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Relativistic description of the double P-wave charmonium production in e^+e^- annihilation

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On the basis of perturbative QCD and the relativistic quark model we calculate relativistic and bound state corrections in the production processes of a pair of $\mathscr{P}$-wave charmonium states. Relativistic factors in the production amplitude connected with the relative motion of heavy quarks and the transformation law of the bound state wave function to the reference frame of the moving $\mathscr{P}$-wave mesons are taken into account. For the gluon and quark propagators entering the production vertex function we use a truncated expansion in the ratio of the relative quark momenta to the center-of-mass energy $\sqrt{s}$ up to the second order. Relativistic corrections to the quark bound state wave functions in the rest frame are considered by means of the Breit-like potential. It turns out that the examined effects change essentially the nonrelativistic results of the cross section for the reaction $e^+ + e^- \to h_c + \chi_{cJ}$ at the center-of-mass energy $\sqrt{s} = 10.6$ GeV.

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1. Introduction

The large value of the exclusive double charmonium production cross section measured at the Belle and BABAR experiments [1, 2] reveals definite problems in the theoretical description of these processes [3, 4]. Many theoretical efforts were made in order to improve the calculation of the production cross section $e^+ + e^- \to J/\Psi + \eta_c$. They included the analysis of other production mechanisms for the state $J/\Psi + \eta_c$ [5, 6] and the calculation of different corrections which could change essentially the initial nonrelativistic result [7, 8, 9, 10]. Despite the evident successes achieved on the basis of nonrelativistic quantum chromodynamics (NRQCD), the light cone method, quark potential models for correcting the discrepancy between the theory and experiment, the double charmonium production in e^+e^- annihilation remains an interesting task. On the one hand, there are other production processes of the $\mathscr{P}$- and $\mathscr{D}$-wave charmonium states which can be investigated in the same way as the production of $\mathscr{S}$-wave states. Recently the Belle and BABAR collaborations discovered new charmonium-like states in e^+e^- annihilation [11, 12]. The nature of these numerous resonances remains unclear to the present. Some of them are considered as a $\mathscr{P}$- and $\mathscr{D}$-wave excitations in the system $(c\bar{c})$. On the other hand, the variety of the used approaches and the model parameters in this problem raises the question about the comparison of the obtained results, that will lead to a better understanding of the quark-gluon dynamics and different mechanisms of the charmonium production. Two sources of the changing of the nonrelativistic cross section for the double charmonium production are revealed to the present: the radiative corrections of order $O(\alpha_s)$ and relative motion of c-quarks forming the bound states. An actual physical processes of the charmonium production require formation of hadronic particles in final states (bound states of a charm quark c and a charm anti-quark $\bar{c}$), for which perturbative quantum chromodynamics can not provide high precision description. Further investigation of charmonia production can improve our understanding of heavy quark production and the formation of quark bound states.

This work continues our study of the exclusive double charmonium production in e^+e^- annihilation in the case of a pure $\mathscr{P}$-wave $(c\bar{c})$ quarkonium on the basis of a relativistic quark model (RQM) [9, 13, 14, 15, 16].

2. General formalism

We investigate the quarkonium production in the lowest-order perturbative quantum chromodynamics. The usual color-singlet mechanism is considered as a basic one for the pair charmonium production. We analyze the reactions $e^+ + e^- \to h_c + \chi_{cJ}$, where the final state consists of a pair of $\mathscr{P}$-wave (χ_{c0}, χ_{c1}, χ_{c2}) and h_c charm mesons. The diagrams that give contributions to the amplitude of these processes in leading order of the QCD coupling constant α_s are presented in Fig. 1. Two other diagrams can be obtained by corresponding permutations. There are two stages of the production process. In the first stage, which is described by perturbative QCD, the virtual photon γ^* produces four heavy c-quarks and $\bar{c}$-antiquarks with the following four-momenta:

$$p_{1,2} = \frac{1}{2}P \pm p, \quad (p \cdot P) = 0; \qquad q_{1,2} = \frac{1}{2}Q \pm q, \quad (q \cdot Q) = 0, \tag{2.1}$$

where $P(Q)$ are the total four-momenta, $p = L_P(0,\mathbf{p})$, $q = L_P(0,\mathbf{q})$ are the relative four-momenta obtained from the rest frame four-momenta $(0,\mathbf{p})$ and $(0,\mathbf{q})$ by the Lorentz transformation to the

