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Threshold corrections to the MSSM effective Higgs potential: gaugino and higgsino contributions

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One-loop corrections to the effective Higgs potential induced by interactions of Higgs bosons with superpartners of gauge and Higgs fields are calculated. Numerical comparison of these corrections with threshold corrections from the sector of scalar quarks calculated earlier in [1] is performed.

The XXth International Workshop High Energy Physics and Quantum Field Theory
September 24 - October 1, 2011
Sochi, Russia

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PoS(QFTHEP2011)068

1. Introduction

Gauge supermultiplets of the minimal supersymmetric standard model (MSSM) include gluons and their superpartners - gluinos, and $SU(2)_L \times U(1)$ gauge bosons with their fermionic superpartners - gauginos. Higgs supermultiplet includes two complex doublets of Higgs fields and their fermionic superpartners - higgsinos, and corresponding antiparticles. Supermultiplets of the matter fields contain three generations of left and right quarks and leptons, their scalar superpartners (scalar quarks and leptons) and antiparticles corresponding to them. One-loop corrections to the MSSM effective potential parameters λ_i (i =1,...7) strongly influence Higgs boson masses and couplings [2]. Main contribution induced by interactions of Higgs bosons with the third generation of scalar quarks has been evaluated in [1]. However, additional contributions from superpartners of gauge and Higgs bosons in some cases may have substantial values close to leading contributions of squarks. Gaugino and higgsino contributions are the main ones in the framework of "split supersymmetry" scenario [3] when unification of couplings at the multiTeV scale [4] can be achieved with only two light CP even scalars, while other scalars acquire masses much larger than the electroweak scale. Only gaugino fields and light CP even Higgs boson fields remain in the effective low-energy Higgs potential. These types of MSSM scenarios are promising for consideration of general problems like FCNC, large CP violation beyond the CKM or proton lifetime in GUT's.

In this proceeding we evaluate the one-loop corrections induced by interactions of gaugino and higgsino with Higgs bosons and compare them with corrections arising from interactions of scalar quarks [1].

2. Effective potential

General model for Higgs sector with two doublets Φ_1, Φ_2

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix}, \tag{2.1}$$

$$\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{pmatrix} \tag{2.2}$$

is defined by means of a gauge invariant renormalized hermitian potential

$$\begin{aligned} U(\Phi_1,\Phi_2) = &-\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \mu_{12}^{2\,*}(\Phi_1^\dagger\Phi_2)+ \\ &+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)+ \\ &+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\lambda_5^*}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1)+ \\ &+\lambda_6(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_1) + \lambda_6^*(\Phi_2^\dagger\Phi_1)(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2). \end{aligned} \tag{2.3}$$

Vacuum expectation values in the minimum are chosen to be

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \tag{2.4}$$

At the supersymmetry mass scale the tree level parameters of the effective potential are real and can be expressed as boundary conditions in terms of the electroweak gauge symmetry $SU(2)\times U(1)$ couplings g_1 and g_2:

$$\begin{gathered}\lambda_1(M_{\rm SUSY})=\lambda_2(M_{\rm SUSY})=\tfrac{1}{4}\left(g_2^2(M_{\rm SUSY})+g_1^2(M_{\rm SUSY})\right),\\ \lambda_3(M_{\rm SUSY})=\tfrac{1}{4}\left(g_2^2(M_{\rm SUSY})-g_1^2(M_{\rm SUSY})\right),\quad \lambda_4(M_{\rm SUSY})=-\tfrac{1}{2}g_2^2(M_{\rm SUSY}),\\ \lambda_5(M_{\rm SUSY})=\lambda_6(M_{\rm SUSY})=\lambda_7(M_{\rm SUSY})=0.\end{gathered}\tag{2.5}$$

In the following we calculate the one-loop corrections to $\lambda_1-\lambda_7$ (threshold corrections) induced by interaction of Higgs bosons with gauginos and higgsinos. Some examples of box diagrams can be found in the following section.

