Low-energy particle physics and chiral extrapolations

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In this review I address the rôle of chiral extrapolations for the determination of several phenomenologically relevant quantities, including light quark masses, meson decay constants and the axial charge of the nucleon. In particular, I discuss whether chiral extrapolations are sufficiently controlled in order to rightfully claim the accuracy which is quoted in recent compilations of these quantities. While this is the case for the masses of the light quarks and the ratio f_K/f_π of decay constants, small inconsistencies in the chiral and continuum behaviour of individual decay constants f_K and f_π, as well as the hadronic radii r_0 and r_1 remain and must be clarified. In the case of the nucleon axial charge, g_A, the chiral behaviour is still poorly understood due to the presence of other systematic effects.

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1. Introduction

Lattice calculations are becoming increasingly important for particle physics phenomenology. They address and quantify the "hadronic uncertainties" which still afflict many quantities that constrain the validity of the Standard Model [1–6]. Almost every talk on lattice QCD delivered to a more general audience during the past 10–15 years contained the phrase that "lattice calculations are performed at unphysical quark masses." What we usually mean by this statement is that for any given discretisation, there is *a priori* no way of knowing which values of the bare quark masses correspond to those of the physical quarks. In most, if not all, cases it turns out that the physical light quark masses lie outside the regime which is directly accessible using the currently available algorithms and machines. Likewise, the physical values of the heavy quarks are dangerously close to, if not above, the affordable cutoff scale.

Chiral extrapolations are thus required in order to make contact with the physical light quark masses. Chiral Perturbation Theory (ChPT) provides theoretical constraints on the quark mass dependence of observables, based on the underlying dynamics associated with chiral symmetry breaking. The following quotation from the FLAG report [1] serves as a reminder that improving the control over the chiral behaviour is mandatory in order to make further progress:

> "Although [light quark masses] are decreasing very significantly with time [. . .] it remains true that [the chiral] extrapolation is one of the most significant sources of systematic error."

Some collaborations have produced lattice data at or even below the physical pion mass [7, 8] which may render extrapolations guided by ChPT soon obsolete. Instead, one can resort to some sort of analytic *ansatz* to interpolate lattice data to the physical point. However, even if ChPT were not required to perform chiral extrapolations, a comparison of the quark mass dependence determined on the lattice with the predictions of an effective theory would provide useful information, since it allows for the determination of the low-energy constants (LECs) which parameterise ChPT. Furthermore, as simulation algorithms still show a significant increase in computational cost when the light quark masses are tuned to their physical values, it is still difficult to disentangle the chiral behaviour from systematic effects arising from finite volume and/or coarse lattice spacings.

The vast majority of lattice estimates for phenomenologically relevant quantities is still dominated by systematic errors. In this review I try to investigate whether chiral extrapolations are sufficiently well controlled in order to rightfully claim the accuracy which is quoted in the recent compilations. Here I shall focus on three different types of observables: Lattice estimates of the light quark masses are discussed in the next section. In section 3 I will study the systematics of chiral fits applied to meson decay constants. Section 4 contains a discussion of the chiral behaviour of the axial charge of the nucleon. Summary and conclusions are provided in section 5. For the purpose of this review all lattice results are taken at face value. Issues such as "rooting", induced non-localities, or the freezing of topology will not be discussed here.