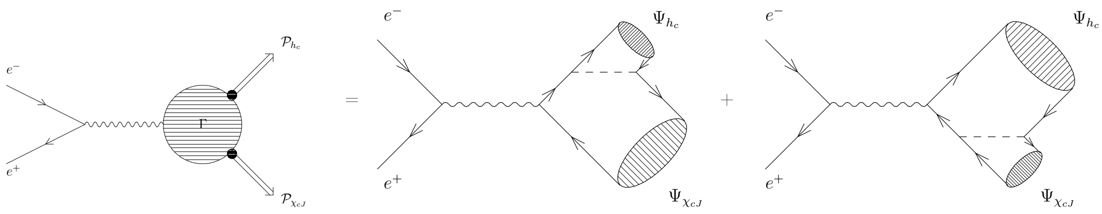

Figure 1: The production amplitude of a pair of $\mathscr{P}$-wave charmonium states in e^+e^- annihilation. $\mathscr{P}_{h_c}$ denotes the $\mathscr{P}$-wave meson h_c and $\mathscr{P}_{\chi_{cJ}}$ denotes the $\mathscr{P}$-wave meson χ_{cJ}. The wavy line shows the virtual photon and the dashed line corresponds to the gluon. Γ is the production vertex function.

system moving with the momenta P, Q. In the second nonperturbative stage, quark-antiquark pairs form the final mesons.

Let consider the production amplitude of the $\mathscr{P}$-wave vector state h_c and $\mathscr{P}$-wave states χ_{cJ} ($J=0,1,2$), which can be presented in the form [9, 14, 16]:

$$\mathscr{M}(p_-,p_+,P,Q)=\frac{8\pi^2\alpha\alpha_s(4m^2)\mathscr{Q}_c}{3s}\bar{v}(p_+)\gamma^\beta u(p_-)\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\times \tag{2.2}$$
$$\times Sp\left\{\overline{\Psi}^{\mathscr{P}}_{h_c}(p,P)\Gamma_1^{\beta\nu}(p,q,P,Q)\overline{\Psi}^{\mathscr{P}}_{\chi_{cJ}}(q,Q)\gamma_\nu+\overline{\Psi}^{\mathscr{P}}_{\chi_{cJ}}(q,Q)\Gamma_2^{\beta\nu}(p,q,P,Q)\overline{\Psi}^{\mathscr{P}}_{h_c}(p,P)\gamma_\nu\right\},$$

where a superscript $\mathscr{P}$ indicates the $\mathscr{P}$-wave meson, $\alpha_s(4m^2)$ is the QCD coupling constant, α is the fine structure constant and $\mathscr{Q}_c$ is the c-quark electric charge, $\Gamma_{1,2}$ are the vertex functions defined below.

The production processes $e^++e^-\to h_c+\chi_{cJ}$ contain the quark bound states. The transition of free quarks to the $(c\bar{c})$ mesons is described by specific wave functions. The relativistic $\mathscr{P}$-wave functions of the bound quarks $\Psi^{\mathscr{P}}$ accounting for the transformation from the rest frame to the moving one with four momenta P, Q are

$$\begin{aligned}\overline{\Psi}^{\mathscr{P}}_{h_c}(p,P) &= \frac{\overline{\Psi}^{h_c}_0(\mathbf{p})}{\left[\frac{\varepsilon(p)}{m}\frac{(\varepsilon(p)+m)}{2m}\right]}\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\varepsilon(p)+m)}-\frac{\hat{p}}{2m}\right]\\ &\times\gamma_5(1+\hat{v}_1)\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\varepsilon(p)+m)}+\frac{\hat{p}}{2m}\right],\end{aligned} \tag{2.3}$$