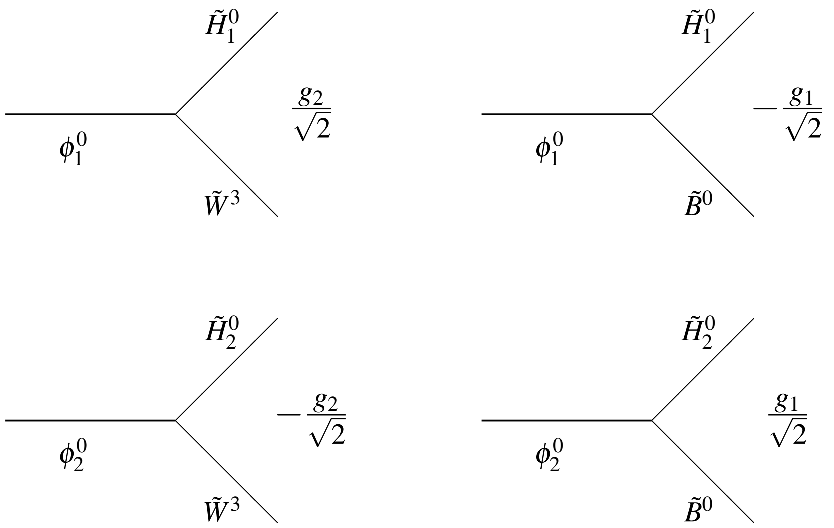
3. Gaugino and higgsino contributions

The gaugino - higgsino - Higgs Lagrangian terms are taken following a standard convention, see e.g. [5, 6]:

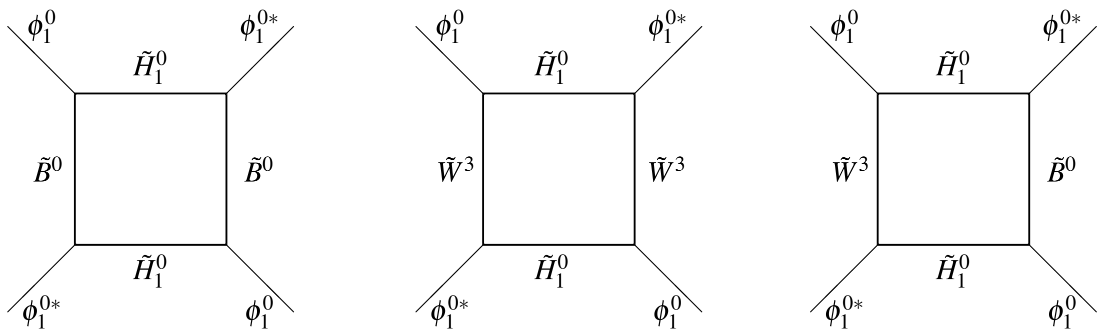
$$\begin{aligned}V_{H\tilde{H}\tilde{V}}=&-\left(g_2\tilde{H}_1^0\tilde{W}^- -\frac{g_2}{\sqrt{2}}\tilde{H}_1^-\tilde{W}^3-\frac{g_1}{\sqrt{2}}\tilde{H}_1^-\tilde{B}^0\right)\phi_1^+ +\\ &+\left(g_2\tilde{H}_2^0\tilde{W}^+ +\frac{g_2}{\sqrt{2}}\tilde{H}_2^+\tilde{W}^3+\frac{g_1}{\sqrt{2}}\tilde{H}_2^+\tilde{B}^0\right)\phi_2^- +\\ &+\left(g_2\tilde{H}_1^-\tilde{W}^+ +\frac{g_2}{\sqrt{2}}\tilde{H}_1^0\tilde{W}^3-\frac{g_1}{\sqrt{2}}\tilde{H}_1^0\tilde{B}^0\right)\phi_1^0 +\\ &+\left(g_2\tilde{H}_2^+\tilde{W}^- -\frac{g_2}{\sqrt{2}}\tilde{H}_2^0\tilde{W}^3+\frac{g_1}{\sqrt{2}}\tilde{H}_2^0\tilde{B}^0\right)\phi_2^0+h.c.,\end{aligned}$$

