PROCEEDINGS OF SCIENCE

Localization of scalar fields on thick branes with asymmetric geometries in the bulk

Vladimir Andrianov*
St.Petersburg State University
E-mail: v.andriano@rambler.ru

Alexander A. Andrianov[a,b]
[a] *St.Petersburg State University, Russia*
[b] *Departament d'ECM and ICCUB-Institut de Ciencies del Cosmos, University of Barcelona, Spain*
E-mail: andrianov@icc.ub.edu, espriu@ecm.ub.edu

The model of a domain wall ("thick brane") in noncompact five-dimensional space-time with asymmetric geometries of AdS type aside the brane is proposed. Asymmetric geometries in the bulk are provided by a space defect in the scalar field potential and a related defect of cosmological constant. The possibility of localization of scalar modes on such "thick branes" is investigated.

The XXth International Workshop High Energy Physics and Quantum Field Theory
September 24 - October 1, 2011
Sochi, Russia

*Speaker.

PoS(QFTHEP2011)080

1. Introduction

Different scenarios of domain wall generation and their applications to elementary particle physics and cosmology can be found in a number of reviews, in particular, in [1] and references therein. The influence of gravity is especially interesting, which plays an important role in a (de) localization of matter fields on the brane [2] - [8], [9]. As regarding to gravity the question arises under what circumstances the localization of spin-zero matter fields on a brane is still possible when the minimal interaction with gravity is present? This work is partially devoted to answer this question.

In our talk we consider a model of the domain wall formation with finite thickness ("thick" branes) and gravity in five-dimensional noncompact space-time [10], but with asymmetric behavior of a warped factor in the bulk on both sides of the brane, i.e. with different anti-de Sitter geometries. The formation of "thick" brane with the localization of light particles on it was obtained earlier in [11] with the help of background scalar (and gravitational) fields, when their vacuum configurations have nontrivial topology. Appearance of scalar states with (almost) zero mass on a brane has happened to be possible. However, as it was previously shown [9], the existence of the centrifugal potential leads to absence of localized modes on a brane when background gravity realizes a symmetric geometry of Anti-de Sitter type on both sides of the brane. Therefore, in this situation it makes sense to explore the mechanisms of generation of localized states when the symmetry of the geometries on both sides of the brane is manifestly violated by the introduction of a defect in the direction of extra dimension.

2. Formulation of the model

Consider the five-dimensional space, providing it with a pseudo Riemann metric,

$$X^A = (x^\mu, z), \quad x^\mu = \left(x^0, x^1, x^2, x^3\right), \quad \eta^{AB} = \mathrm{diag}\,(+,-,-,-,-)$$

with extra spatial coordinate z. It is assumed that the size of extra dimension is large or infinite.

We introduce a gravitational field with the five-dimensional metric tensor g_{AB}, which is reduced to η^{AB} in flat space and for the rectangular coordinate system. We define the dynamics of real scalar field $\Phi(X)$ with a minimal interaction to gravity with the help of the action functional,

$$S[g,\Phi,] = \int d^5X \sqrt{|g|}\mathscr{L}(g,\Phi,), \quad \mathscr{L} = -\frac{1}{2}M_*^3 R + \frac{1}{2}(\partial_A\Phi\partial^A\Phi) - V(\Phi,), \tag{2.1}$$

where R is a scalar curvature, $|g|$ is a determinant of the metric tensor, and M_* is a five-dimensional gravitational Planck scale.

The equations of motion are

$$R_{AB} - \frac{1}{2}g_{AB}R = \frac{1}{M_*^3}T_{AB}, \quad D^2\Phi = -\frac{\partial V}{\partial \Phi}, \tag{2.2}$$

where $D^2 = D_C D^C$ is a covariant D'Alambertian, and the energy-momentum tensor reads,

$$T_{AB} = \partial_A\Phi\partial_B\Phi - g_{AB}\left(\frac{1}{2}\partial_C\Phi\partial^C\Phi - V(\Phi,)\right). \tag{2.3}$$

Let us confine ourselves to the study of classical vacuum configurations, which do not violate four-dimensional Poincare invariance. In this part it is more convenient to present a metric in the conformally flat form, $g_{AB} = A^2(z)\,\eta_{AB}$.