$$\begin{aligned}\overline{\Psi}^{\mathscr{P}}_{\chi_{cJ}}(q,Q) &= \frac{\overline{\Psi}^{\chi_{cJ}}_0(\mathbf{q})}{\left[\frac{\varepsilon(q)}{m}\frac{(\varepsilon(q)+m)}{2m}\right]}\left[\frac{\hat{v}_2-1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\varepsilon(q)+m)}+\frac{\hat{q}}{2m}\right]\\ &\times\hat{\varepsilon}^*_{\mathscr{P}}(Q,S_z)(1+\hat{v}_2)\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\varepsilon(q)+m)}-\frac{\hat{q}}{2m}\right],\end{aligned} \tag{2.4}$$

where the hat is a notation for the contraction of the four vector with the Dirac matrices, $v_1=P/M_{h_c}$, $v_2=Q/M_{\chi_{cJ}}$; $\varepsilon_{\mathscr{P}}(Q,S_z)$ is the polarization vector of the spin-triplet state χ_{cJ}, $\varepsilon(p)=\sqrt{p^2+m^2}$ and m is the c-quark mass.

At leading order in α_s the vertex functions $\Gamma^{\beta\nu}_{1,2}(p,P;q,Q)$ can be written as

$$\Gamma_1^{\beta\nu}(p,P;q,Q)=\gamma_\mu\frac{(\hat{l}-\hat{q}_1+m)}{(l-q_1)^2-m^2+i\varepsilon}\gamma_\beta D^{\mu\nu}(k_2)+\gamma_\beta\frac{(\hat{p}_1-\hat{l}+m)}{(l-p_1)^2-m^2+i\varepsilon}\gamma_\mu D^{\mu\nu}(k_2), \tag{2.5}$$

PoS(QFTHEP2011)051

$$\Gamma_2^{\beta\nu}(p,P;q,Q)=\gamma_\beta\frac{(\hat{q}_2-\hat{l}+m)}{(l-q_2)^2-m^2+i\varepsilon}\gamma_\mu D^{\mu\nu}(k_1)+\gamma_\mu\frac{(\hat{l}-\hat{p}_2+m)}{(l-p_2)^2-m^2+i\varepsilon}\gamma_\beta D^{\mu\nu}(k_1), \quad (2.6)$$

where the gluon momenta are $k_1=p_1+q_1$, $k_2=p_2+q_2$ and $l^2=s=(P+Q)^2=(p_-+p_+)^2$, p_-, p_+ are four momenta of the electron and positron. The dependence on the relative momenta of c-quarks is presented both in the gluon propagator $D_{\mu\nu}(k)$ and quark propagator as well as in the relativistic wave functions (2.3), (2.4). Taking into account that the ratio of the relative quark momenta p and q to the energy $\sqrt{s}$ is small, we expand the inverse denominators of quark and gluon propagators. In this expansions we keep terms of third order in relative momenta p and q. We preserve relativistic factors entering the denominators of the relativistic wave functions (2.3)–(2.4), but in the numerator of the amplitude (2.2) we take into account corrections of second order in $|\mathbf{p}|/m$ and $|\mathbf{q}|/m$ relative to the leading order result. This provides the convergence of the resulting momentum integrals.

For a specific $\mathscr{P}$-wave state, summing over S_z and L_z in the amplitude (2.2) can be further simplified as [17]

$$\sum_{S_z,L_z}\langle 1,L_z;1,S_z|J,J_z\rangle\varepsilon^*_{\mathscr{P}\alpha}(Q,L_z)\varepsilon^*_{\mathscr{P}\beta}(Q,S_z)=\begin{cases}\frac{1}{\sqrt{3}}(g_{\alpha\beta}-v_{2\alpha}v_{2\beta}), & J=0,\\ \frac{i}{\sqrt{2}}\varepsilon_{\alpha\beta\sigma\rho}v_2^\sigma\varepsilon^{*\rho}(Q,J_z), & J=1,\\ \varepsilon^*_{\alpha\beta}(Q,J_z), & J=2,\end{cases} \quad (2.7)$$

where $\langle 1,L_z;1,S_z|J,J_z\rangle$ are the Clebsch-Gordon coefficients. Calculating the trace in the amplitude (2.2) by means of expressions (2.3)–(2.4) and the system FORM [18], we find that the tensor parts of four amplitudes describing the production of $\mathscr{P}$-wave charmonium states have the following structure:

