

Standard Model Heavy Flavor physics on the Lattice

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Lattice QCD calculations in charm and bottom physics are particularly important because they can provide the hadronic weak decay matrix elements needed for key constraints on the CKM Unitarity Triangle. I will summarise recent results in this area, paying particular attention to sources of error, comparison between methods and tests of results against experiment, for example, in the spectrum. Updated world averages for decay constants this year are : $f_{D_s}=248.6(2.4)$ MeV; $f_D = 212.1(3.4)$ MeV; $f_{B_s} = 227(4)$ MeV; $f_B = 190(4)$ MeV. Note that B decay constants are clearly lower than the corresponding D decay constants. Improved D semileptonic form factors, both shape and normalisation, now allow the direct determination of V_{cs} and V_{cd} to 3% and 5% respectively. This year we also have a clear demonstration that dependence of form factors on the spectator quark mass between light and strange is very small. Apart from the phenomenology implications, this has practical application to the normalisation of branching fractions in experiment. The current best Standard model rate for a key LHC mode sensitive to new physics - $B_{(s)} \rightarrow \mu^+ \mu^-$ - is derived from lattice QCD calculations on B/B_s mixing rates. I will discuss the current result and prospects for improving lattice QCD errors.

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1. Introduction

Heavy quark physics has turned out to be one of the ‘killer applications’ of lattice QCD and results in this area have done much to persuade the particle physics community that we now have a serious tool for calculating strong interaction effects that can provide accurate phenomenology not available with any other method. Particularly important are various heavy meson weak decay matrix elements that are key to constraining the vertex of the Unitarity Triangle derived from the Cabibbo-Kobayashi-Maskawa (CKM) matrix for a stringent test of the self-consistency of the Standard Model. Calculations of these matrix elements must not be seen in isolation, however. One of the key features of (lattice) QCD is the small number of free parameters - a mass for each quark and an overall scale parameter or equivalently a coupling constant. Once these are fixed a myriad of parameter-free tests against experiment become possible. This is particularly true in the heavy quark sector where there are many gold-plated states in the spectrum. It is still possible to make predictions ahead of experiment, which are many times more valuable than postdictions in terms of credibility. In addition new methods for heavy quarks have allowed us to leverage improved accuracy in light quark physics, for example for quark masses.

Here I will discuss the current status of lattice QCD calculations in charm and bottom physics, comparing formalisms, providing world averages and discussing prospects for the future. An important theme will be how to improve the errors *and* how to test that we have improved the errors, since precision from lattice QCD is critical for the Unitarity Triangle tests discussed above.

2. Heavy quark physics on the lattice

In discretising the QCD Lagrangian onto a space-time lattice we inevitably generate discretisation errors that appear as some power of the lattice spacing, a . Physical results, for example hadron masses, then depends on the lattice spacing as:

$$m(a) = m_{a=0} [1 + A(\Lambda a)^i + B(\Lambda a)^j + \dots]. \quad (2.1)$$

Here, for a light hadron, we would expect Λ to be the typical dimensionful scale of QCD, say a few hundred MeV. Then A, B are $\mathcal{O}(1)$. Since the lattice spacing of a modern lattice QCD calculation is of size $a^{-1} \approx 1 - 3$ GeV, $\Lambda a \ll 1$. Good discretisations, with small errors, have leading power $i = 2$ and higher powers, j , starting at 4.

For heavy quarks the scale for the discretisation errors will typically be set by the heavy quark mass, m_Q , which makes controlling the discretisation errors harder. For a lattice spacing $a \approx 0.1$ fm then $m_c a \approx 0.4$, $m_b a \approx 2$. For charm quarks this indicates that, although discretisation errors will be larger than those for light hadrons, good results are possible with a highly improved action on fine lattices. For example, using the Highly Improved Staggered Quark action [1] where the leading errors are $\mathcal{O}(\alpha_s a^2)$, discretisation errors of 2% are seen on 0.09fm lattices (where $m_c a = 0.4$) in the decay constant of the η_c meson, when using the heavy quark potential parameter, r_1 , to fix the lattice spacing [2]. For light meson decay constants at the same lattice spacings, the discretisation errors are barely visible [3]. For b quarks the situation is worse and this is why methods using a nonrelativistic expansion of the Dirac Lagrangian were developed. By removing m_Q as a dynamical scale in the calculation they allow the scale of discretisation errors to be set again by Λ . A price

has to be paid in terms of complexity and in other sources of systematic error coming from the nonrelativistic expansion. However, these methods have been the workhorses of b physics in the past because they enabled calculations to be done at the lattice spacing value that were available. As we will see here, this is beginning to change but it is likely that we will continue to need a mix of methods into the future.

There are a variety of relativistic actions in use for light quarks and charm quark physics programmes are now being developed with all of them, sometimes with some modifications.

- The Highly Improved Staggered Quark action developed by the HPQCD collaboration [1] includes a further smearing level beyond the asqtad improved staggered quark action to give discretisation errors at $\mathcal{O}(\alpha_s a^2)$ and $\mathcal{O}(a^4)$, with small taste-changing errors. When used for heavy quarks, the coefficient of the ‘Naik’ 3-link improvement term is modified to have a coefficient which is calculated as an expansion in ma (starting at $(ma)^2$) to remove the leading $(ma)^4$ errors. Results from extending this action to b quarks are now available [4, 5].
- The twisted mass action developed by the European Twisted Mass Collaboration [6] uses a doublet of Wilson-like quarks that have an additional mass term multiplying τ_3 in flavor space. This is used to introduce a u/d doublet in the sea and work is underway on configurations that also include a s/c doublet [7]. For valence c and s quarks ETMC use a separate doublet for each flavor [8]. The leading discretisation errors are $\mathcal{O}(a^2)$ at maximal twist [6]. This action is also being used now for heavier quarks to extrapolate up to b [9].
- Clover actions use a clover term to remove the tree-level $\mathcal{O}(a)$ errors from the Wilson action and have been in use for many years. In principle, unless the clover term is tuned nonperturbatively, these actions have $\alpha_s a$ discretisation errors. In newer variants, including various kinds of smeared links, discretisation errors can be made quite small and effectively $\mathcal{O}(a^2)$ and $\mathcal{O}(a^3)$ [10]. The clover action is being used for c quarks on lattices with a fine temporal lattice spacing and relatively coarse spatial lattice spacing by the Hadron Spectrum collaboration [11].
- The good chiral properties of domain wall and overlap quarks enforce discretisation errors starting at $\mathcal{O}(a^2)$. This is an expensive method for charm physics, but is being tested. See, for example [12].

The advantages of a relativistic action for c quarks, if discretisation errors can be made small, are:

- The hadron mass is simply and precisely obtained from the energy of the zero momentum hadron correlator. With a non-relativistic approach it is necessary to calculate finite-momentum correlators and extract the ‘kinetic mass’ from the momentum dependence of the energy which is much less precise.
- In formalisms with enough chiral symmetry (HISQ, twisted mass and domain wall/overlap in the list above) the PCAC relation protects the axial current from renormalisation. This means, for example, that pseudoscalar decay constants can be obtained directly with the correct normalisation and no error from a Z factor is required.

- Using the same action for c quarks as for u/d and s allows some cancellation in ratios. For example the HPQCD collaboration obtained m_c/m_s to 1% (value 11.85(16)) using the HISQ action for both quarks [13]. ETMC obtain 12.0(3) [8] for this ratio and 11.34(45) is a preliminary result with ‘Brillouin-improved’ Wilson quarks [14].

The contrasting approaches that incorporate nonrelativistic ideas are:

- NonRelativistic QCD (NRQCD) is a discretised version of a nonrelativistic effective theory which can be matched at a given order in v_h , the velocity of the heavy quark, to full QCD [15]. The heavy quark mass is tuned nonperturbatively and this fixes the $\mathcal{O}(v_h^2)$ kinetic energy term. Higher order terms have coefficients that are in principle calculable in perturbation theory (and will typically diverge as $m_h a \rightarrow 0$) but in most work to date have taken tree level values. The same action can be used for heavy-heavy and heavy-light physics.
- The discretisation of Heavy Quark Effective Theory onto the lattice starts from the static (infinite mass) approximation and adds $1/m_h$ and higher corrections through the calculation of matrix elements, with coefficients determined nonperturbatively, rather than incorporating kinetic terms in the dynamics. It can be used for heavy-light physics where a systematic programme has been developed by the Alpha collaboration [16].
- The Fermilab method, as originally implemented, uses the tadpole-improved clover action with a heavy quark mass (b or c) but removing the leading discretisation errors by fixing the quark mass from the meson kinetic energy [17]. The field is also ‘rotated’ to remove tree-level $\mathcal{O}(a)$ errors. As the lattice spacing is reduced this becomes the standard clover action. Extensions of this method called ‘Relativistic Heavy Quarks’ (RHQ) tune further coefficients nonperturbatively [18], for example fixing the clover term from a hyperfine splitting and an asymmetry between time and space so that static and kinetic masses are equal [19].