2. Light quark masses

Quark masses are fundamental parameters of the Standard Model whose values determine many important quantities in particle phenomenology. The recent compilation of lattice results for
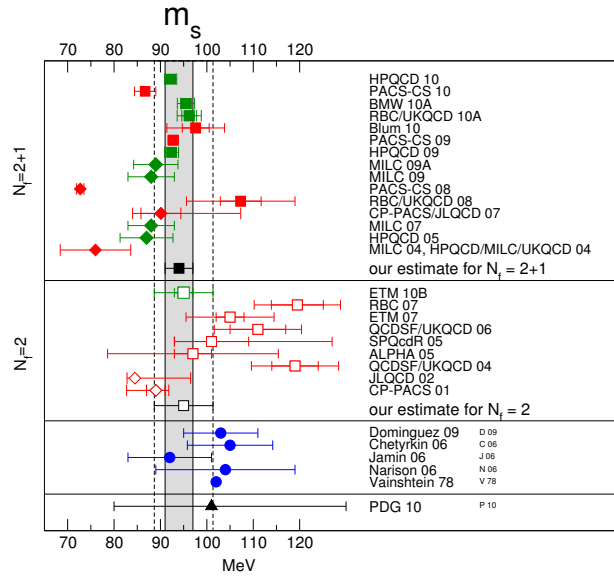
Figure 1: Results for the strange quark mass in the $\overline{\text{MS}}$-scheme at 2 GeV, obtained in lattice QCD with $N_{\rm f} = 2 + 1$ and $N_{\rm f} = 2$ flavours of dynamical quarks [1]. Green points represent lattice results which are free of any red tags according to the FLAG criteria. Blue circles denote the results from sum rule calculations. The grey band and the vertical dotted lines denote the global estimate for the $N_{\rm f} = 2 + 1$ and two-flavour theory, respectively. The PDG estimate is shown at the bottom.

the light quark masses and their conversion into "global" averages in the FLAG report [1] is based on a set of "quality criteria". Using a simple colour code, they are meant to assess the quality of a given calculation regarding a number of different systematic effects. Obviously, these criteria must be adjusted over time, in order to reflect the true state-of-the-art. In the current FLAG review, a green star ($\bigstar$) is awarded if the systematic error is "convincingly shown to be under control". An amber ball ($\bullet$) signifies that a "reasonable attempt" at estimating a particular systematic error has been made. Finally, a red box ($\blacksquare$) indicates that no attempt was undertaken to quantify a systematic effect. To set the scene for the discussions to follow, the criteria for the colour code referring to chiral extrapolations and the related finite-volume effects are repeated here:

Chiral extrapolation: Finite-volume effects:

- $\bigstar$ $m_\pi^{\rm min} < 250\,{\rm MeV}$ $\bigstar$ $m_\pi^{\rm min} L > 4$ or at least 3 volumes
- $\bullet$ $250\,{\rm MeV} \le m_\pi^{\rm min} \le 400\,{\rm MeV}$ $\bullet$ $m_\pi^{\rm min} L > 3$ and at least 2 volumes
- $\blacksquare$ $m_\pi^{\rm min} > 400\,{\rm MeV}$ $\blacksquare$ otherwise, or if $(L_{\rm min} < 2\,{\rm fm})$

Moreover, the FLAG rules stipulate that results which are classified with at least one red tag and/or without a journal reference be excluded from global estimates. As an example we show the compilation of results for the strange quark mass from the FLAG report in Fig. 1. One observes that lattice estimates for m_s obtained with $N_{\rm f} = 2 + 1$ flavours of dynamical quarks appear to be somewhat smaller compared to the two-flavour theory, although this may be attributed to the fact that the results for $N_{\rm f} = 2$ are typically older and may be more strongly affected by other systematic effects. A striking feature of the plot is that lattice estimates are broadly consistent, despite the fact

	Ref.	a [fm]	m_π^{min} [MeV]	$m_\pi^{\mathrm{min}} L$	$m_s^{\overline{\mathrm{MS}}}$ [MeV]	m_s/m_{ud}
PACS-CS	[12]	0.09	156	2.3	72.7 ± 0.8	28.8 ± 0.4
	[8, 13]				$86.7 \pm 2.3^\dagger$	$31.2 \pm 2.7^\dagger$
BMW	[7, 15]	0.116	136	3.9	95.5 ± 1.9	27.53 ± 0.22
		0.093	131	3.9		
		0.077	120	3.0		
		0.065	182	3.8		
		0.054	219	3.8		

Table 1: Results for the strange quark mass, $m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$, and the ratio m_s/m_{ud} from simulations near the physical pion mass. Results marked by a dagger were obtained using mass reweighting. Numbers in red are based on non-perturbatively determined renormalisation factors.

that they have been obtained for several different discretisations of the quark action. Moreover, the quoted uncertainties are much smaller than those attributed to sum rule results and the PDG average PDG2010. The FLAG report quotes the following global estimates, based on the results of refs. [9, 10] and [11]:

$$m_{ud}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 3.43(11)\,\mathrm{MeV}, \quad m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 94(3)\,\mathrm{MeV}, \quad m_s/m_{ud} = 27.4 \pm 0.4. \quad (2.1)$$

This high level of accuracy raises the question whether systematic effects, in particular those associated with the chiral extrapolation, are indeed controlled.