Then the equations of motion are simplified,

$$\left(\frac{A'}{A^2}\right)' = -\frac{\Phi'^2}{3M_*^3 A}, \quad -2A^5 V(\Phi) = 3M_*^3\Big(A^2A'' + 2A(A')^2\Big), \quad \big(A^3\Phi'\big)' = A^5\frac{\partial V}{\partial \Phi}, \tag{2.4}$$

One can prove [11] , that only three of these equations are independent.

3. Fluctuations around the background metric

The action (2.1) is invariant under diffeomorphisms. Infinitesimal diffeomorphisms correspond to the Lie derivative along an arbitrary vector field $\tilde{\zeta}^A(X)$, defining the coordinate transformation $X \to \tilde{X} = X + \tilde{\zeta}(X)$.

Let us introduce the fluctuations of the metric $h_{AB}(X)$ and of the scalar fields $\phi(X)$ around the background solutions, of the equations of motion,

$$g_{AB}(X) = A^2(z)\,(\eta_{AB} + h_{AB}(X))\,; \quad \Phi(X) = \Phi(z) + \phi(X)\,.. \tag{3.1}$$

Since 4D Poincare symmetry is not broken, we define the corresponding 4D part of the metric metric fluctuations as $h_{\mu\nu}$ and introduce the notation for gravivectors $h_{5\mu} \equiv v_\mu$ and graviscalars $h_{55} \equiv S$. Let's rescale the vector fluctuations $\tilde{\zeta}_\mu = A^2\zeta_\mu$ and the scalar ones $\tilde{\zeta}_5 = A\zeta_5$.

Now we expand the action to quadratic order in fluctuations. The full action after this procedure is a sum,

$$\mathscr{L}_{(2)} = \mathscr{L}_h + \mathscr{L}_\phi + \mathscr{L}_S + \mathscr{L}_V, \tag{3.2}$$

where

$$\sqrt{|g|}\mathscr{L}_h \equiv -\frac{1}{2}M_*^3A^3\left\{-\frac{1}{4}\,h_{\alpha\beta,\nu}h^{\alpha\beta,\nu} - \frac{1}{2}\,h^{\alpha\beta}_{,\beta}\,h_{,\alpha} + \frac{1}{2}\,h^{\alpha\nu}_{,\alpha}\,h^{\beta}_{\nu,\beta} + \frac{1}{4}\,h_{,\alpha}h^{,\alpha} + \frac{1}{4}h'_{\mu\nu}h'^{\mu\nu} - \frac{1}{4}h'^2\right\},$$

$$\sqrt{|g|}\mathscr{L}_\phi \equiv \frac{1}{2}A^3(\phi_{,\mu}\phi^{,\mu} - \phi'^2) - \frac{1}{2}\,A^5\Big(\frac{\partial^2 V}{\partial\Phi^2}\phi^2\Big) + \frac{1}{2}A^3h'(\Phi'\phi),$$

$$\sqrt{|g|}\mathscr{L}_S \equiv \frac{1}{4}\Big(-A^5VS^2 + S\Big(M_*^3A^3\,(h^{\mu\nu}_{,\mu\nu} - h^{,\mu}_{,\mu}) + M_*^3\,(A^3)'\,h' + 2\big(A^3(\Phi'\phi)\big)' - 4A^3(\Phi'\phi')\Big)\Big),$$

$$\sqrt{|g|}\mathscr{L}_V \equiv -\frac{1}{8}M_*^3A^3v_{\mu\nu}v^{\mu\nu} + \frac{1}{2}v^\mu\Big[-M_*^3A^3\,(h^{,\nu}_{\mu\nu} - h_{,\mu})' + 2A^3(\Phi'\phi_{,\mu}) + M_*^3\Big(A^3\Big)'S_{,\mu}\Big], \tag{3.3}$$

where $v_{\mu\nu} = v_{\mu,\nu} - v_{\nu,\mu}$, $h = h_{\mu\nu}\eta^{\mu\nu}$.