It can be helpful for an accurate comparison of b and c physics to use the same action for both. However, so far this has really only been made an advantage in results using the relativistic HISQ action for all 5 quarks [4].

A key aim of flavor physics from lattice QCD is to calculate simple meson weak matrix elements that allow the determination of elements of the CKM matrix from experimental rates for leptonic or semileptonic decays or (for neutral B/K mesons) oscillations. In Fig. 1 I give the CKM matrix and the simple processes that allow the determination of that CKM element by combining experiment and lattice QCD. The processes are dominated by those for B and D meson decay, hence the importance of b and c quark physics in lattice QCD. Here I will review lattice calculations for a number of these processes and summarise the direct tests of CKM unitarity that result. Further discussion of the impact of these tests on Beyond the Standard Model scenarios is given in Enrico Lunghi’s talk [20].

3. Charm physics results

3.1 Spectroscopy

The spectroscopy of charmonium, D and D_s mesons and charmed baryons has had a resurgence of interest from experiment in recent years, particularly following the discovery of a number of new

$$a) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \rightarrow l\nu & K \rightarrow l\nu & B \rightarrow l\nu \\ & K \rightarrow \pi l\nu & B \rightarrow \pi l\nu \\ V_{cd} & V_{cs} & V_{cb} \\ D \rightarrow l\nu & D_s \rightarrow l\nu & B_c \rightarrow l\nu \\ D \rightarrow \pi l\nu & D \rightarrow Kl\nu & B \rightarrow Dl\nu \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle & \end{pmatrix}$$

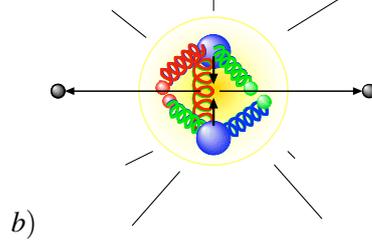


Figure 1: a) The Cabibbo-Kobayashi-Maskawa matrix in the Standard Model and the simple weak decay processes from which the elements can be determined given accurate lattice QCD calculations. b) an illustration of the leptonic decay of a charged pseudoscalar meson through annihilation of its valence quark and antiquark to a W boson.

states in the charmonium spectrum by the B factories. The LHC will produce charmed and doubly charmed baryons in huge numbers, giving lattice QCD the opportunity to predict some masses ahead of experiment.

This year saw new general spectroscopy results from a variety of clover actions. The clover action, as discussed above, is not highly improved and so discretisation errors are not minimised, but it is a simple and relatively efficient action to use when calculating a lot of hadron correlators with multiple source and sink operators. Examples include promising first results from the Hadron Spectrum Collaboration [11] giving a large number of states in the charmonium spectrum. The use of an anisotropic lattice with a finer spacing in the time than in spatial directions enables improved access to excited states, which can otherwise disappear rapidly from the correlator as a function of time from the source. In addition many operators are used for each irrep of the lattice rotation group, allowing identification of the expected continuum multiplets and the separation of quark model and hybrid states. All of this requires an enormous number of correlators to be calculated, however, and so results at only one lattice spacing and sea u/d quark mass (on $n_f = 2 + 1$ configurations) are available so far. The issue of overlap with multi-hadron channels for the excited ‘non-gold-plated’ states has also not yet been addressed. The $X(3872)$ [21] is still safe, but perhaps not for much longer, from the straitjacket of an unambiguous lattice QCD identification.

The aims of calculations using SLiNC and 2-HEX smeared clover quarks on isotropic $n_f = 2 + 1$ configurations by [22] are similar but also include preliminary results on the D and D_s spectrum. Results for charmed baryons were given by [23] using an RHQ action on the new ‘second generation’ MILC configurations that include 2+1+1 flavors of sea HISQ quarks. The conclusions back up earlier indications that the mass of the Ξ_{cc} found by SELEX [24] is unlikely to be correct.

The calculations above are fairly general ones aimed at mapping out a large part of the spectrum. I now want to turn to the ‘precision calculations’ that are being done to provide weak decay rates for determination of the CKM matrix. These calculations focus on ground state mesons only and accuracy is critical. I will argue that accuracy must still be judged in tandem with results for the spectrum and other quantities that can be compared with experiment or between calculations.

One variable that provides an excellent test in charm physics is the difference in mass, Δ , between the D_s meson and one half of the mass of the η_c . Typically one of these meson masses is

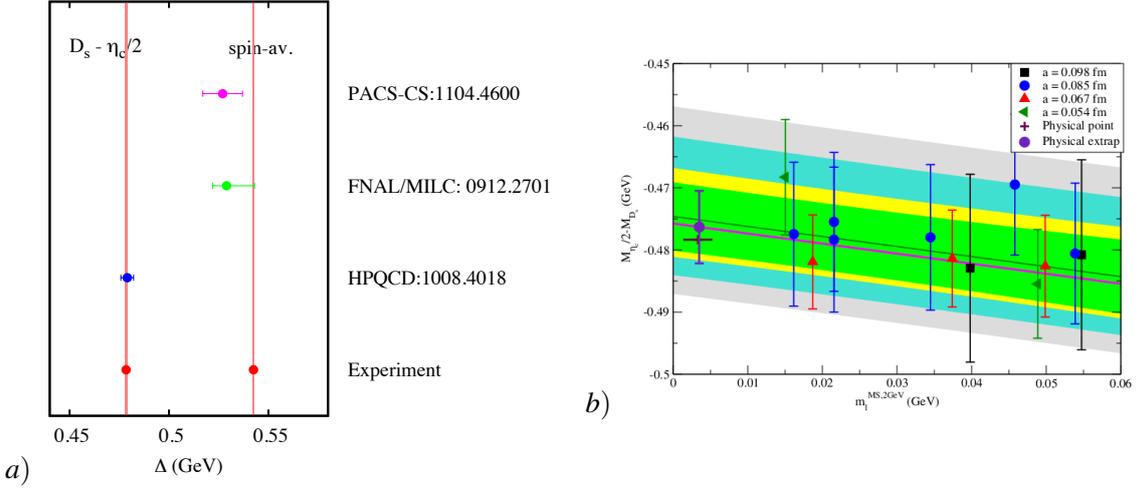


Figure 2: a) Comparison of lattice QCD results including u , d and s sea quarks [26, 25, 2] to experiment for the mass difference $\Delta = m(D_s)/m(\eta_c)/2$ or (on the right) a spin-averaged version of it. b) Results from ETMC for Δ on $n_f = 2$ configurations with a chiral and continuum extrapolation (marked as physical extrapol.) [27] compared to experiment (marked as physical point).

used to fix the c quark mass and the other is then a parameter-free determination from the lattice QCD calculation (assuming that the s quark is determined from a light meson such as the K). HPQCD use the η_c , Fermilab/MILC use the D_s and ETMC have tried both. Δ can be calculated in either case and is much more precisely determined than the masses themselves. This is because the difference is much smaller than either mass and so the absolute size of lattice spacing errors is much reduced. An error of a few percent is readily achieved in modern lattice QCD calculations and this would be impossible with any other method. Δ is the difference in binding energy between a heavy-light and a heavy-heavy meson and there are no good approximate treatments that apply to both systems. Certainly there are none that could be relied on for errors of a few percent.

Fig. 2 summarises results for Δ from different calculations. The Fermilab Lattice/MILC result [25] includes vector meson masses to make a ‘spin-averaged’ mass difference. This removes a systematic error of $\mathcal{O}(\alpha_s a)$ resulting from the clover term in their action. Their total error is around 2% (10 MeV), dominated on the upward side by lattice spacing uncertainties from an old determination which has now been improved. The PACS-CS collaboration [26] using a similar RHQ action also give a spin-averaged splitting. The PACS-CS result is from a calculation at one value of the lattice spacing only, with no estimate of lattice spacing errors. The HISQ result from HPQCD [2] is much more accurate, with errors of less than 1% (3 MeV). Small statistical errors enable the discretisation errors to be clearly identified and extrapolated away, underlining the advantages of a relativistic formalism. Note that the value and the error quoted by HPQCD includes an estimate of effects from electromagnetism, annihilation of the η_c , and missing c quarks in the sea.