We begin by discussing two recent calculations which do not rely on chiral extrapolations. The PACS-CS Collaboration [8, 12, 13] has used non-perturbatively $\mathrm{O}(a)$ improved Wilson fermions and the Iwasaki gauge action at a fixed value of the lattice spacing to determine the light quark masses at the physical pion mass via mass reweighting. The BMW Collaboration [7, 15] has performed simulations with smeared tree-level improved Wilson quarks at pion masses as low as 120 MeV. Quark masses were obtained by an interpolation to the physical pion mass. Results and some simulation details are shown in Table 1.

At $m_\pi^{\mathrm{min}} = 156\,\mathrm{MeV}$ the PACS-CS Collaboration is almost at the physical point. Via a short chiral extrapolation, PACS-CS have obtained the results shown in the first row of Table 1 [12]. In a subsequent work they have proceeded to simulate with hopping parameters $(\kappa_{ud}^*, \kappa_s^*)_{\mathrm{ext}}$ which, according to the chiral extrapolation of the results in [12], correspond to the physical pion mass. In order to compensate the slight observed mismatch between the targeted and actually measured pion mass, they have reweighted their ensembles using the single-histogram method [14]. The second line in Table 1 indicates that the resulting estimate for the strange quark mass is quite different from the value obtained from an extrapolation. However, this increase can be largely attributed to the use of non-perturbative renormalisation factors in [8] which were found to be 30% larger than their perturbative counterparts in ref. [12]. This is consistent with the observation that the difference between extrapolation and reweighting is much less pronounced for the ratio m_s/m_{ud} in which the renormalisation factors cancel. One concludes that reweighting allows one to avoid chiral extrapolations at the expense of incurring a larger statistical error. Despite the large overall uncertainty, the reweighted results by PACS-CS do not agree well with the global estimates
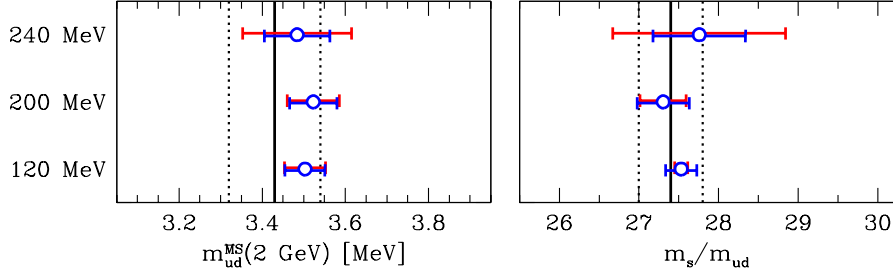
Figure 2: The effect of cuts applied to the lower limit of the pion mass interval on the light quark mass (left) and the ratio m_s/m_{ud} (right) [16]. Red error bars denote the systematic uncertainty. The vertical bands represent the global results of eq. (2.1).

in eq. (2.1). This may be explained by the presence of other systematic errors, most notably lattice artefacts and finite-volume effects, which are not yet sufficiently controlled.

For Wilson-type fermions, the BMW Collaboration has realised the lowest pion masses so far, at least at the three coarsest lattice spacings. However, their results for the light quark masses were not included in the global estimates according to the FLAG rules, since [7, 15] had not been published by the time when ref. [1] was completed. BMW have performed a comprehensive investigation of systematic effects, including studies of the stability of their results under variations in the *ansatz* for the fit function. To this end they compared chiral fits based on either SU(2) ChPT at NLO or on a Taylor expansion, viz.

$$m_{ud} = \frac{m_\pi^2}{2B}\left\{1 - \frac{1}{2}\frac{m_\pi^2}{(4\pi f_\pi)^2}\ln\frac{m_\pi^2}{\Lambda_3^2}\right\}(1+c_s\Delta) \quad \text{versus} \quad m_{ud} = c_1 + c_2 m_\pi^2 + c_3 m_\pi^4 + c_4\Delta, \quad (2.2)$$

where Δ parameterises the deviation from the physical strange quark mass. BMW have also applied cuts on the pion mass, by limiting its maximum value to either 340 or 380 MeV.

