the energy (in the logarithmic scale) and the location of differences $|\frac{d\tilde{Y}_i}{dz} - (\frac{d\tilde{Y}_i}{dz})_{appr}| \gtrsim 10^{-3}$, $i = 1, 2$ (the magenta shaded region). The magenta solid line shows the location of the wall. The dotted blue lines show location of zeros for the charged wall.

change z_a found from formal conditions (2.4). We can check that the formal z_a in fact solves also the regularized condition if the regularization is smooth enough. So, we take z_a and substitute it in the LHS of regularized condition (2.17). We define

$$F_{a,reg} \Big|_{z=z_a} = \frac{z_a}{2L} \left(\frac{d\psi_a}{dz} \Gamma_1 + \frac{d\psi_b}{dz} \Gamma_2 \right) \Big|_{z=z_a} = 1 + \delta_1,$$

$$F_{b,reg} \Big|_{z=z_b} = \frac{z_b}{2L} \left(\frac{d\psi_a}{dz} \Gamma_1 + \frac{d\psi_b}{dz} \Gamma_2 \right) \Big|_{z=z_b} = -1 + \delta_2.$$

We can calculate $F_{a,reg}$. The deviation of $F_{a,reg}$ from 1 will show how the regularization changes conditions (2.4). In the following table we present calculations of $F_{a,reg}$ for the wide range of parameter of the theory.

We choose the parameter R as minimally needed to make δ_1 and δ_2 negligible at energies $10^{-4} < E < 10^2$ TeV. Using the direct numerical calculations we choose $R = 10^4$. We perform

numerical calculations at $R = 10^4$ and get the following table:

E, TeV	$Q, \text{fm}^{1/2}$	z_a, fm	$z_b, \text{fm},$	F_a	F_b
118.2	0	0.04399350434	$4.015208900 \cdot 10^6$	1.00000	-1.00000
30	0	0.06948439782	$1.019088495 \cdot 10^6$	1.00000	-1.00000
0.03	0	0.6948439783	1018.393720	1.00000	-1.00000
0.00025	0	4.260706906	4.340400579	0.99999	-0.99999

Thus, from the table evidently $F_a \approx 1, F_b \approx -1$.

As has been mentioned above, strictly speaking one may not consider infinite surface as a trapped surface of any kind. Nevertheless it is possible to assume that transversal size of colliding objects is finite but very large, and therefore boundary conditions do not affect the process of gravitational interactions of inner parts of sources. If we are interested only in the specific area of the formed trapped surface in respect to the unit of shock wave area, we may define it as

$$\mathcal{A} \approx \lim_{L \rightarrow \infty} \frac{A_{\text{trap}}(L)}{A_{\text{source}}(L)}, \quad (2.19)$$

and the approximate equality takes place due to negligibility of boundary effects. As often happens, we can get answers for finite physical systems performing calculations for infinite non-physical objects.

3. Charged wall

Let us note that the form of the J_{uu}^{WP} in (2.1) can be obtained by spreading out the energy-momentum tensor of an ultrarelativistic point, i.e. over the transversal surface.

The Einstein equation for the charged wall (membrane) has the form

$$\left(\partial_z^2 - \frac{3}{z}\partial_z\right)\phi(z) = -16\pi G_5 \left(J_{uu}^{WP} + J_{uu}^{WQ}(Q, z)\right). \quad (3.1)$$

where J_{uu}^{WP} is given by (2.1) and we suppose that $J_{uu}^{WQ}(Q, z)$ can be obtained in the similar way by spreading the energy-momentum tensor of the ultrarelativistic charged point T_{uu}^{pQ} over the transversal surface. In the previous calculations:

$$J_{uu}^{WQ} = \frac{\int_{\mathcal{M}} J_{uu}^{pQ} \mathcal{D}x_{\perp}}{\int_{\mathcal{M}} \mathcal{D}x_{\perp}} \quad (3.2)$$

here the subscript "pQ" means the electromagnetic part of the energy momentum tensor of the charged point particle and " $\mathcal{D}x_{\perp}$ " means that we integrate over the induced metrics on the orthogonal surface \mathcal{M} .