Fig. 2 also includes results from the ETM collaboration using the twisted mass formalism for c quarks on gluon configurations including only u/d quarks in the sea. Since heavy-heavy mesons and heavy-light mesons are sensitive to different momentum scales it might be expected that Δ would ‘see’ the incorrect running of the strong coupling constant between scales that is a consequence of using only 2 flavors in the sea. Challenged to test this at the lattice conference

ETM produced the right hand plot [27] in fig. 2 which shows that in fact a continuum and chiral extrapolation of Δ agrees well with experiment with errors of 2% ((6)(4) MeV, where the first error is statistical and fitting and second an estimate of systematics).

Using time moments of their statistically very precise η_c correlators the HPQCD collaboration has developed a method with continuum QCD theorists [28] that enables the c quark mass to be extracted very accurately, using the high-order continuum perturbation theory that is available for the charm quark polarisation function. The result in the \overline{MS} scheme, $\overline{m}_c^{(4)}(3\text{GeV}) = 0.986(6)$ GeV [4], agrees well with that determined from e^+e^- cross-sections in the charm region using continuum methods. The ETM collaboration are now also applying this method and preliminary results presented here [29] look promising. The c quark mass is then another quantity that can be compared accurately between lattice QCD calculations (and with continuum results) and it would be good to have numbers from other formalisms. The accuracy in the c mass can then be cascaded down to lighter masses using mass ratios [13].

3.2 Leptonic decays

The annihilation rate of the charged D and D_s mesons to leptons via a W boson is parameterised by the decay constant, $f_{D(s)}$. This is defined (here for the D_s at rest) as the matrix element between the meson and the vacuum of the temporal axial current that couples to the W (the vector part of the W interaction does not contribute):

$$\langle 0 | \bar{c} \gamma_0 \gamma_5 s | D_s \rangle = f_{D_s} M_{D_s}. \quad (3.1)$$

In a formalism with a partially conserved axial current, we can also use

$$(m_s + m_c) \langle 0 | \bar{c} \gamma_5 s | D_s \rangle = f_{D_s} M_{D_s}^2. \quad (3.2)$$

The decay constant is a property of the meson related to the internal configuration of its valence quark and antiquark affected by their strong interaction. It is typically calculated in lattice QCD from the amplitudes in the same multiexponential fit to the meson correlator that gives the meson masses from the exponents. When the same operator, \mathcal{O} , is used to create and destroy the meson, the fit function for the correlator is:

$$C_{2pt} = \sum_i a_i^2 f(E_i, t); \quad f(E_i, t) = e^{-E_i t} + e^{-E_i(T_p - t)}. \quad (3.3)$$

Here E_i are the energies of different radial excitations - the ground state will be denoted E_0 . T_p is the time length of the lattice and the form of the time dependence allows for the meson to go round the lattice either way. For mesons containing staggered quarks there are typically additional oscillating terms that must be fitted. The amplitudes $a_i = \langle 0 | \mathcal{O} | i \rangle / \sqrt{2E_i}$, allowing the decay constants to be extracted from eqs. 3.1 and 3.2 if appropriate local operators are used for \mathcal{O} . The decay constants we will discuss here are all for ground-state mesons and therefore extracted from a_0 . For formalisms with a PCAC relation (such as HISQ or twisted mass) the decay constant is absolutely normalised. For formalisms without this level of chiral symmetry (such as the clover formalism) a renormalisation factor must be calculated to convert the lattice decay constant to a continuum value.

The branching fraction for D_s leptonic decay is then given by:

$$\mathcal{B}(D_s \rightarrow l\nu_l) = \frac{G_F^2 |V_{cs}|^2 \tau_{D_s} f_{D_s}^2 m_{D_s} m_l^2}{8\pi} \left(1 - \frac{m_l^2}{m_{D_s}^2}\right)^2. \quad (3.4)$$

This has been determined by the BaBar, Belle and CLEO-c experiments and in both μ and τ modes. Given a value for V_{cs} , for example from assuming unitarity of the CKM matrix, then an experimental value for f_{D_s} can be extracted. Alternatively the comparison of theory and experiment can be used to determine V_{cs} .

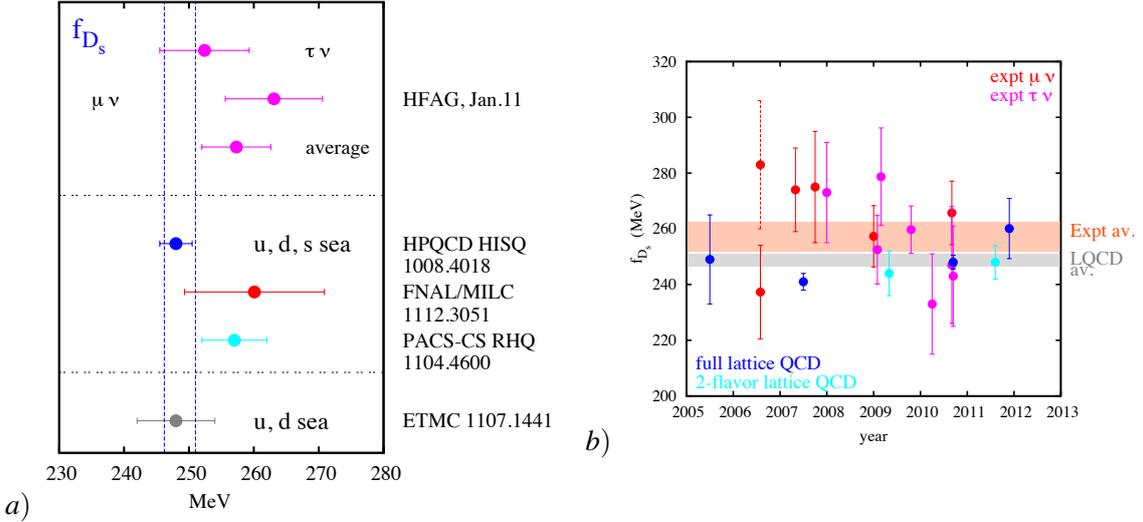


Figure 3: a) Comparison of lattice QCD results for f_{D_s} to experimental averages extracted from leptonic decay rates (also showing $\mu\nu$ and $\tau\nu$ modes separately) using $V_{cs} = 0.97345(16)$ [30]. The lattice QCD world average is 248.6(2.4) MeV, shown with dashed lines; the experimental average is 257.3(5.3) MeV. b) A ‘history’ plot for experimental and lattice results for f_{D_s} , updated from [2].

Fig. 3a shows the current results for f_{D_s} from both lattice QCD and experiment. The experimental averages are from January 2011 [30], but there have been no new experimental results since. The experimental average over all modes gives $f_{D_s} = 257.3(5.3)$ MeV, using $V_{cs} = 0.97345(16)$.

f_{D_s} determination was specifically targeted by the CLEO-c and the B factory experiments as a good test of lattice QCD and this is why the subject has been vigorously pursued both by experimentalists and lattice QCD theorists. Because the D_s has no valence light quarks, f_{D_s} is relatively insensitive to the extrapolation of $m_{u/d}$ to the physical point and so the key issue for an accurate result is how well the valence charm quarks can be handled. Full lattice QCD results for f_{D_s} date back to 2005 when a prediction of 249(16) MeV was given by the Fermilab Lattice/MILC collaboration ahead of experimental results starting in 2006. They used the Fermilab action for charm quarks and the asqtad improved staggered action for the strange quark. We have seen from the discussion in the introduction that nonrelativistic methods for heavy quarks reduce discretisation errors. They have several disadvantages, however, and I would claim that it was fairly conclusively demonstrated by the HPQCD collaboration in 2007 [31] that an improved relativistic action (HISQ) was superior. HPQCD’s result for f_{D_s} had a 1.5% error, similar to those possible in the same calculations for the light decay constants f_K and f_π . This is not surprising - statistical and scale uncertainty errors are

fairly similar in all cases, there is no normalisation uncertainty, and the chiral extrapolation for f_K and f_π is over a similar range to the a extrapolation of the D_s results.