where $\tilde{H}_i^0,\tilde{W}^3,\tilde{B}^0$ are higgsino, wino and bino fields, respectively, with mass parameters $m_{\tilde{B}^0}=M_1$, $m_{\tilde{W}^3}=M_2$, $m_{\tilde{H}_i^0}=\mu$. The choice of M_1,M_2,μ variation range in the MSSM parameter space respects the constraints from [7]. Contours of neutralino masses for degenerate M_1,M_2 are presented in Fig. 1.

The couplings to be taken into account are the following:



The general scheme of the one-loop calculation is illustrated by the following example of λ_1 evaluation:



Calculation of all corrections at the one-loop gives the following result for the shift $\Delta\lambda_i$ (see λ_i normalization in [1]) of effective parameters (2.5)

$$\frac{1}{2}\Delta\lambda_1 = \frac{g_1^4}{4}J_2(m_{\tilde{H}_1^0}, m_{\tilde{B}^0}) + \frac{g_2^4}{4}J_2(m_{\tilde{H}_1^0}, m_{\tilde{W}^3}) + \frac{g_1^2 g_2^2}{4}J_3(m_{\tilde{H}_1^0}, m_{\tilde{W}^3}, m_{\tilde{B}^0}), \tag{3.1}$$

$$\frac{1}{2}\Delta\lambda_2 = \frac{g_1^4}{4}J_2(m_{\tilde{H}_2^0}, m_{\tilde{B}^0}) + \frac{g_2^4}{4}J_2(m_{\tilde{H}_2^0}, m_{\tilde{W}^3}) + \frac{g_1^2 g_2^2}{4}J_3(m_{\tilde{H}_2^0}, m_{\tilde{W}^3}, m_{\tilde{B}^0}), \tag{3.2}$$

$$\Delta\lambda_3 + \Delta\lambda_4 = \frac{g_1^4}{4}J_3(m_{\tilde{B}^0}, m_{\tilde{H}_1^0}, m_{\tilde{H}_2^0}) + \frac{g_2^4}{4}J_3(m_{\tilde{W}^3}, m_{\tilde{H}_1^0}, m_{\tilde{H}_2^0}) + \frac{g_1^2 g_2^2}{4}J_4(m_{\tilde{H}_1^0}, m_{\tilde{H}_2^0}, m_{\tilde{W}^3}, m_{\tilde{B}^0}), \tag{3.3}$$

where we have to deal with the following integrals over fermion loops

$$J_2(m_1, m_2) = \mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\frac{i}{(\hat{k}-m_1)(\hat{k}-m_2)(\hat{k}-m_1)(\hat{k}-m_2)} = \tag{3.4}$$

$$= 4\frac{1}{16\pi^2}B_0(m_1^2, m_1^2) + 4(m_1^2 + 3m_2^2 + 4m_1 m_2)\frac{1}{16\pi^2}C_0(m_2^2, m_1^2, m_1^2) +$$

$$+8m_2^2(m_1+m_2)^2\frac{1}{16\pi^2}D_0(m_1^2, m_1^2, m_2^2, m_2^2) \tag{3.5}$$

$$J_3(m_1, m_2, m_3) = \mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\frac{i}{(\hat{k}-m_1)(\hat{k}-m_2)(\hat{k}-m_1)(\hat{k}-m_3)} = \tag{3.6}$$

$$= 4\frac{1}{16\pi^2}B_0(m_1^2, m_1^2) + 4(3m_1^2 + m_2 m_3 + 2m_1(m_2+m_3))\frac{1}{16\pi^2}C_0(m_1^2, m_2^2, m_3^2) +$$

$$+[(4m_1^2(2m_1^2 + 2m_1(m_2+m_3)) + 8m_1^2 m_2 m_3]\frac{1}{16\pi^2}D_0(m_2^2, m_3^2, m_1^2, m_1^2). \tag{3.7}$$