4. Separation of equations for physical degrees of freedom

For a better understanding of physical states in the model it is convenient to decompose the ten gravitational fields $h_{\mu\nu}$ in terms of a traceless-transverse tensor, a transverse vector and scalar components [5, 12],

$$h_{\mu\nu} = b_{\mu\nu} + F_{\mu,\nu} + F_{\nu,\mu} + E_{,\mu\nu} + \eta_{\mu\nu}\psi, \tag{4.1}$$

where $b_{\mu\nu}$ and F_μ obey the relation $b^{;\mu}_{\mu\nu} = b\mu^\mu = 0 = F^{;\mu}_\mu$. Obviously, the gravitational fields $b_{\mu\nu}$ are gauge invariant and thereby describe graviton fields in the 4-dim space.

Using the parametrization (4.1) we can calculate components of the quadratic action,

$$\begin{aligned}
\mathscr{L}_{(2)} \;=\; & \frac{1}{8}M_*^3 A^3 \left\{\, b_{\mu\nu,\sigma}b^{\mu\nu,\sigma} - (b')_{\mu\nu}(b')^{\mu\nu} - f_{\mu\nu}f^{\mu\nu}\right\} \\
& + \frac{3}{4}M_*^3 A^3 \left\{ -\psi_{,\mu}\psi^{,\mu} + \psi_{,\mu}S^{,\mu} + 2(\psi')^2 + 4\frac{A'}{A}\psi' S\right\} \\
& + \frac{1}{2}A^3 \left\{ \phi_{,\mu}\phi^{,\mu} - (\phi')^2 - A^2\frac{\partial^2 V}{\partial\Phi^2}\phi^2 \right. \\
& \left. - \frac{1}{2}A^2 V(\Phi) S^2 + 4\psi'\Phi'\phi + S\Big(-\Phi'\phi' + A^2\frac{\partial V}{\partial\Phi}\phi\Big)\right\} \\
& + \frac{3}{4}M_*^3 A^3\, \Box(E' - 2\eta)\Big(\frac{A'}{A}S + \psi' + \frac{2}{3M_*^3}\Phi'\phi\Big),
\end{aligned} \tag{4.2}$$

where $f_\mu \equiv F'_\mu - v^\perp_\mu, \quad f_{\mu\nu} \equiv f_{\mu,\nu} - f_{\nu,\mu}$. Obviously, in the quadratic approximation graviton, gravivector and graviscalar are decoupled from each other. From the last line it follows that the scalar E' is a Lagrange multiplier and generates a gauge-invariant constraint,

$$\frac{A'}{A}S + \psi' = -\frac{2}{3M_*^3}(\Phi'\phi). \tag{4.3}$$

Thus taking this constraint into account and with the further choice of a gauge, only one independent scalar field remains.

5. Scalar sector in gauge $\phi = 0$

This gauge in fact can be related to the choice of gauge invariant variables. After substitution of the decomposed fluctuation metric tensor and assuming that $\phi = 0$, one can represent the scalar part of the lagrangian (4.2) as a sum of noninteracting contributions: tensor, vector and scalar parts,

$$\mathscr{L}_{(2)}(v_\mu = \phi = 0) = \mathscr{L}_b + \mathscr{L}_f + \mathscr{L}_{S,\psi}, \tag{5.1}$$

where

$$\sqrt{|g|}\mathscr{L}_{\phi=0} = \frac{3}{4}M_*^3 A^3\Big(-\psi_{,\mu}\psi^{,\mu} + 2(\psi')^2 + \psi_{,\mu}S^{,\mu} + 4\frac{A'}{A}\psi' S + \frac{2(A')^2 + AA''}{2A^2}S^2\Big). \tag{5.2}$$

In its derivation the identity $M_*^3(A^3)'' = -2A^5 V(\Phi)$ is used, and integration by parts is performed in the action.