In 2008 the experimental results for f_{D_s} were around 275 MeV, although individual results had fairly large errors of 15-20 MeV. This gave a very exciting picture for a while, as mapped out in the history plot in fig. 3b. Although the 3σ discrepancy between lattice QCD and experiment that existed then has now fallen to 1.6σ [2] as the experimental values moved down and the lattice QCD result moved up, it showed very clearly how precision lattice QCD calculations can have an impact on experiment.

The current picture is summarised in fig. 3. The HPQCD result was updated to 248.0(2.5) MeV in 2010 [2] with a recalibration of the lattice spacing. This now includes results from MILC asqtad $n_f = 2 + 1$ configurations at 5 values of the lattice spacing from 0.15fm down to 0.04fm and multiple $m_{u/d}$ values. Absolute normalisation of the decay constant is also possible in the twisted mass formalism and ETMC gave a result of 244(8) MeV in 2009 [32] using $n_f = 2$ twisted mass configurations. This was updated in 2011 to 248(6) MeV [9] using 4 values of the lattice spacing from 0.1fm down to 0.05fm and multiple $m_{u/d}$ values. In 2011 The PACS-CS collaboration gave 257(5) MeV [26], including only a partial estimate of errors. They used an RHQ action at one value of the lattice spacing with $n_f = 2 + 1$ flavors of clover sea quarks, but having reweighted a small ensemble of configurations to the physical $m_{u/d}$ point.

The Fermilab Lattice/MILC collaboration have also recently updated their analysis [33] using the Fermilab action on the 0.09 and 0.12 fm MILC 2+1 asqtad lattices. They obtain an improved result of 260.1(10.8) MeV where the error is dominated by heavy quark effects, i.e. estimates of the mixed relativistic/discretisation corrections from matching the Fermilab action to continuum QCD. The error from the overall renormalisation constant, $Z_{A_{Qq}}^4$, to be applied to convert from the lattice decay constant to the continuum one is taken to be 1.5%. The renormalisation is done using a method which combines perturbative and nonperturbative techniques. Z for the local temporal vector current can be determined nonperturbatively in both the heavy-heavy case and the light-light case using the normalisation condition:

$$1 = \langle H_{q\bar{q}} | Z_{V_{qq}}^4 | H_{q\bar{q}} \rangle. \quad (3.5)$$

Then the temporal axial current renormalisation needed for f_{D_s} is defined by:

$$Z_{A_{Qq}}^4 = \rho_{A_{Qq}}^4 \sqrt{Z_{V_{qq}}^4 Z_{V_{Qq}}^4} \quad (3.6)$$

and $\rho_{A_{Qq}}^4$ is calculated through $\mathcal{O}(\alpha_s)$ in lattice QCD perturbation theory. A surprising and significant result is found - the coefficient of α_s is very small (< 0.1 , but not zero) for heavy clover quark masses up to around $ma = 1$ after which it rises linearly with ma . This is shown in Fig. 3 of [34] but note that this is a plot of the coefficient of g^2 . The x -axis is the quark mass $m_1 a = \log(1 + m_0 a)$ where $m_0 a$ is the standard mass. The relevant region, even for b quarks, is then quite restricted: $m_0 a = 3$ corresponds to $m_1 a = 1.4$. It is not clear why the α_s coefficient in ρ_{Qq} is so small. Some cancellation of perturbative corrections between Z_V and Z_A in eq. 3.6 seems reasonable but it would be good to understand if the tiny one-loop coefficient near clover masses of zero is accidental or whether it is telling us something. Does it hold for the combination of clover quarks with other light quarks in general? It seems to hold, but to a somewhat lesser extent, for other types of current, such as the spatial vector [34]. It is not clear to me what this says about the next order in

perturbation theory and it would be good to see some nonperturbative tests of this. A simple test would be to compare the amplitudes of mixed clover-staggered pseudoscalar correlators to those of absolutely normalised staggered-staggered correlators at the same mass, for example that of the η_s . At these lower masses discretisation errors are less of an issue and a connection could be made to calculations using relativistic clover quarks for light decay constants, which could be interesting (for a review of issues here see [35]). Fermilab/MILC take an error on f_{D_s} from uncalculated higher orders in perturbation theory in $\rho_{A_{Qq}^4}$ of $0.1\alpha_s^2$. This seems optimistic to me without further tests, but on the other hand this is academic at present because even a much more pessimistic error would have little impact on the total error.

The current HPQCD and Fermilab/MILC results supersede their earlier results and are the only two using 2+1 flavors of sea quarks with a complete error budget. Even though they both use the MILC configurations, there is little overlap in the key ensembles in the two cases, so I take their errors to be independent. I then combine them into a world-average value of f_{D_s} from lattice QCD of 248.6(2.4) MeV, not surprisingly dominated by the HPQCD result.

In the ratio f_{D_s}/f_D Z factors cancel and so the errors are similar at 2-3% between HPQCD results using HISQ (1.164(18) [2], updating the error from [31]) and Fermilab/MILC (1.188(25) [33]). This gives a world average of 1.172(15) where I have now taken a 100% correlation between the statistical errors of the two calculations since the f_D calculations used the same ensembles. This value is not yet accurate enough to tell whether it agrees or disagrees with the similar ratio f_K/f_π which has been calculated to be 1.193(5) from lattice QCD [35]. Note also that f_{η_s}/f_K , another ratio in which the numerator and denominator differ by the substitution of an s quark for a u/d , is 1.165(8) from lattice QCD [3, 36]. Surprisingly the PACS-CS result of $f_{D_s}/f_D = 1.14(3)$ [26], which used $m_{u/d}$ values at the physical point, is lower in its central value than either HPQCD or Fermilab/MILC, who have to extrapolate upwards to that point. I have not included this number in the average since it was only obtained on configurations at one lattice spacing. To improve results for f_{D_s}/f_D clearly needs more chiral lattices; there is no particular need for finer lattices. The new sets of lattices with u/d sea quark masses at the chiral point which are being generated should allow f_{D_s}/f_D to be calculated with 1% accuracy. The experimental average for f_{D_s}/f_D is poorly determined at 1.26(5) [37] because there is no cancellation of errors in the ratio. The experimental result for f_D is 206.1(8.9) MeV from CLEO-c [37] using V_{cd} from CKM unitarity = 0.2252(7) [38]. Combining the average lattice f_{D_s} with the average ratio of f_{D_s}/f_D gives $f_D = 212.1(3.4)$ MeV.

3.3 Semileptonic decays

The analysis of semileptonic decays in which one meson changes into another and emits a W boson gives us access to more detailed information about meson internal structure than the one number represented by the decay constant. The information from QCD in these decay processes is parameterised by form factors that are functions of q^2 , the square of the 4-momentum transfer from the initial to the final meson. Calculation of the form factors in lattice QCD allows the q^2 -dependence of the rate for such decays to be compared to experiment as well as CKM matrix elements to be extracted. Since the lattice calculation corresponds to a specific final state meson, the experimentalists must identify this meson in their sample of decays to give the ‘exclusive’ (as opposed to the ‘inclusive’) rate. The simplest case is that of pseudoscalar to pseudoscalar decay in which only the vector piece of the current coupling to the W contributes, and there are two form

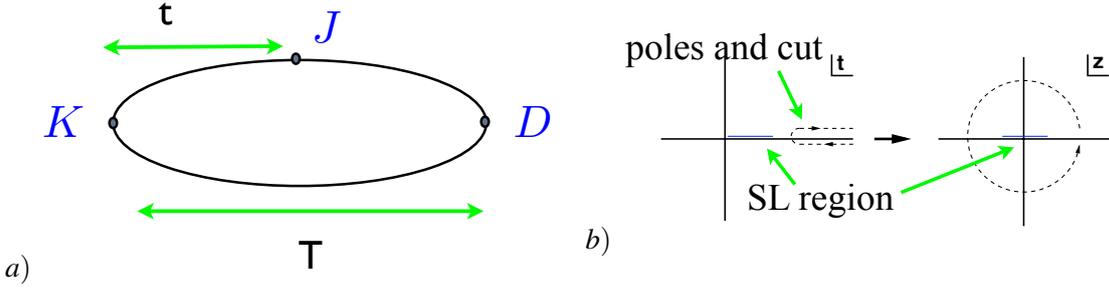


Figure 4: a) Sketch of a 3-point function. The operators for D and K are a distance T apart on the lattice and the current operator is inserted between them. The 3-point correlator should be calculated for all t from 0 to T and for several different T values. b) Transformation of the complex $t = q^2$ plane to the z plane for semileptonic form factors.