In this proceeding we have considered the degenerate higgsino masses $m_{\tilde{H}_{1,2}^0} = \mu$. In such case the parametrization of integrals in (3.3) is reduced, and integrals become simpler $J_4(m_1, m_2, m_3, m_4) = J_3(m_1, m_2, m_3)$ and $J_3(m_1, m_2, m_3) = J_2(m_1, m_2)$.

The values of effective potential parameters are collected in Table 1, calculated at some choice of parameter set, including M_1, M_2. One-loop contributions to parameters of the effective potential

i	1	2	$3+4$
λ_i^{SUSY}	0.14	0.14	-0.13
$\Delta\lambda_i^{squark}(\tilde{t},\tilde{b})$	0.01	-0.17	-0.04
$\Delta\lambda_i^{gaugino}(\tilde{H}_i^0,\tilde{W}^3,\tilde{B}^0)$	0.028	0.028	0.014
$\lambda_i^{SUSY}+\Delta\lambda_i^{gaugino}$	0.17	0.17	- 0.12
$\lambda_i^{SUSY}+\Delta\lambda_i^{squark}+\Delta\lambda_i^{gaugino}$	0.18	0.00	- 0.16

Table 1: Comparison of different corrections $\Delta\lambda_i$ at the scale m_{top} and $m_Z = 91.19$ GeV, $m_W = 79.96$ GeV, $m_b = 3$ GeV, $m_t = 175$ GeV, $g_2 = 0.6517$, $g_1 = 0.3573$, $v = 245.4$ GeV, $\tan\beta = 5$, $A = 1000$ GeV, $|\mu| = 2000$ GeV, $m_{\tilde{Q}} = 1500$ GeV, $m_{\tilde{U}} = 1200$ GeV, $m_{\tilde{D}} = 1000$ GeV, $M_1 = 2400$ GeV, $M_2 = 2500$ GeV.

increase weakly when mass parameters are increasing. Choice of a range for these parameters corresponds to constraints in [7]. where neutralino masses and linear combinations of unphysical wino, bino and higgsino states were analysed. For gaugino and higgsino mass parameters $m_{\tilde{B}^0} = M_1, m_{\tilde{W}^3} = M_2, m_{\tilde{H}_1^0} = m_{\tilde{H}_2^0} = \mu$ the neutralino masses were evaluated, contourplots for them are presented in Fig. 1.

4. Summary

Calculation of the one-loop effects induced in the effective Higgs potential by interactions of Higgs fields with gaugino and higgsino demonstrates that the sector of gauge and Higgs superpartners gives an important contribution comparable with stop and sbottom contributions at nearly degenerate mass parameters in the gaugino-higgsino sector. Explicit forms of gaugio and higgsino contributions to $\lambda_1 - \lambda_4$ are calculated at the one-loop, while $\lambda_5 - \lambda_7$ do not acquire corrections.

Work was partially supported by RFBR grant 10-02-00525-a. E. R. and I. E. thank the Dynasty Foundation and ICFPM for partial financial support.

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5. Appendix

In d dimensions the two-point loop integral B_0 is defined as

$$\frac{1}{(4\pi)^2}B_0(m^2,M^2) \;=\; \mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{i}{(k^2-m^2)\,[k^2-M^2]}. \tag{5.1}$$

The 3- and 4-point loop integrals at vanishing external momenta are defined as

$$\frac{1}{(4\pi)^2}C_{2n}(m_1^2,m_2^2,m_3^2) \;=\; \mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{ik^{2n}}{\prod_i^3(k^2-m_i^2)}, \tag{5.2}$$

$$\frac{1}{(4\pi)^2}D_{2n}(m_1^2,m_2^2,m_3^2,m_4^2) \;=\; \mu^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{ik^{2n}}{\prod_i^4(k^2-m_i^2)}. \tag{5.3}$$