Given the availability of ensembles with pion masses as low as 120 MeV, it is interesting to study the question of the impact of interpolations in the pion mass, compared to relying on extrapolations. In other words, how do the results for quark masses and their uncertainties vary when the *lower* limit of the pion mass interval to which the chiral fit is applied, increases gradually? Figure 2 shows the results for the light quark mass m_{ud} and the ratio m_s/m_{ud} obtained after applying mass cuts at $m_\pi = 120$, 200 and 240 MeV, respectively [16]. One clearly sees that the results are quite stable and consistent within errors, indicating that chiral extrapolations are under good control. Unsurprisingly, the error increases for longer extrapolations. A look at Table 2 shows that the error budget is, in fact, increasingly dominated by the fit *ansatz* when the minimum pion mass is shifted to larger values. It would be helpful to discount the possibility that these findings are obscured by lattice artefacts, since below-physical pion masses have only been simulated for the three coarsest lattice spacings. As a suggestion for an improved future analysis of the BMW data, the effects of imposing lower mass cuts should be investigated after the data have been extrapolated to the continuum limit for fixed values of m_π. The above discussion shows that simulations at or below the physical pion mass allow for a systematic investigation into the quality of chiral fits. In order to rightfully claim an overall accuracy of a few percent in lattice estimates of the light quark masses, minimum pion masses of 250 MeV appear to be sufficient.

cut	m_{ud}	σ_{stat}	σ_{syst}	plateau	scale	fit form	mass cut	renorm.	cont.
120 MeV	3.503	0.048	0.049	0.330	0.034	0.030	0.157	0.080	0.926
200 MeV	3.523	0.057	0.063	0.354	0.078	0.470	0.236	0.087	0.765
240 MeV	3.484	0.079	0.131	0.316	0.092	0.807	0.341	0.046	0.349

Table 2: Error budget for the light quark mass $m_{ud}^{\overline{\text{MS}}}(2\,\text{GeV})$ after applying cuts to the minimum pion mass of 120, 200 and 240 MeV. The first row corresponds to the original results in [7]. The last six colums represent the relative contributions of individual systematic effects to the overall systematic error σ_{syst}.

3. Systematics of chiral fits: meson decay constants

Masses and decays constants of pseudoscalar mesons belong to the set of quantities whose dependence on the quark mass has been studied most extensively. Chiral fits using lattice data and the expressions of ChPT give access to the effective coupling constants (low-energy constants – LECs) of ChPT. These include the pion decay constant in the chiral limit, f, the quark condensate Σ, and also some of the LECs which enter at NLO in the chiral expansion (e.g. $\bar{\ell}_3$ and $\bar{\ell}_4$). Furthermore, the decay constants of the physical pion and kaon, f_π, f_K, as well as the ratio f_K/f_π are determined in this way. While the individual decay constants are often used to set the lattice scale, the ratio f_K/f_π is important for constraining the ratio $|V_{us}|/|V_{ud}|$ of CKM matrix elements. Both aspects will be covered in this section. The discussion here is restricted to lattice calculations in the p-regime.

Lattice results for the ratio f_K/f_π are in general quite stable and consistent among different collaborations. Examples of chiral extrapolations are shown in Fig. 3. The FLAG report provides separate global estimates for QCD with $N_f = 2$ and $2 + 1$ flavours, i.e.

$$f_K/f_\pi = 1.193 \pm 0.005 \quad (N_f = 2 + 1), \qquad f_K/f_\pi = 1.210 \pm 0.006 \pm 0.017 \quad (N_f = 2). \quad (3.1)$$

The result for $N_f = 2 + 1$ is based on refs. [18, 20, 21], while the two-flavour result is identical to the value quoted in [22].