factors, the vector form factor f_+ and the scalar form factor f_0 . Illustrating this for $D \rightarrow K\bar{l}\nu$ decay we have:

$$\langle K|V^\mu|D\rangle = f_+(q^2) \left[p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_D^2 - m_K^2}{q^2} q^\mu. \quad (3.7)$$

The matrix element on the left-hand side is determined from the amplitude of a "3-point correlator" sketched in Fig. 4. The source and sink mesons are taken a distance T apart on the lattice and the vector current is inserted a distance t from the source. t is allowed to run over all values from 0 to T and the correlator is fit simultaneously as a function of t and T along with the standard "2-point" meson correlators for the source and sink mesons, for which the fit function is given in eq. 3.3. The 3-point function fit is :

$$C_{3pt} = \sum_{i,j} a_i b_j V_{ij} f(E_{a,i}, t) f(E_{b,j}, T - t) \quad (3.8)$$

where a_i and b_j are the same amplitudes and $E_{a,i}$ and $E_{b,j}$ are the same energies as in the 2-point function fits for mesons a and b respectively and V_{ij} is the matrix element of the vector current between them. The result we typically want, and that I will discuss here, is that between the ground states, V_{00} . When staggered quarks are used there are generally additional oscillating pieces that need to be fit.

To cover the range of q^2 values in the decay one or both of the source or sink mesons can be given a spatial momentum. It is simplest to work in the rest frame of the source meson (for example the D above). Then, when the K is at rest, q^2 is a maximum at $(m_D - m_K)^2$, and the lepton and antineutrino emerge back-to-back. This is the easiest kinematics to reproduce on the lattice, but the experimental rate at this point is zero (see eq. 3.9 below). The other extreme is $q^2 = 0$ when the lepton and antineutrino balance the K momentum. Lattice QCD errors grow as spatial momentum of the K increases. The experimental errors are best for small q^2 (but not necessarily 0) where the rate is larger.

When the W decay to leptons is folded in with eq. 3.7, the contribution to the rate from $f_0(q^2)$ appears multiplied by lepton masses and so for e and μ channels is negligible. Then :

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 p_K^3}{24\pi^3} |V_{cs}|^2 |f_+(q^2)|^2. \quad (3.9)$$

There is a useful kinematic constraint from eq. 3.7 at $q^2 = 0$: $f_+(0) = f_0(0)$. This is helpful because the scalar form factor can be absolutely normalised from the PCVC relation $\partial_\mu V^\mu = (m_1 - m_2)S$ when the same formalism is used for the two quarks in the weak decay (c and s for $D \rightarrow K$ decay) and they differ in mass. Then:

$$\langle K|S|D \rangle = f_0(q^2) \frac{m_D^2 - m_K^2}{m_c - m_s}, \quad (3.10)$$

where m_c and m_s are the bare lattice quark masses. This method has recently been introduced by the HPQCD collaboration [39], reducing lattice errors significantly for $f_+(0)$.

Continuum theorists have developed a good understanding of the behaviour of form factors based on their pole and cut structure in the wider complex q^2 plane (see, for example, [40]). The q^2 region for semileptonic decay runs from $q^2 = 0$ up to (for $D \rightarrow K$) $q_{max}^2 = t_- = (m_D - m_K)^2$. Above $t_+ = (m_D + m_K)^2$ where a real D and K can be exchanged, there is a cut. In addition there may be poles if there are isolated resonances with the right quantum numbers with masses between t_- and t_+ . For example, the $D \rightarrow K$ vector form factor has a pole at $q^2 = m_{D_s^*}^2$ where $m_{D_s^*} - m_D = 243$ MeV and the scalar form factor has a pole at $m_{D_{s0}} - m_D = 448$ MeV (ignoring the suppressed $D_s\pi$ cut). The form factor diverges at the pole, but this is outside the physical region for semileptonic decays so what is seen is a rise of the form factor as q^2 increases to q_{max}^2 .

It is useful to remove this pole behaviour from the form factor so that $\tilde{f}(q^2) = f(q^2)/(1 - q^2/m_{pole}^2)$ and then to transform \tilde{f} into z -space where

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}. \quad (3.11)$$

Eq. 3.11 maps the line around the cut from $q^2 = \infty$ to $q^2 = t_+$ and back again to the circle with $|z| = 1$, as shown in Fig. 4b. The physical semileptonic region is then inside this circle, where \tilde{f} is finite and well-behaved. The exact position of the physical region depends on the value for t_0 (since $z(q^2 = t_0) = 0$). Often the region is made symmetric about $z = 0$ to minimise the maximum z value but it can be more convenient to take $t_0 = 0$. The form factor \tilde{f} in the physical region can be described by a simple power series in z . Allowing the coefficients of the expansion to depend on lattice spacing and $m_{u/d}$ provides a straightforward way to extrapolate to the continuum and chiral limits which, for example, reduces the confusion between q^2 dependence and $m_{u/d}$ dependence. The continuum and chiral limit form factor $f(q^2)$ can then easily be reconstituted at the end.

Lattice QCD calculations for semileptonic form factors have improved hugely over the last few years as techniques have developed. State-of-the-art calculations now have: high statistics, including the use of random wall sources to improve this further; multi-exponential fits to the 3-point function for multiple T values and not simply a "plateau" search in a ratio of 3-point and 2-point functions; use of a phase rotation of the gluon field to tune spatial momenta accurately to find the $q^2 = 0$ point [41] and use of the z expansion [42, 39] as described above to fit the form factor shape and improve extrapolation to the chiral/continuum limit.

Experimentalists often quote their results, for example for $D \rightarrow K$, in the form of a value for $V_{cs}f_+(0)$, obtained from a fit to their data which includes a parameterisation of the form factor. A lattice calculation of $f_+(0)$ can then be used to determine the CKM element. Fig. 5 shows the current status of lattice QCD calculations for $f_+(0)$ for $D \rightarrow K$ and $D \rightarrow \pi$, along with experimental

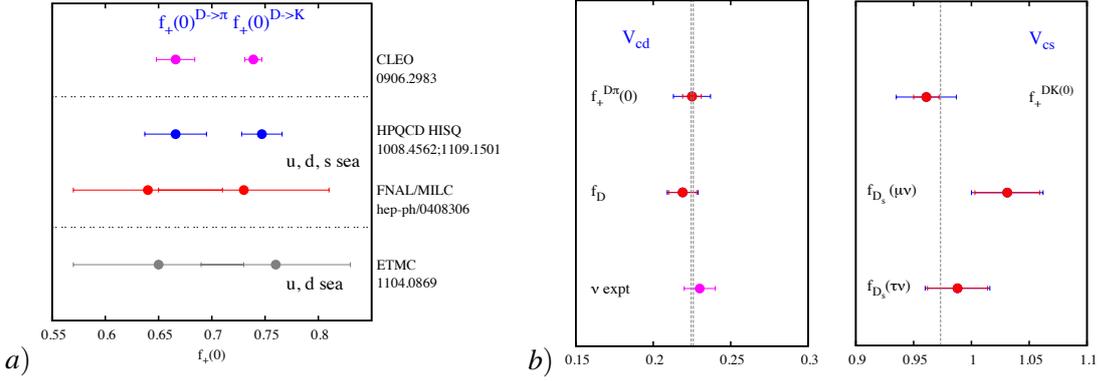


Figure 5: a) Lattice QCD results for $f_+(0)$ from $D \rightarrow K$ and $D \rightarrow \pi$ decay, compared to results from fitting experimental semileptonic rates as a function of q^2 [46], determining $f_+(0)V_{cx}$ and then taking $V_{cs}=0.97345(16)$ and $V_{cd}=0.2252(7)$. b) V_{cd} and V_{cs} determined from combining results for semileptonic and leptonic decay rates with lattice QCD world average values for form factors and decay constants given in the text. The inner (red) error bar is from experiment; the outer error bar is the total error bar including that from lattice QCD. The V_{cd} plot includes the direct determination from neutrino experiments [38]. The dashed grey lines give the values from assuming CKM unitarity [38].

results obtained by using unitarity values for V_{cs} and V_{cd} . The HPQCD/HISQ results [39, 43] are obtained from $f_0(0)$ which, as discussed above, is absolutely normalised. This shows a big improvement over earlier results in that $D \rightarrow K$ and $D \rightarrow \pi$ values are clearly distinguished, with the accuracy improved to the level of a few percent. Since these are the only results in full QCD to date using more than one lattice spacing we simply quote these numbers as the ‘world average’: $f_+^{D \rightarrow K}(0) = 0.747(19)$ and $f_+^{D \rightarrow \pi}(0) = 0.666(29)$.