For this purpose we take

$$J_{uu}^{pQ}(z, z_0) = \frac{L}{z} \rho^{pQ}, \quad \rho^{pQ} = \frac{5Q_n^2}{\pi 24 \cdot 64 L^5} \frac{1}{[q(q+1)]^{5/2}}, \quad q = \frac{(x_{\perp}^1)^2 + (x_{\perp}^2)^2 + (z - z_0)^2}{4zz_0}. \quad (3.3)$$

According to our prescription (3.2) we integrate over all transversal coordinates

$$J_{uu}^{pQ, II} = \frac{\frac{L}{z} \int_0^{\infty} \rho^{pQ}(q) \frac{L^2}{z_0^2} \frac{1}{2} dr^2}{\int_0^{\infty} \frac{L^2}{z_0^2} r dr}$$

and obtain

$$J_{uu}^{pQ} = \mathcal{X} \mathcal{J}, \quad \mathcal{J} = \frac{64}{3} z z_0 \left(1 - \frac{z_0^6 - 3z^2 z_0^4 - 3z^4 z_0^2 + z^6}{|z_0^2 - z^2|^3} \right), \quad \mathcal{X} = \frac{5}{256} \frac{Q_n^2}{\pi z L^6} = \frac{5}{256} \frac{Q^2}{z L^6}. \quad (3.4)$$

We see divergency at $z = z_0$, as it should be for the energy-momentum tensor of a charged plane. We introduce regularization by adding the ε factor in the denominator.

4. Quasi trapped surface for charged wall collision

To find the quasi TS formation condition in the wall-wall collision one has to solve Einstein equation

$$\left(\partial_z^2 - \frac{3}{z} \partial_z \right) \phi(z) = -16\pi G_5 (J_{uu}^{pW}(z) + J_{uu}^{QW}(Q, z)), \quad J_{uu}^{pW}(z) = \frac{E}{L^2} \frac{z_0^3}{L^3} \delta(z - z_0), \quad (4.1)$$

$$J_{uu}^{QW}(Q, z) = \frac{128 \mathcal{X}}{3} z z_0 \frac{z^4 (-z^2 + 3z_0^2) \theta(z_0 - z) + z_0^4 (-3z^2 + z_0^2) \theta(z - z_0)}{(-z^2 + z_0^2 + \varepsilon^2)^3} \quad (4.2)$$

with the following boundary conditions

$$\mathbf{1) \quad} \phi(z_a) = \phi(z_b) = 0, \quad \phi_a(z_0) = \phi_b(z_0) \quad (4.3)$$

$$\mathbf{2) \quad} \left(\psi'(z_a) \frac{z_a}{L} \right) = 2, \quad \left(\psi'(z_b) \frac{z_b}{L} \right) = -2, \quad (4.4)$$

where z_a and z_b are the boundaries of the TS and ψ is related to $\phi(z) = \frac{z}{L} \psi$.

We search for a solution to the Einstein equation with a charged source in the form of the sum of the "neutral" solution and a correction proportional to Q^2 $\phi = \phi_n + \phi_q$, where ϕ_n denotes the solution of the neutral case.

As in the neutral case it is convenient to consider domains $z < z_0$, $z > z_0$ separately and we have

$$\left(\partial_z^2 - \frac{3}{z} \partial_z \right) \phi_{qz_0 > z} = -16\pi G_5 \mathcal{X} \frac{128}{3} z z_0 \frac{z^4 (-z^2 + 3z_0^2)}{(-z^2 + z_0^2 + \varepsilon^2)^3}, \quad z_0 > z; \quad (4.5)$$

$$\left(\partial_z^2 - \frac{3}{z} \partial_z \right) \phi_{qz > z_0} = -16\pi G_5 \mathcal{X} \frac{128}{3} z z_0 \frac{z_0^4 (-3z^2 + z_0^2)}{(-z^2 + z_0^2 - \varepsilon^2)^3}, \quad z > z_0. \quad (4.6)$$