Note that denominators of integrals under consideration are spherically symmetric. For the case of degenerate masses well-known general integral can be expressed using Γ function:

$$\int\frac{d^nk}{(2\pi)^n}\frac{(k^2)^b}{(k^2-A^2)^a} = \frac{i}{(4\pi)^{n/2}}(-1)^{a+b}(A^2)^{b-a+n/2}\frac{\Gamma(b+n/2)\Gamma(-b+a-n/2)}{\Gamma(n/2)\Gamma(a)}, \tag{5.4}$$

where the left-hand side is an integral in n-dimensional Minkowski space. In the special case $b=0$

$$\int\frac{d^nk}{(2\pi)^n}\frac{1}{(k^2-A^2)^a} = \frac{i}{(4\pi)^{n/2}}(-1)^a(A^2)^{-a+n/2}\frac{\Gamma(a-n/2)}{\Gamma(a)}. \tag{5.5}$$

Explicit formulae at $d\to 4$ are listed below:

$$B_0(x,y) \;=\; -\Delta+\log\frac{x}{\mu^2}-\frac{y}{x-y}\log\frac{y}{x}, \tag{5.6}$$

$$C_0(x,y,z) \;=\; \frac{1}{y-z}\left[B_0(x,y)-B_0(x,z)\right] = \frac{y\log\frac{y}{x}}{(x-y)(z-y)}+\frac{z\log\frac{z}{x}}{(x-z)(y-z)}, \tag{5.7}$$

$$C_0(x,y,y) \;=\; \frac{\partial}{\partial y}B_0(x,y) = \frac{1}{y-x}\left[1+\frac{x\log\frac{x}{y}}{y-x}\right], \tag{5.8}$$

Equal arguments correspond to equivalent masses. The divergent part is

$$\Delta_{\{\mu;1\}} = \frac{2}{4-d}+\log(4\pi\{\mu^2;\,1\})-\gamma_E+1$$

where μ is the renormalization scale.

Explicit formulae for the four-point integrals at vanishing external momenta are

$$D_0(x,y,z,t) = \qquad \frac{1}{z-t}\left(C_0(x,y,z)-C_0(x,y,t)\right) =$$

$$= \frac{y\log\frac{y}{x}}{(y-x)(y-z)(y-t)}+\frac{z\log\frac{z}{x}}{(z-x)(z-y)(z-t)}+\frac{t\log\frac{t}{x}}{(t-x)(t-y)(t-z)}, \tag{5.9}$$

$$D_0(x,y,z,z) = \frac{\partial}{\partial z}C_0(z,y,x) = \frac{\partial}{\partial z}\left(\frac{y\log\frac{y}{z}}{(z-y)(x-y)} + \frac{x\log\frac{x}{z}}{(z-x)(y-x)}\right) =$$

$$= \frac{1}{(x-z)(y-z)} + \frac{x\log\frac{x}{z}}{(x-y)(x-z)^2} + \frac{y\log\frac{y}{z}}{(y-x)(y-z)^2}, \tag{5.10}$$

The case with two pairs of equivalent masses in D_0 can be reduced using the following algebraic relation:

$$\frac{1}{(k^2-A^2)^2(k^2-B^2)^2} =$$

$$= \frac{1}{(A^2-B^2)^2}\left(\frac{1}{(k^2-A^2)^2} + \frac{1}{(k^2-B^2)^2}\right) - \frac{2}{(A^2-B^2)^3}\left(\frac{1}{k^2-A^2} - \frac{1}{k^2-B^2}\right), \tag{5.11}$$

or by derivatives:

$$D_0(x,x,y,y) = \frac{\partial^2}{\partial x\partial y}B_0(x,y) = \frac{\partial}{\partial x}C_0(x,y,y) = \frac{2}{(x-y)^2} + \frac{x+y}{(y-x)^3}\log\frac{x}{y}. \tag{5.12}$$