For the normalization of the kinetic term it is useful to redefine the field $\psi = \Omega^{-1}\hat{\psi}$,

$$\sqrt{|g|}\mathscr{L}_{\psi} = \hat{\psi}_{,\mu}\hat{\psi}^{,\mu} - \hat{\psi}(-\partial_z^2 + V(z))\hat{\psi}, \qquad V(z) = \frac{\Omega''}{\Omega}. \tag{5.3}$$

The equations (5.3) describe the quadratic action for scalar particles, branons and allow to calculate their mass spectrum.

To simplify analytical calculations let's proceed to the gaussian normal coordinates x_μ, y,

$$ds^2 = A^2(z)\left(dx_\mu dx^\mu - dz^2\right) = \exp(-2\rho(y))\, dx_\mu dx^\mu - dy^2. \tag{5.4}$$

In these coordinates and after redefining , $\hat{\psi} = \exp(-\rho/2)\tilde{\psi}$ and $\exp(-\rho)\tilde{\psi}_m = \Psi_m$, the equation for the mass spectrum has a form,

$$\left(-\partial_y^2 + V(y) - m^2 \exp(2\rho)\right)\Psi_m = 0, \tag{5.5}$$

$$V = \frac{1}{4}\rho'^2 - \frac{1}{2}\rho'' + \frac{\rho'}{\exp(-3\rho/2)\Phi'}\left(\partial_y^2 - \rho'\partial_y\right)\left(\frac{\exp(-3\rho/2)\Phi'}{\rho'}\right), \tag{5.6}$$

where $\hat{D}_y\tilde{\psi}_m = m^2\tilde{\psi}_m$. One can see that the existence of a state with mass m depends on an existence of zero-mode in the potential $\tilde{V} = V - m^2\exp(2\rho)$.

These formulas allow to calculate the spectrum of quadratic fluctuations of the boson field minimally interacting to gravity.

6. Branon mass spectrum in the theory with potential ϕ^4, induced by five-dimensional fermions

Let us study the formation of a brane in the theory with ϕ^4 potential and with the wrong-sign mass term, which are presumably induced by self-interacting of five-dimensional fermions [10] and admit a kink-type solution. The effective action has the form,

$$S_{eff}(\Phi, g) = \frac{1}{2}M_*^3 \int d^5X\sqrt{|g|}\left\{-R + 2\lambda + \frac{3\kappa}{M^2}\left(\partial_A\Phi\partial^A\Phi + 2M^2\Phi^2 - \Phi^4\right)\right\}, \tag{6.1}$$

where the normalization of the kinetic term of scalar fields κ is inherited from the low-energy effective action of composite scalar fields and reflects the dynamics of fundamental interaction of five-dimensional pre-fermions [10]. In our case, we assume that κ is a small parameter, which characterizes the interaction of gravity and matter fields.

We use the warped metric in gaussian normal coordinates,

$$ds^2 = \exp(-2\rho(y))\, dx_\mu dx^\mu - dy^2. \tag{6.2}$$

After variation of the action (6.1) in respect to the metric g_{AB} and to the scalar field we obtain a system of three equations,

$$\Phi'' = -2M^2\Phi + 4\rho'\Phi' + 2\Phi^3, \tag{6.3}$$

$$\rho'' = \frac{\kappa}{M^2}\Phi'^2, \qquad \lambda + 6\rho'^2 = \frac{3\kappa}{2M^2}\left\{\Phi'^2 + 2M^2\Phi^2 - \Phi^4\right\}. \tag{6.4}$$