As was pointed out by Marciano [23], a precise lattice estimate of f_K/f_π in conjunction with accurate experimental measurements of the leptonic decay rates of $K_{\ell 2}$ decays provides a stringent constraint on the ratio $|V_{us}|/|V_{ud}|$ of CKM matrix elements, since

$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to e\bar{\nu}_e(\gamma))} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2 m_K}{f_\pi^2 m_\pi}. \tag{3.2}$$

An additional constraint on $|V_{us}|$ is provided by the form factor $f_+(q^2)$ which appears in the expression for the rate of the decay $K \to \pi\ell\nu$. Lattice calculations for $f_+(q^2)$ are consistent and equally precise compared with the effective field theory result of ref. [24] in which $f_+(0)$ was determined by invoking the Ademollo-Gatto theorem [25], which states that the corrections due to isospin and SU(3) flavour breaking are second order. Global lattice estimates for $f_+(0)$ are quoted in [1] as

$$f_+(0) = 0.9597 \pm 0.0038 \quad (N_f = 2 + 1), \qquad f_+(0) = 0.9604 \pm 0.0075 \quad (N_f = 2). \tag{3.3}$$

The current accuracy of lattice results for f_K/f_π and $f_+(0)$ allows for a precision test of first-row unitarity of the CKM matrix, i.e.

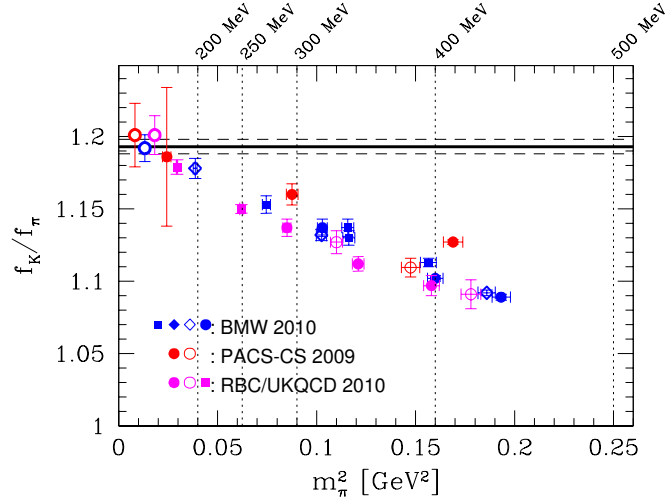$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \tag{3.4}$$

Figure 3: The ratio f_K/f_π plotted versus the squared pion mass from refs. [10, 12, 18, 19]. Open circles denote the extrapolated values at the physical pion mass which are shifted for clarity. The horizontal lines represent the global estimate from the FLAG report [1] for $N_f = 2+1$.

The contribution from the b-quark can be dropped, since $|V_{ub}|^2 = \mathrm{O}(10^{-5})$, which is below the relevant level of accuracy in the following discussion. The experimental branching fractions are

$$|V_{us}|f_+(0) = 0.2163(5), \qquad \left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right| = 0.2758(5). \tag{3.5}$$

Combining these values with the global lattice estimates for $N_f = 2+1$ in eqs. (3.1) and (3.3) yields

$$|V_{ud}|^2 + |V_{us}|^2 = 1.002 \pm 0.015. \tag{3.6}$$

The precision of this test can be considerably enhanced by including another experimental constraint, namely the determination of V_{ud} from super-allowed nuclear β-decays. Using the lattice result for the ratio f_K/f_π which fixes $|V_{us}|/|V_{ud}|$ gives [1]

$$|V_{ud}|^2 + |V_{us}|^2 = 0.9999 \pm 0.0006. \tag{3.7}$$

In this way, first-row unitarity is confirmed with per-mille accuracy, using experimental information and lattice estimates alone. The unitarity test is equally precise if the lattice estimate is provided by $f_+(0)$ instead of f_K/f_π.