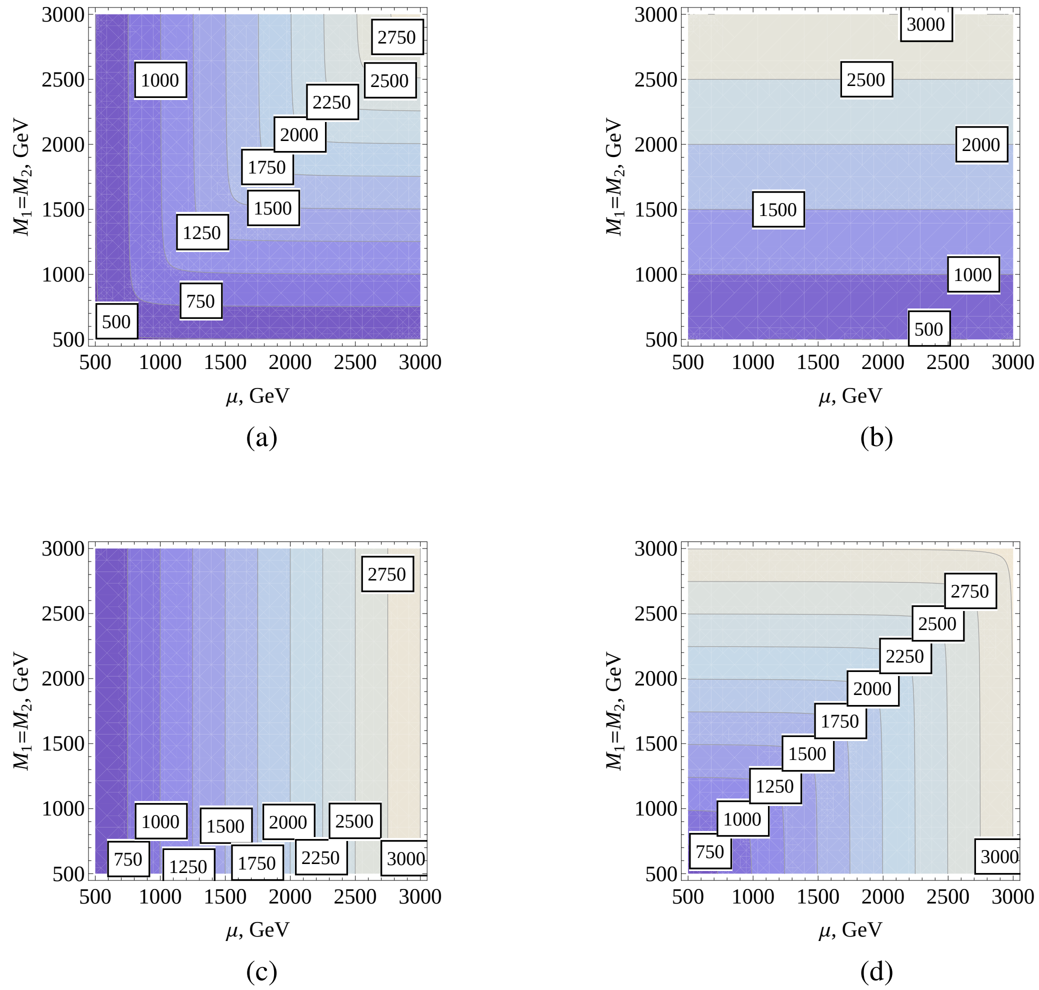

Figure 1: Contour plots of neutralino masses for the case $M_1 = M_2$, $\tan\beta = 5$. (a) $m_{\tilde{\chi}_1^0}$; (b) $m_{\tilde{\chi}_2^0}$; (c) $m_{\tilde{\chi}_3^0}$; (d) $m_{\tilde{\chi}_4^0}$. We use the conventions of [7].