$$S_{0,\beta}(h_c+\chi_{c0})=A_0\varepsilon_{\mu\nu\alpha\beta}v_1^\mu v_2^\nu\varepsilon_{h_c}^{*\alpha}, \quad (2.8)$$

$$S_{1,\beta}(h_c+\chi_{c1})=B_1v_{1\beta}(v_1\cdot\varepsilon^*_{\chi_{c1}})(v_2\cdot\varepsilon^*_{h_c})+B_2v_{2\beta}(v_1\cdot\varepsilon^*_{\chi_{c1}})(v_2\cdot\varepsilon^*_{h_c})+B_3\varepsilon^*_{\chi_{c1}\beta}(v_2\cdot\varepsilon^*_{h_c})+ \quad (2.9)$$
$$+B_4\varepsilon^*_{h_c\beta}(v_1\cdot\varepsilon^*_{\chi_{c1}})+B_5v_{1\beta}(\varepsilon^*_{\chi_{c1}}\cdot\varepsilon^*_{h_c})+B_6v_{2\beta}(\varepsilon^*_{h_c}\cdot\varepsilon^*_{\chi_{c1}}),$$

$$S_{2,\beta}(h_c+\chi_{c2})=\varepsilon^*_{\alpha\gamma}\Big[C_1\varepsilon_{\sigma\rho\beta\gamma}v_1^\alpha v_1^\sigma v_2^\rho(v_2\cdot\varepsilon^*_{h_c})+C_2\varepsilon_{\sigma\rho\beta\gamma}v_1^\alpha v_2^\sigma\varepsilon_{h_c}^{*\rho}+C_3\varepsilon_{\sigma\rho\beta\gamma}\varepsilon_{h_c}^{*\alpha}v_1^\sigma v_2^\rho+ \quad (2.10)$$
$$+C_4g_{\alpha\beta}\varepsilon_{\sigma\rho\omega\gamma}v_1^\sigma v_2^\rho\varepsilon_{h_c}^{*\omega}+C_5\varepsilon_{\sigma\rho\lambda\beta}v_1^\alpha v_1^\gamma v_1^\sigma v_2^\rho\varepsilon_{h_c}^{*\lambda}+C_6\varepsilon_{\sigma\rho\lambda\gamma}v_1^\alpha v_1^\sigma v_{1\beta}v_2^\rho\varepsilon_{h_c}^{*\lambda}+,$$
$$+C_7\varepsilon_{\sigma\rho\lambda\gamma}v_1^\alpha v_1^\sigma v_2^\rho v_{2\beta}\varepsilon_{h_c}^{*\lambda}+C_8\varepsilon_{\sigma\lambda\beta\gamma}v_1^\alpha v_1^\sigma\varepsilon_{h_c}^{*\lambda}\Big],$$

where the coefficients A_i, B_i, C_i can be presented as sums of terms containing the factors $u=M_{\chi_{cJ}}/(M_{h_c}+M_{\chi_{cJ}})$, $\kappa=m/(M_{h_c}+M_{\chi_{cJ}})$ and $C_{ij}=c^i(p)c^j(q)=[(m-\varepsilon(p))/(m+\varepsilon(p))]^i[(m-\varepsilon(q))/(m+\varepsilon(q))]^j$, preserving terms with $i+j\le 2$, and $r^2=(M_{h_c}+M_{\chi_{cJ}})^2/s$. Exact analytical expressions for these coefficients are sufficiently lengthy, and so they are omitted here.