Fermilab/MILC gave a progress report on their calculation of $D \rightarrow \pi$ $f_+(q^2)$ this year [44] showing results from 11 ensembles and a shape that agrees well with CLEO-c data. They project future errors on $f_+(0)$ of less than 5%, dominated by heavy quark and chiral extrapolation errors. The renormalisation of the clover-staggered vector current is done in the same way as for decay constants discussed in section 3.2.

HPQCD also presented new results on $f_+(q^2)$ obtained using a spatial vector current normalised in the symmetric case using eq. 3.5 since the Z factor does not depend significantly on mass [45]. This is found to agree with a local temporal charm-strange vector current which can be normalised by the fact that at q_{max}^2 the vector matrix element in eq. 3.7 is equal to $f_0(q_{max}^2)(m_D + m_K)$. The phase technique [41] is used to tune across the physical q^2 range, including at $q^2 = 0$. f_+ and f_0 are fit together in z -space, using $t_0 = 0$ so that the constraint $f_+(0) = f_0(0)$ can be maintained. The final form factor shapes agree well with experiment and we expect a further factor of two improvement in the determination of V_{cs} from this method.

These results [45] showed up an interesting fact, not apparently noticed before, that the form factors for charmed meson decay are very insensitive to the spectator quark mass as it is varied between light and strange. This is within statistical errors of 1-2% at high q^2 and 2-5% at $q^2 = 0$. The insensitivity has significant consequences. One is that $D_s \rightarrow K$ and $D \rightarrow \pi$ form factors are the same [45], which can be tested experimentally. Another is that this would then be expected to hold also for B meson decays so that $B_s \rightarrow D_s$ and $B \rightarrow D$ form factors would be equal. This is useful

for experimental normalisation and will be discussed further in section 4.4.

Further results this year came from QCDSF [47] looking at the disconnected contributions necessary to determine $D_{(s)} \rightarrow \eta^{(\prime)} l \nu$ rates (using a relativistic smeared clover action for all quarks) and from HPQCD on axial and vector form factors for $D_s \rightarrow \phi l \nu$ [48] (using HISQ). These form factors add to the range of quantities that can be determined from essentially the same lattice QCD calculation and tested against experiment with no free parameters.

3.4 V_{cs} and V_{cd}

Fig. 5b shows the current status of the direct determination of V_{cs} and V_{cd} using leptonic decays of D and D_s mesons and semileptonic $D \rightarrow \pi$ and $D \rightarrow K$ decays, combining experimental results with lattice QCD world averages given in sections 3.2 and 3.3. The results will be summarised in terms of unitarity tests of the CKM matrix in section 5.

4. Bottom physics results

4.1 Spectroscopy

This year has seen progress on improving methods for b physics. The HPQCD collaboration has now taken the NRQCD action to the next stage by determining the $\mathcal{O}(\alpha_s)$ corrections to the sub-leading (v^4) terms [49]. Improved analysis of the bottomonium spectrum was shown [36, 50] including the hyperfine splitting (the mass difference between the Υ and the η_b) and predictions for D -wave states [51]. These calculations have been done on MILC configurations including u , d , s and c HISQ quarks in the sea and with a further improved gluon action. Heavy-light physics is underway on these configurations. The improvements to the action should reduce NRQCD systematic errors to the level of 5 MeV when determining the difference between, say, the B_s meson mass and one half the mass of the Υ . This is not enough to distinguish the effects of c quarks in the sea since they are expected from a perturbative analysis [52] to shift the Υ by around 5 MeV but have little effect on the B_s . It might, however, be enough to see the effect of quenching s quarks since, based on old results from quenching all 3 light quarks, this could affect this mass splitting at the 10-20 MeV level [53], as well as affecting radial and orbital Υ splittings at the 2-3% level (relative to the $2S - 1S$ splitting). It would be interesting to do a systematic analysis on $n_f = 2$ configurations to see if an error in the comparison to experiment shows up here, since no effect has so far been seen anywhere else. This would help to quantify the consequences of quenching s quarks, since these cannot be easily estimated.

Both ETMC [9] and HPQCD [5] have given new results, to be discussed further below, using relativistic approaches (twisted mass and HISQ respectively) for heavy quarks and extrapolating in the heavy quark mass up to the b . The RBC/UKQCD collaboration have tuned their RHQ action for b quarks and are now using that action for bottomonium and heavy-light physics [54].

A useful comparison between lattice QCD calculations is in the determination of the b quark mass. The Alpha [55] and ETMC [9] collaborations presented new results on m_b from configurations that include u and d quarks in the sea. Alpha use HQET through $1/m_b$ on the lattice and find $m_b^{(n_f=2)}(m_b)$ in the \overline{MS} scheme to be 4.23(15) GeV in agreement with the ETMC result of 4.29(14) GeV. The ETMC result uses the twisted mass formalism for the heavy quarks, extrapolating up to the b using a function with a known static limit.

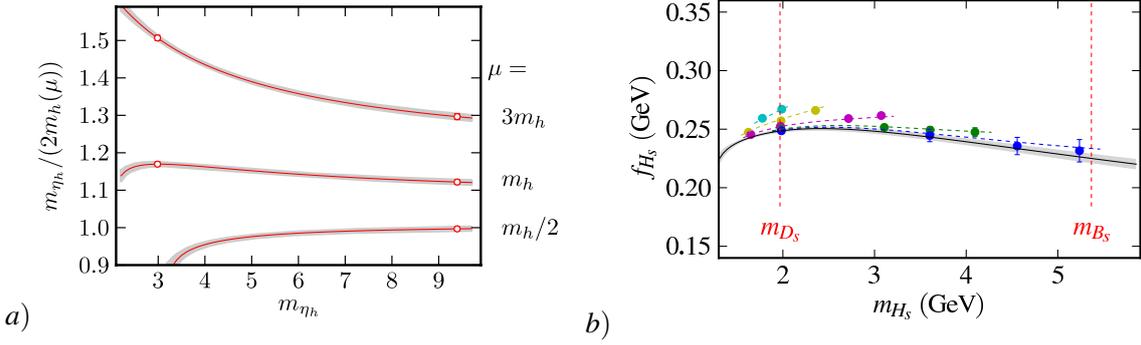


Figure 6: Covering the region from charm to bottom: a) The ratio of the quark mass in the \overline{MS} scheme at a given μ to one half the pseudoscalar heavyonium mass as a function of the pseudoscalar heavyonium mass (as a proxy for the heavy quark mass) [4]. Note how flat the curve for $\mu = m_h$ is. b) The Heavy-strange decay constant as a function of heavy-strange meson mass (as proxy for heavy quark mass) [5]. Note that the curve peaks at the D_s .

These results can be compared to the HPQCD result of $m_b(m_b)^{n_f=5} = 4.164(23)$ GeV [4], obtained using current-current correlator methods with HISQ quarks on MILC $n_f = 2 + 1$ configurations and perturbatively matching to $n_f = 5$. Here a range of heavy quark masses were explored from c upwards at 5 different values of the lattice spacing, allowing the physical curve of the ratio of the quark mass to one half of the pseudoscalar heavyonium mass to be mapped out. This is interesting because it allows lattice QCD to ‘fill in’ the values between the two results at c and b on which we have experimental information. What we see, reproduced in Fig. 6a, is that $m_{\eta_h}/2m_h(\mu)$ gives a very flat curve with value falling between 1.2 and 1.1 as m_h is increased when $\mu = m_h$. This is a well-defined and accurately quantifiable version of the hand-waving statement that “the heavy quark mass is roughly half the heavyonium meson mass” and only possible from lattice QCD.

It would be useful to have accurate results for m_b from NRQCD, Fermilab and RHQ methods for comparison - these are underway.

4.2 Leptonic decays (to $l\nu_l$)

Direct annihilation to a W is only possible for charged pseudoscalars. However it is still possible and useful to calculate the equivalent matrix element for the neutral B_s meson in lattice QCD and compare its value to that for the B . There are several new results this year for f_B and f_{B_s} and these are summarised in Fig. 7.