The above discussion suggests that lattice calculations of pseudoscalar meson decay constants are under very good control. Table 3 contains a compilation of recent estimates for decay constants, the hadronic radii r_0 and r_1 and certain combinations thereof. Although the ratio f_K/f_π is consistent among different calculations within the quoted errors, this is not necessarily true for the absolute values of decay constants and hadronic radii. On the assumption that there are no significant differences between two- and three-flavour QCD within the presently quoted errors, one finds that the r_0 determination from ETMC [22] (which uses the physical value of f_π to set the scale) contradicts the estimate quoted by RBC/UKQCD [10]. On the other hand, RBC/UKQCD,

	scale	f_π	f_K	f_K/f_π	r_0 [fm]	r_1 [fm]	$f_K r_0$	$f_K r_1$
RBC / UKQCD [10]	m_Ω	124(5)	149(5)	1.204(26)	0.487(9)	0.333(9)	0.368(9)	0.251(10)
PACS-CS [12]	m_Ω	134(4)	159(3)	1.189(20)	0.492($^{10}_6$)		0.397($^{11}_9$)	
MILC [26]	f_π	./.	157($^1_3$)	1.197($^7_{13}$)		0.311($^3_8$)		0.246(5)
ETMC [22]	f_π	./.	158(2)	1.210(18)	0.438(10)		0.351(10)	

Table 3: Results for decay constants and the hadronic radii r_0 and r_1 from simulations with $N_f = 2+1$ dynamical flavours (RBC/UKQCD, PACS-CS, MILC) and $N_f = 2$ (ETMC). The renormalisation of the axial current in [12, 26] was done perturbatively.
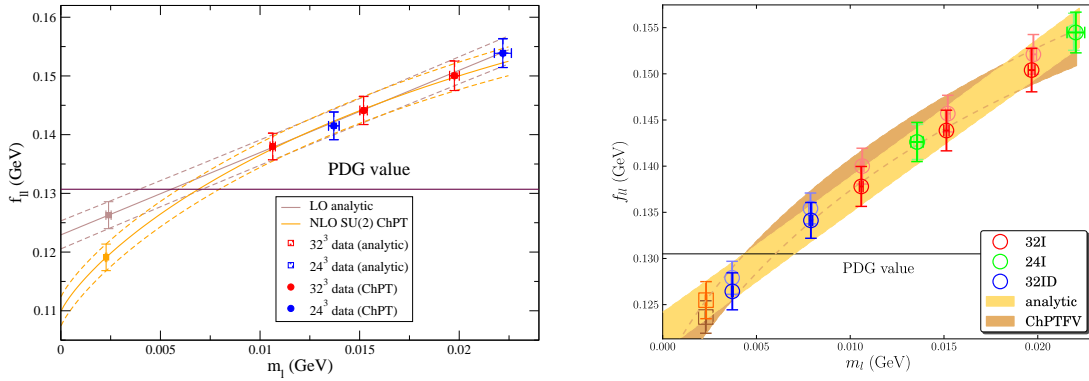

Figure 4: Chiral extrapolations of f_π from RBC/UKQCD. The left panel shows the comparison of SU(2) ChPT with an analytic *ansatz*, using a minimum pion mass of 290 MeV at $a \approx 0.11$ fm [10]. In the right panel two additional points with $m_\pi = 170$ and 250 MeV at $a \approx 0.014$ fm have been included [27].

who determine the lattice scale from the mass of the Ω baryon, find a value for f_π which is smaller than the experimental value. Similar observations apply to the results for f_K, r_1 and $f_K r_1$ quoted by RBC/UKQCD and MILC [26].

One may suspect that the observed differences in the absolute values of f_K and f_π are linked to the normalisation of the axial current. In fact, only the correctly normalised matrix elements can be expected to approach the continuum limit with a rate proportional to the leading lattice artefacts. For quantities such as r_0 and r_1 the situation is not much better, because little is known about the chiral behaviour one is to expect. In view of the importance of decay constants and hadronic radii for the overall scale setting, one must make an effort to understand the observed differences. Several collaborations have reported new results for these quantities at this conference [27–32].