We find it useful to present the charmonium production cross sections in the following form ($k=0,1,2$ corresponds to χ_{c0}, χ_{c1} and χ_{c2}):

$$\sigma(h_c+\chi_{cJ})=\frac{2\alpha^2\alpha_s^2(4m^2)\mathscr{Q}_c^2\pi r^6\sqrt{1-r^2}\sqrt{1-r^2(2u-1)^2}}{9\kappa^4u^{11}(1-u)^{11}}\frac{|\tilde{R}'_{h_c}(0)|^2|\tilde{R}'_{\chi_{cJ}}(0)|^2}{s(M_{\chi_{cJ}}+M_{h_c})^{10}}\sum_{i=0}^{7}F_i^{(k)}(r^2)\omega_i, \quad (2.11)$$

State H_1H_2	σ (fb) [3]	$\sigma_{non-rel}$ (fb) Our result	σ_{rel} (fb) Our result (2.11)
$h_c+\chi_{c0}$	0.053 ± 0.019	0.135	0.076 ± 0.030
$h_c+\chi_{c1}$	0.258 ± 0.064	0.601	0.096 ± 0.037
$h_c+\chi_{c2}$	0.017 ± 0.002	0.035	0.0024 ± 0.0009

Table 1: Comparison of the obtained results with previous theoretical predictions.

where the functions $F_i^{(k)}$ ($k=0,1,2$) are written explicitly in [19]. The parameters ω_i can be expressed in terms of momentum integrals J_n for the states h_c and χ_{cJ} as follows:

$$J_n=\int_0^\infty q^3R_{\mathscr{P}}(q)\frac{(\varepsilon(q)+m)}{2\varepsilon(q)}\left(\frac{m-\varepsilon(q)}{m+\varepsilon(q)}\right)^n dq,\qquad \tilde{R}'_{\mathscr{P}}(0)=\frac{1}{3}\sqrt{\frac{2}{\pi}}J_0, \tag{2.12}$$

$$\omega_0=1,\quad \omega_1=\frac{J_1(h_c)}{J_0(h_c)},\quad \omega_2=\frac{J_2(h_c)}{J_0(h_c)},\quad \omega_3=\omega_1^2, \tag{2.13}$$

$$\omega_4=\frac{J_1(\chi_{cJ})}{J_0(\chi_{cJ})},\quad \omega_5=\frac{J_2(\chi_{cJ})}{J_0(\chi_{cJ})},\quad \omega_6=\omega_4^2,\quad \omega_7=\omega_1\omega_4.$$

On the one side, in the potential quark model the relativistic corrections, connected with the relative motion of heavy c-quarks, enter the production amplitude (2.2) and the cross section (2.11) through the different relativistic factors. They are determined in the final expression (2.11) by the specific parameters ω_i. The momentum integrals which determine the parameters ω_i are convergent and we can calculate them numerically, using the wave functions obtained by the numerical solution of the Schrödinger equation. Nevertheless, we introduce new cutoff parameter $\Lambda\approx m$ in (2.12) for momentum integrals $J_{1,2}$ at high momenta q because we don't know exactly the bound state wave functions in the region of the relativistic momenta. The exact form of the wave functions $\Psi_0^{h_c}(\mathbf{p})$ and $\Psi_0^{\chi_{cJ}}(\mathbf{q})$ is important for improving the accuracy of the calculation of the relativistic effects. It is sufficient to note that the double charmonium production cross section $\sigma(s)$ in the nonrelativistic approximation contains the factor $|R'_{h_c}(0)|^2|R'_{\chi_{cJ}}(0)|^2$. Small changes of the numerical values of the bound state wave functions at the origin lead to substantial changes of the final results. In the framework of NRQCD this problem is closely related to the determination of the color-singlet matrix elements for the charmonium [20]. Thus, on the other side, there are relativistic corrections to the bound state wave functions $\Psi_0^{h_c}(\mathbf{p})$, $\Psi_0^{\chi_{cJ}}(\mathbf{q})$. In order to take them into account, we suppose that the dynamics of a $c\bar{c}$-pair is determined by the QCD generalization of the standard Breit Hamiltonian in the center-of-mass reference frame. Starting with this Hamiltonian we construct the effective potential model based on the Schrödinger equation and find its numerical solutions in the case of $\mathscr{P}$-wave charmonium [21]. Then we calculate the matrix elements entering in the expressions for the parameters ω_i (2.13) and obtain the value of the production cross sections at $\sqrt{s}=10.6$ GeV.