Solutions to (4.5) and (4.6) can be presented as :

$$\psi_{qz_0 > z} = z^3 C_1 + \frac{C_2}{z} - \frac{NLz_0 z^3}{4(-z^2 + z_0^2 + \varepsilon^2)}, \quad z_0 > z, \quad (4.7)$$

$$\psi_{qz > z_0} = \frac{C_3}{z} + z^3 C_4 + \frac{NLz_0^5}{4z(-z^2 + z_0^2 + \varepsilon^2)}, \quad z > z_0 \quad (4.8)$$

Here $N = \frac{40}{3} \frac{\pi G_5 Q^2}{L^6}$. The first two terms in (4.7) and (4.8) are solution to the Lin and Shuryak equation (55) in [21]. If one assumes that they satisfy condition **1**, i.e. $\psi_n(z_a) = \psi_n(z_b) = 0$,

$\Psi_{na}(z_0) = \Psi_{nb}(z_0)$, one gets [25]:

$$\Psi_n = \begin{cases} \Psi_{na} = C \left(\frac{z^3}{z_a^3} - \frac{z_a}{z} \right), C = -\frac{4\pi G_5 E}{L^4} \frac{\left(\frac{z_0^4}{z_b^4} - 1 \right) z_b}{\frac{z_b^4 - z_a^4}{z_a^3 z_b^3}}, z < z_0 \\ \Psi_{nb} = D \left(\frac{z^3}{z_b^3} - \frac{z_b}{z} \right), D = -\frac{4\pi G_5 E}{L^4} \frac{\left(\frac{z_0^4}{z_a^4} - 1 \right) z_a}{\frac{z_b^4 - z_a^4}{z_a^3 z_b^3}}, z_0 < z \end{cases} \quad (4.9)$$

In the neutral case one find z_a and z_b from the 2-nd condition $(\Psi'_{na}(z_a) \frac{z_a}{L}) = 2$, $(\Psi'_{nb}(z_b) \frac{z_b}{L}) = -2$, here z_a and z_b are the boundaries of the TS.

As to (4.7) and (4.8), choosing $C_1 = \frac{NLz_0}{4(z_a^2 - z_0^2)}$, $C_2 = 0$, $C_3 = \frac{NLz_0^5}{4(z_b^2 - z_0^2)}$, $C_4 = 0$, we obtain

$$\begin{cases} \Psi_{aq} = -\frac{NLz_0 z^3}{4} \frac{-z_a^2 + z^2 - \varepsilon^2}{(-z^2 + z_0^2 + \varepsilon^2)(-z_a^2 + z_0^2)}, z < z_0 \\ \Psi_{bq} = \frac{NLz_0^5}{4z} \frac{-z_b^2 + z^2 + \varepsilon^2}{(-z^2 + z_0^2 - \varepsilon^2)(-z_b^2 + z_0^2)}, z_0 < z \end{cases} \quad (4.10)$$

Note that for the constructed solution the condition $\Psi(z_a) = \Psi(z_b) = 0$ is satisfied automatically. Using the the second requirement (4.4) and supposing $z_0 = L$ we obtain the equations system for numerical analyze:

$$F_a = -\frac{8\pi G_5 E (z_0^4 - z_b^4) z_a^3}{z_0^5 (z_b^4 - z_a^4)} - \frac{10 \pi G_5 Q^2}{3} \frac{z_0 z_a^5}{z_0^6 (-z_a^2 + z_0^2)^2} = 1, \quad (4.11)$$

$$F_b = -\frac{8\pi G_5 E (z_0^4 - z_a^4) z_b^3}{z_0^5 (z_b^4 - z_a^4)} + \frac{10 \pi G_5 Q^2}{3} \frac{z_0 z_b^5}{z_0^6 (-z_b^2 + z_0^2)^2} = -1. \quad (4.12)$$

The system allows to get TS boundary points z_a, z_b .

5. Conclusion

In the present paper the regularization of the ultrarelativistic point energy-momentum tensor spreading over the transversal surface has been considered. The solution of regularized charged wall-on-wall Einstein equation with special boundary conditions has been constructed and applied to construction of TS.

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