RBC/UKQCD have investigated why the individual decay constants f_π and f_K are lower than experiment, while their ratio agrees with other simulations. To this end they have supplemented their existing data sets by two more ensembles with pion masses of 250 and 170 MeV [27]. In order to keep $m_\pi^{\min} L > 4$ for $L/a = 32$, lower pion masses could be simulated at the expense of having to use coarser lattice spacings ($a \approx 0.14$ fm) compared to [10]. Figure 4 shows the impact of the additional data points on the chiral extrapolation of f_π. Clearly, the ambiguity associated with the *ansatz* for the chiral behaviour is reduced: Extrapolations based either on SU(2) ChPT or on a Tay-
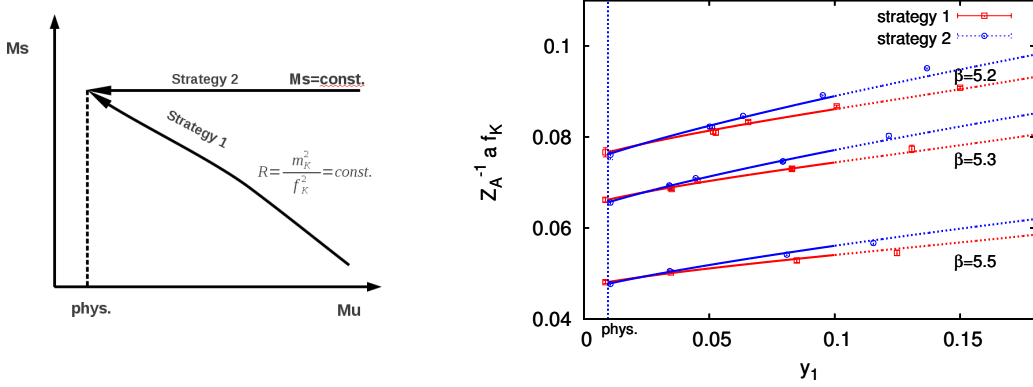
Figure 5: Chiral extrapolation of f_K by ALPHA [31]. Left: schematic view of the paths of two different extrapolations in the (m_{ud}, m_2)-plane. Right: chiral extrapolations of $a f_K / Z_A$ in the variable $y_1 \equiv m_\pi^2 / 8\pi^2 f_K^2$ for three different bare couplings.

lor expansion produce results for f_π which agree very well within errors. However, while the ratio f_K/f_π is consistent with the earlier result, the new and preliminary value of $f_\pi = 125(2)(3)\,\text{MeV}$ still appears to be smaller than the experimental value. It should be kept in mind that the entire range of pion masses which RBC/UKQCD have access to, involves different lattice spacings. The systematics of chiral fits can only be investigated reliably provided that the dependence on the lattice spacing is well understood.

The consistency of different chiral fit *ansätze* has also been studied by the ALPHA Collaboration for $O(a)$ improved Wilson quarks [31]. In order to check the robustness of the extrapolation of f_K to the physical pion mass, they have compared two different fit strategies. Denoting the hopping parameters by κ_1 and κ_2, where $\kappa_1 = \kappa_{\text{sea}}$, the first strategy amounts to adjusting κ_2 until m_K^2/f_K^2 is equal to the experimental value. Repeating this for each sea quark mass defines, at leading order, a sequence of data points with $m_s + m_{ud} = \text{const.}$, which can be extrapolated to the physical pion mass using the expressions from partially quenched ChPT. In the second strategy the strange quark mass is held fixed: at each value of κ_{sea}, the hopping parameter κ_2 is tuned such that the PCAC mass reproduces the fixed value of $\mu \equiv m_s$. The resulting values of f_K can then be extrapolated in the pion mass using SU(2) ChPT at NLO. A schematic view how the physical point in the (m_{ud}, m_s)-plane is approached with the two strategies is shown in Fig. 5 (left). The right panel of the figure compares the chiral extrapolation of the (bare) f_K for three different values of β. Even though they are based on quite different variants of ChPT, both strategies converge to the same values at the physical pion mass, which enhances the credibility of the chiral fits. After taking the renormalisation of the axial current into account the results can be used to set the lattice scale using f_K. In this way ALPHA find that the three β-values correspond to $a \approx 0.075, 0.066$ and $0.049\,\text{fm}$, respectively. These preliminary results agree well within errors with the scale determination via the mass of the Ω baryon [35] on the same ensembles.