The two using nonrelativistic methods on the MILC $n_f = 2 + 1$ asqtad configurations can be directly compared. These are the HPQCD results using NRQCD b quarks (without the further improvements discussed above) with HISQ valence light quarks [56] and the Fermilab/MILC results using Fermilab b quarks and asqtad light quarks [33]. The HPQCD NRQCD-HISQ results of $f_{B_s} = 227(10)$ MeV and $f_B = 191(9)$ MeV supersede earlier NRQCD-asqtad calculations [57] using the same methods - the error is improved by 50% because of better statistics and smaller discretisation errors but the central values change very little. The Fermilab/MILC results of $f_{B_s} = 242(10)$ MeV and $f_B = 197(9)$ MeV are calculated on essentially the same set of $a = 0.12$ and 0.09 fm gluon fields and also update earlier work. The Fermilab/MILC and HPQCD results agree reasonably well - f_{B_s} values differ by 1.5σ allowing for correlated statistical errors. In both cases the error budgets

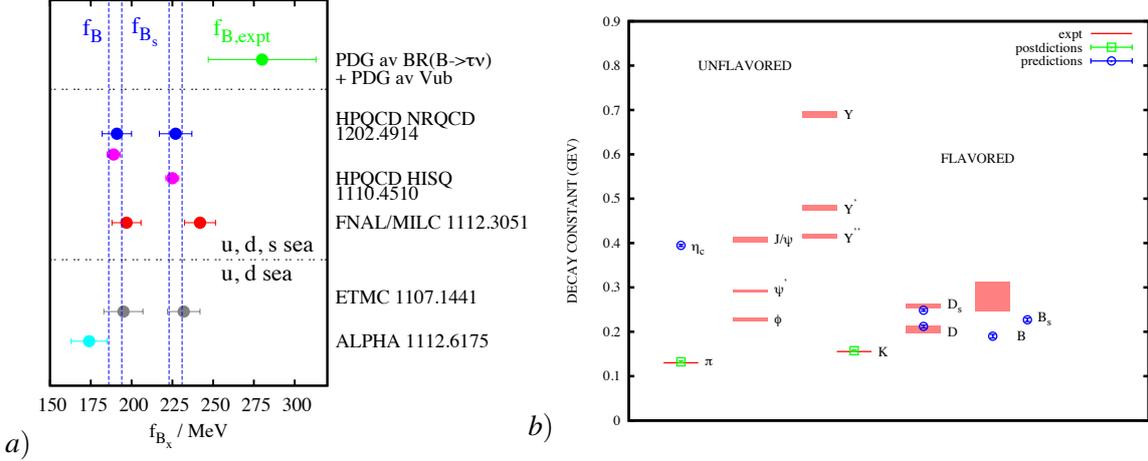


Figure 7: a) Comparison of lattice QCD results for f_{B_s} and f_B and the experimental average for f_B from $B \rightarrow \tau \nu$ decay, using $V_{ub} = 3.47 \times 10^{-3}$. The lattice QCD world averages are $f_B=190(4)$ MeV and $f_{B_s}=227(4)$ MeV (see text), shown with dashed lines. b) The ‘spectrum’ of decay constants, using $D/D_s/B/B_s$ world averages from here, π/K from [31] and η_c , surprisingly close to J/ψ from experiment, from [2].

are dominated by other errors, presumably uncorrelated. For the NRQCD case the problem is the unknown $\mathcal{O}(\alpha_s^2)$ pieces of the perturbative matching of the axial current to the continuum. The error allows for a coefficient at $\mathcal{O}(\alpha_s^2)$ of 0.4 (the one-loop coefficient is around 0.1), uncorrelated between the two lattice spacing values since it can vary with $m_b a$ (the $\mathcal{O}(\alpha_s)$ coefficient varies from 0.1 to 0.15). For the Fermilab case the dominant errors are heavy-quark discretisation and tuning and statistics. Their perturbative matching error allows for $0.1\alpha_s^2$ in the ratio of axial to vector renormalisations, as described earlier for the D/D_s case.

f_{B_s}/f_B is determined to around 2% in both cases with values 1.188(18) for NRQCD-HISQ and 1.229(26) for Fermilab-asqtad. Again this represents agreement at the 1.5σ level allowing for correlated statistical errors. Now statistical errors are a more significant part of the total since the renormalisation and heavy-quark discretisation effects largely cancel. Neither result for f_{B_s}/f_B is sufficiently accurate to distinguish it from f_{D_s}/f_D or f_K/f_π although the statistical and systematic correlations in the Fermilab/MILC results between the B and D calculations presumably mean that the difference between their values of 1.229(26) for f_{B_s}/f_B and 1.188(25) for f_{D_s}/f_D is significant. The Fermilab/MILC results will improve with values on additional finer MILC asqtad ensembles; the HPQCD results will be updated with improved NRQCD on the HISQ 2+1+1 configurations, exploring also improvements in renormalisation that may be possible with heavy-light current-current correlator methods [58].

The Alpha collaboration gave an updated result from their HQET method for f_B only, obtaining $f_B=174(11)$ MeV [55]. This is done on the CLS $n_f = 2$ lattices with 3 values of a from 0.075 fm to 0.05 fm and m_π values down to 270 MeV. The b quark is included in the static (infinite mass) formalism using an action with smeared links. This improves the poor signal/noise for static quarks which arises from the unphysical energy of the heavyonium state (with no kinetic term) that contributes to the noise. Matrix elements of operators that give rise to $1/m_b$ corrections to the static limit are included after a nonperturbative determination of their coefficients. The result for f_B is

1.5 σ lower than those from the 2+1 flavor nonrelativistic results discussed above.

Relativistic methods have yielded well-controlled results this year for the first time and these look encouraging for the future. ETMC use HQET to extrapolate up to the b quark from quark masses around $2m_c$ using 4 values of a down to 0.05fm on $n_f = 2$ lattices [9]. They include a static result to bound the upper limit of the extrapolation. They average over two different methods for f_{B_s} that give results 225(8) MeV and 238(10) MeV, possibly worryingly far apart given that the (correlated) statistical errors dominate. The final result for f_{B_s} is 232(10) MeV and a separate determination of the ratio f_{B_s}/f_B is used to obtain $f_B = 195(12)$ MeV. HPQCD use HISQ quarks [5] with masses from slightly below m_c up to close to m_b to map out the heavy-strange decay constant as a function of the heavy-strange meson mass (as a physical proxy for the heavy quark mass). They use 5 values of a down to 0.045 fm on $n_f = 2 + 1$ configurations and limit $m_b a$ to values below 0.85. The method relies on the finding that f_{D_s} is very insensitive to sea quark masses [2] to avoid an extensive study as a function of sea light quark mass. The extrapolation to the b uses (and tests) HQET formulae and obtains $f_{B_s} = 225(4)$ MeV, significantly more accurate than previous results. Surprisingly it provides the first really solid demonstration (although most earlier results have indicated this) that $f_{B_s} < f_{D_s}$ with the ratio $f_{B_s}/f_{D_s} = 0.906(14)$. In fact f_{D_s} seems to be close to the maximum of the heavy-strange decay constant curve (see Fig. 6b), which shows a rapid rise from lower masses up to f_{D_s} and then a rather slow fall down to f_{B_s} , much slower than that of leading-order HQET. Since the ratios of heavy-strange and heavy-light decay constants for B and D differ very little, note that it is also true that $f_B < f_D$.

To obtain a world-average value of f_{B_s} I take an error-weighted average of the three $n_f = 2 + 1$ results, allowing 100% correlation between the statistical errors of the Fermilab and NRQCD results. The HPQCD-HISQ result I treat as independent since it relies on results on finer lattices. The world-average is then $f_{B_s} = 227(4)$ MeV. For a world-average value of f_B I average the Fermilab result with the HPQCD value of $f_B = 189(4)$ MeV [56] that uses the HISQ value for f_{B_s} and the NRQCD result for f_{B_s}/f_B . This gives 190(4) MeV. Both of these new world-average values are lower than last year by 1-2 σ (see for example [59]) and the error has improved by a factor of 3. Note that the lattice QCD result for f_B is now clearly below 200 MeV.