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These equations contain terms which have different order in small parameter κ, and accordingly they can be solved by perturbation theory assuming that, $\frac{|\rho'(y)|}{M} = O(\kappa) = \frac{|\rho''(y)|}{M^2}$. Then in the leading order in κ the equations for the field $\Phi(y)$ and the metric are,

$$\Phi'' = -2M^2\Phi + 2\Phi^3 + O(\kappa), \quad \frac{\rho''}{M^2} = \frac{\kappa}{M^4}\Phi'^2 + O\left(\kappa^2\right). \tag{6.5}$$

The solution of these equations has the form of a kink, $\Phi_0 = M\tanh(My) + O(\kappa)$, and allows to find the conformal factor,

$$\rho_0(y) = \frac{2\kappa}{3}\left\{\ln\cosh(My) + \frac{1}{4}\tanh^2(My) + tMy\right\} + O\left(\kappa^2\right). \tag{6.6}$$

We notice that in this order of the curvature κ solutions triggering an asymmetric brane are possible, that corresponds to $t \neq 0$. Later on we'll show that in general such solutions are not possible in the case of spontaneous creation of the brane, but become accessible if there is a manifest breaking of the space symmetry,i.e. if there is a defect of the cosmological constant.

Let's substitute the solutions of the equations of motion into the formula for calculation of the mass spectrum (5.6). In the case of a symmetric ($t = 0$) brane it turns out that in this potential a centrifugal barrier exists being located at zero and non-vanishing when gravity is switched off,

$$V(y, t = 0, \kappa = 0) = M^2\left(4 + \frac{2}{\sinh^2(My)} + 8\frac{1 - 4\cosh^2(My)}{\left(1 + 2\cosh^2(My)\right)^2}\right)\Bigg|_{y\to 0} \sim \frac{2}{y^2}. \tag{6.7}$$

Numerical calculations show that at the leading order in the gravitational constant there are neither zero-modes, no resonances at $m^2 > 0$. Thus, localized scalar states don't exist near a symmetric brane with potential (6.1) .

Next, let's consider the case $t \neq 0$, where the metric on both sides of the brane is different. This solution seems to be possible in the leading order in κ. In this order we can calculate the mass spectrum. The asymmetry parameter is assumed to be small as well which corresponds to the case $t \sim 1$. In the main approximation in κ the potential with asymmetric brane has the form (where $u = My$),

$$V(t, \kappa = 0) = 2M^2\left(2 + \frac{-12(1+t^2)\cosh^4 u + 12t\cosh u\sinh u(1 - 2\cosh^2 u) + 24\cosh^2 u - 3}{4(1+t^2)\cosh^6 u + 4t\cosh^3 u\sinh u(1 + 2\cosh^2 u) - 3\cosh^2 u - 1}\right).$$

Numerical calculations show that at zero mass normalizable localized states don't arise, but localized states with nonzero mass arise when $t > t_{\min}$, $t_{\min} = 0.21\ldots$. They are resonances, since $V - m^2\exp(2\rho)$ exponentially decreases at infinity and the barrier is penetrable, although the probability of its penetration is very small [10].

7. Asymmetric background solutions and a defect of the cosmological constant

For different asymptotics one must introduce an asymmetry in the cosmological constant and/or break the symmetry under the reflection $\Phi \to -\Phi$, which generates an asymmetry of kink-type solutions of equations of motion in respect to $y \to -y$. It can be implemented by adding a defect to the action (6.1),

$$\mathscr{L}_{def} = 6\kappa M_*^3 M\eta(y)\Phi(X), \tag{7.1}$$

where η is a dimensionless function, which has the following limits $\eta \to \eta_\pm$ when $y \to \pm\infty$. The equations of motion with the defect read,

$$\Phi_0{}'' = -2M^2\Phi_0 + 4\rho'\Phi_0' + 2\Phi_0^3 + M^3\eta, \tag{7.2}$$

$$\rho'' = \frac{\kappa}{M^2}\,\Phi_0'^{\,2}, \qquad \lambda + 6\rho'^2 = \frac{3\kappa}{2M^2}\left\{\Phi_0'^{\,2} + 2M^2\Phi_0^2 - \Phi_0^4 - 2M^3\eta\Phi_0\right\}. \tag{7.3}$$