3. Numerical results and discussion

Numerical results and their comparison with the previous calculation in NRQCD are presented in Table 1. The exclusive double charmonium production cross section presented in the form (2.11) is convenient for a comparison with the results of NRQCD. Indeed, in the nonrelativistic limit, when $u = 1/2$, $\kappa = 1/4$, $\omega_i = 0$ $(i \geq 1)$, $r^2 = 16m^2/s$, the cross section (2.11) coincides with the calculation in [3]. In this limit the functions $F_0^{(k)}(r^2)$ transform into corresponding functions F_k from [3]. When we take into account bound state corrections working with observed meson masses, we get $u = M_{\chi_{cJ}}/(M_{h_c} + M_{\chi_{cJ}}) \neq 1/2$, $\kappa = m/(M_{h_c} + M_{\chi_{cJ}}) \neq 1/4$. This leads to the modification of the general factor in (2.11) and the form of the functions $F_0^{(k)}$ in comparison with the nonrelativistic theory (see [3]). Different values of the mass of c-quark and nonperturbative parameters $R'_{\mathscr{P}}(0)$ make difficult the direct comparison of our numerical results with predictions of NRQCD. Note that nonrelativistic results obtained in our quark model are the following: $\sigma(\chi_{c0} + h_c) = 0.135$ fb, $\sigma(\chi_{c1} + h_c) = 0.601$ fb, $\sigma(\chi_{c2} + h_c) = 0.035$ fb (compare with predictions of NRQCD in the second column of Table 1). Nevertheless, we can state that in all considered reactions $e^+ + e^- \to h_c + \chi_{cJ}$ the account of all relativistic and bound state effects leads to the decrease of the nonrelativistic cross section obtained in our model. It is necessary to point out once again that the essential effect on the value of the production cross sections $h_c + \chi_{cJ}$ belongs to the parameters $\tilde{R}'_{\mathscr{P}}(0)$ (20), α_s, m. Small changes in their values can lead to significant changes in the production cross sections. In our model the nonrelativistic value $R'_{\mathscr{P}}(0) = 0.26$ GeV$^{5/2}$.

We presented a treatment of relativistic effects in the $\mathscr{P}$-wave double charmonium production in e^+e^- annihilation. We separated two different types of relativistic contributions to the production amplitudes. The first type includes the relativistic v/c corrections to the wave functions and their relativistic transformations. The second type includes the relativistic $p/\sqrt{s}$ corrections appearing from the expansion of the quark and gluon propagators. The latter corrections were taken into account up to the second order. It is important to note that the expansion parameter $p/\sqrt{s}$ is very small. In our analysis of the production amplitudes we correctly take into account relativistic contributions of order $O(v^2/c^2)$ for the $\mathscr{P}$-wave mesons. Therefore the first basic theoretical uncertainty of our calculation is connected with the omitted terms of order $O(\mathbf{p}^4/m^4)$. Since the calculation of the masses of $\mathscr{P}$-wave charmonium states is sufficiently accurate in our model (the error is less then 1%), we suppose that the uncertainty in the cross section calculation due to the omitted relativistic corrections of order $O(\mathbf{p}^4/m^4)$ in the quark interaction operator (the Breit Hamiltonian) is also very small. Taking into account that the average value of the heavy quark velocity squared in the charmonium is $\langle v^2 \rangle = 0.3$, we expect that relativistic corrections of order $O(\mathbf{p}^4/m^4)$ to the cross section (2.11) coming from the production amplitude should not exceed 30% of the obtained relativistic result. Another important part of the total theoretical error is related with radiative corrections of order α_s which were omitted in our analysis. Our approach to the calculation of the amplitude of the double charmonium production can be extended beyond the leading order in the strong coupling constant. Then the vertex functions in (2.2) will have more complicate structure including the integration over the loop momenta. Our calculation of the cross sections accounts for effectively only some part of one loop corrections by means of the Breit Hamiltonian. So, we assume that the radiative corrections of order $O(\alpha_s)$ can cause 20% modification of the production cross sections. We have neglected the terms in the cross section (2.11) containing the product of J_n with summary index > 2 because their contribution has been found negligibly small. There are no another comparable uncertainties related to the other parameters of the model, since their values were fixed from our previous consideration of meson and baryon properties [16, 22].

Our total theoretical errors are written explicitly in Table 1. To obtain this estimate we add the above mentioned uncertainties in quadrature.

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