Experimental observation of B leptonic decay to $l\bar{\nu}_l$ is extremely difficult and the errors on the results consequently rather large. Since the rate is proportional to m_l^2 (eq. 3.4) it is not surprising that the mode that has been seen is $B \rightarrow \tau\nu$. The PDG [38] give the average of the BaBar and Belle branching fractions as $1.65(34) \times 10^{-4}$. Putting in the kinematic factors and B lifetime, this corresponds to:

$$f_B |V_{ub}| = 0.97(10) \text{ MeV}. \quad (4.1)$$

V_{ub} is also rather uncertain with sizeable differences between that extracted from inclusive and exclusive (using lattice QCD) B semileptonic decay modes. The PDG [38] give a result for V_{ub} from requiring unitarity of the CKM matrix as $3.47(16) \times 10^{-3}$. This central value is fairly close to that from exclusive decays - the inclusive result is higher at $4.3(4) \times 10^{-3}$. Combining the unitarity V_{ub} with eq. 4.1 gives $f_{B,\text{expt}} = 280(33)$ MeV which is over 2.5 σ higher than the lattice QCD result above. This is a cause of ‘tension’ in the CKM picture [20] which will need improved experimental results to resolve, although understanding the ambiguity in V_{ub} would also help. From eq. 4.1 and the lattice average for f_B above, we can derive $V_{ub} = 5.1(5) \times 10^{-3}$, not that much worse than other

direct determinations.

Improvements to lattice QCD B/B_s decay constants will continue - it seems likely that relativistic methods will provide the best way forward for the absolute value of f_{B_s} . The ratio of f_{B_s}/f_B may be better calculated with nonrelativistic methods and can certainly be obtained to 1% errors working closer to the chiral limit. However, it should be remembered that the ratio that is really required for combination with experiments on B_s/B_d mixing is the ratio of 4-quark operator matrix elements and this is harder [57].

4.3 $B_{(s)} \rightarrow \mu^+ \mu^-$

An important leptonic mode for neutral B mesons is decay to $\mu^+ \mu^-$. In the Standard Model this proceeds via Z^0 penguin and box diagrams [60] and so is expected to be sensitive to new physics. The effective weak Hamiltonian that gives rise to the decay for B_s is then

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} V_{tb}^* V_{ts} Y(x_t) (\bar{b}s)_{V-A} (\bar{l}l)_{V-A} + h.c. \quad (4.2)$$

where Y is a known function of the mass of the top quark and $x_t = m_t^2/m_W^2$. The 4-fermion operator here has 2 quarks and 2 leptons and so, as far as QCD is concerned, it looks like an operator for quark-antiquark annihilation. The matrix element is then proportional to the decay constant f_{B_s} . The rate, however, depends on CKM elements $V_{tb}^* V_{ts}$ which are derived from the B_s mixing rate (ΔM_s) along with a lattice QCD calculation of the matrix element of the 4-quark operator that corresponds to the box diagram for that process, and is parameterised by $f_{B_s}^2 B_{B_s}$. B_{B_s} is known as the ‘bag parameter’. Buras pointed out [61] that in fact the best way to determine the rate for $B_s \rightarrow l^+ l^-$ was to take a ratio to the mixing rate. Then the CKM elements cancel and so does f_{B_s} and the SM rate for $B_s \rightarrow \mu^+ \mu^-$ is proportional to the bag parameter, B_{B_s} . A determination of this requires lattice QCD (despite the confusion over this in the literature).

Only one such lattice QCD calculation including sea quarks has so far been done [57], by the HPQCD collaboration, as part of a calculation to obtain the mixing matrix elements for ΔM_s . This yields the current best estimate of the B_s bag parameter, $\hat{B}_{B_s} = 1.33(6)$ giving a SM branching fraction for $B_s \rightarrow \mu^+ \mu^-$ of $3.19(19) \times 10^{-9}$. The calculation used NRQCD b quarks and asqtad light quarks and perturbative renormalisation of the 4-quark operators on MILC $n_f = 2 + 1$ asqtad configurations. A significant part of the error was from statistics and will be reduced in new calculations underway on the new $n_f = 2 + 1 + 1$ HISQ configurations.

The observation of $B_s \rightarrow \mu^+ \mu^-$ is a key aim of the LHC experiments. This year [62] LHCb and CMS have improved their limits on the branching fraction to within a factor of 3 of the Standard Model rate from lattice QCD. In the absence of BSM physics, LHC expects to see the process in 2012, and then the comparison of theory and experiment will become more critical, and improved errors on the lattice QCD side may be very important.

4.4 Other decay modes and mixing

The Fermilab Lattice/MILC collaborations discussed their progress on the calculation of 4-quark operator matrix elements for B_s and B_d mixing, using Fermilab b quarks and studying the complete set of 5 $\Delta B = 2$ operators [63]. There was also an update on $B \rightarrow Kl^+ l^-$ form factors which may be sensitive to new physics [64].

The calculation of the ratio of scalar form factors for $B_s \rightarrow D_s$ and $B \rightarrow D$ turns out to be useful for the experimental normalisation of the $B_s \rightarrow \mu^+ \mu^-$ mode [65]. QCD sum rule calculations [66] expect deviations from 1 in this ratio to be related, and of similar size, to the 20% $u/d - s$ effects seen in decay constants. Instead, as discussed in section 3.3, very little spectator quark mass dependence is seen in heavy form factors [45] to a high level of accuracy. Fermilab/MILC have now done an explicit calculation of these B and B_s semileptonic modes [67] and indeed find no significant deviation from 1: $f_0^{B_s}(m_\pi^2)/f_0^{B_d}(m_K^2) = 1.046(46)$.

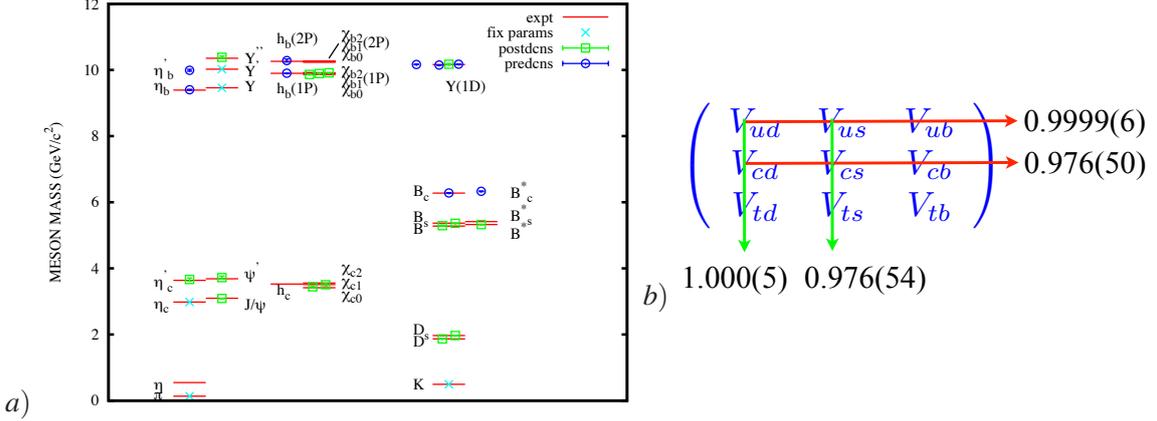


Figure 8: a) The spectrum of gold-plated mesons from lattice QCD, updating [68] to include the prediction of D -wave Upsilon states [51]. b) Tests of unitarity of rows and columns of the CKM matrix now possible using lattice QCD results, except for V_{ud} from nuclear β decay [38]. I have taken direct determinations of V_{us} , V_{cd} and V_{cs} from semileptonic modes [35, 39, 43], although there is no particular reason to prefer those over leptonic modes. V_{ts} and V_{td} come from lattice QCD B_s/B_d mixing calculations [38, 57] and V_{cb} from $B \rightarrow D^*$ decays ($39.7(1.0) \times 10^{-3}$ [69]) but these are too small to have much impact on row/column unitarity. Unitarity triangle tests, from orthogonality of columns 1 and 3, are discussed in [20].

5. Conclusions

Key results can be summarised in three plots, all showing significant progress since last year. Fig. 8a updates the spectrum of gold-plated mesons, which includes many c and b states. Fig. 7b is a similar plot laying out the picture for decay constants, now rather impressive for pseudoscalars. It is clear that similarly accurate results on electromagnetic annihilation of vector mesons would significantly enhance our confidence in lattice QCD results and provide excellent tests of QCD itself. Fig. 8b shows the tests of unitarity of the first and second rows and columns of the CKM matrix that are now possible at the 5% level using lattice QCD and experiment.

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