They imply that the cosmological "constant" should depend on y so that the relation were satisfied on the solutions of the equations of motion,

$$\frac{2M^2}{3\kappa}\lambda' + 2M^3\eta'\Phi_0 = \left((\Phi_0')^2 + 2M^2\Phi_0^2 - \Phi_0^4 - \frac{4M^2(\rho')^2}{\kappa}\right)' - 2M^3\eta\Phi' = 0.$$

This is guarantied if its (fixed) functional dependence of y coincides exactly with the solution $\Phi_0(y)$,

$$\lambda(y) = \lambda_0 + 3\kappa M\int_0^y dy'\eta'(y')\Phi_0(y'), \quad \lambda_0 = const. \tag{7.4}$$

If the defect is not constant, $\eta'(y) \neq 0$, the asymptotics of cosmological functions in the limit $y \to \pm\infty$ are in general different $\lambda(y) \to \lambda_\pm$. Turning back to the asymptotical forms and the dimensionless quantities, $\Phi_\pm = \varphi_\pm M, \quad k_\pm = M\bar{k}_\pm, \quad \lambda_\pm = M^2\bar{\lambda}_\pm$ we obtain,

$$2\bar{k}^2 + \frac{1}{3}\bar{\lambda} = \frac{\kappa}{2}\left\{2\varphi^2 - \varphi^4 - 2\eta\varphi\right\}, \quad 0 = -2\varphi + 2\varphi^3 + \eta.$$

The equation for the field φ has three solutions, one of them (near $\varphi = 0$) realizes an unstable state, it is a maximum. To calculate two other solutions, we assume $\eta \ll 1$ and then obtain,

$$\varphi_\pm = \pm 1 - \frac{\eta_\pm}{4}, \quad 2\bar{k}_\pm^2 = \frac{\kappa}{2} - \frac{1}{3}\bar{\lambda}_\pm \mp \kappa\eta_\pm. \tag{7.5}$$

Therefrom we find the relations between the asymmetry parameter t, the asymptotical form of the defect and the cosmological function,

$$\frac{32}{9}t\kappa^2 = \frac{1}{3}(\lambda_- - \lambda_+) - \kappa(\eta_+ + \eta_-), \quad \frac{16}{9}\kappa^2(1+t^2) = \kappa - \frac{1}{3}(\lambda_- + \lambda_+) + \kappa(\eta_- - \eta_+) > 0.$$

Thus, the asymptotical form of the scalar matter defect and of the cosmological constant $(\lambda_- + \lambda_+)/2)$ completely determine the asymmetry of the conformal factor and of the cosmological function.

8. Conclusions

In this paper we have considered a model of domain wall ("thick brane") in the noncompact five-dimensional space-time with asymmetric geometries on both sides of the brane, generated by self-interacting fermions in the presence of gravity. The asymmetric geometry in the bulk is provided by the asymmetry of scalar field potential and a corresponding defect of the cosmological constant. In the model [10] with a minimal interaction of gravity and scalar fields for the symmetric anti-de Sitter geometry there are no localized states in the vicinity of the brane. In the case of anti-de Sitter geometries asymmetric against reflection of the fifth coordinate such states occur. When the exponent coefficients of a conformal factor for anti-de Sitter spaces on both sides of the brane have different signs there exist only slowly decaying resonance. Asymmetric solutions on the brane were obtained in the leading order of the expansion in a small parameter, corresponding to the curvature of the bulk.

This work was supported by Grants RFBR 12-02-00859-a and by SPbSU grant 11.0.64.2010 as well as from Projects FPA2010-20807, 2009SGR502, CPAN (Consolider CSD2007-00042).

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