Decay constants are often used to calibrate the hadronic radii r_0 and r_1, by determining combinations such as $f_\pi r_0$ at the physical pion mass in the continuum limit. Chiral extrapolations of r_0 and r_1 are usually based on the assumption that they can be described by a polynomial in m_π^2. A detailed investigation of the chiral behaviour of r_0 was presented at this conference [32] (see also
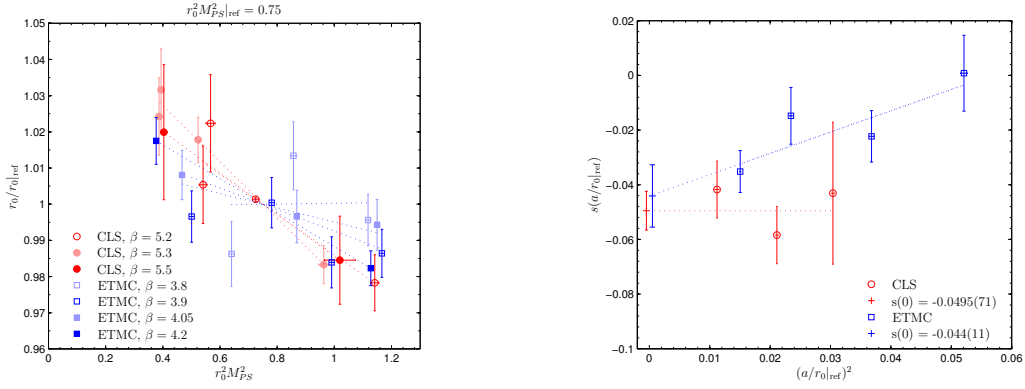
Figure 6: Left: pion mass dependence of $r_0/\left.r_0\right|_{\mathrm{ref}}$ computed on the CLS ensembles [32] (red symbols) compared to the results from ETMC [34] (blue symbols). The quantity $\left.r_0\right|_{\mathrm{ref}}$ denotes the value at a reference pion mass. Right: continuum extrapolation of the slope parameter s in eq. (3.8).

ref. [33]). In Fig. 6 the pion mass dependence of r_0 computed on the CLS ensembles with $\mathrm{O}(a)$ improved Wilson quarks is compared to the results by ETMC [34]. As can be seen from the left panel of the figure, r_0 indeed shows a linear dependence on the squared pion mass at a given value of the bare coupling. By performing a linear fit according to

$$r_0/\left.r_0\right|_{\mathrm{ref}} = A + s \times (m_\pi r_0)^2, \tag{3.8}$$

one can determine the slope parameter s, which is found to depend quite strongly on the lattice spacing in the case of ETMC. The right panel of Fig 6 shows the extrapolation of s to $a = 0$. The rate with which this quantity approaches the continuum limit is indicative of the size of lattice artefacts of $\mathrm{O}(m_q a^2)$. While the slope s from the two collaborations agrees very well in the continuum limit, there are sizeable corrections proportional to $m_q a^2$ for twisted-mass QCD. This discussion serves as a reminder that the problem of mass-dependent lattice artefacts, which are formally of $\mathrm{O}(a^2)$ must be addressed, since they will affect the determination of $f_\pi r_0$ and hence the calibration of r_0.

4. Nucleon axial charge

While hadronic uncertainties in the meson sector could be brought well under control, the situation for baryonic quantities is much less satisfactory. Despite many years of dedicated effort, lattice results for nucleon form factors or moments of structure functions fail to reproduce the experimental values within the quoted uncertainties [36, 37]. A prominent example is the axial charge, g_{A}, of the nucleon. Lattice simulations using pion masses $m_\pi \gtrsim 250\,\mathrm{MeV}$ typically underestimate this quantity by $10-15\,\%$. Even more worrisome is the observation that the gap is stable, i.e. the data show little if no tendency to approach the physical value as the pion mass is decreased. There is a broad concensus that uncontrolled systematic effects must be held responsible.

The axial charge is an ideal observable to study lattice systematics for baryonic quantities: It is defined in terms of a transition matrix element at zero momentum transfer and hence the underlying kinematics is very simple. Second, it can be determined without the evaluation of quark-disconnected diagrams. Among the common sources of systematic error are lattice artefacts, the related issue of the correct normalisation of the axial current, and the influence of finite-volume

effects, which are known to be larger for baryonic systems. An obvious question is whether the chiral behaviour is sufficiently controlled in the calculations performed so far, or whether much smaller pion masses are required in order to make contact with the experimental value. Another issue which has received quite some attention recently, is the possible contamination of baryonic three-point correlation functions by contributions from higher excited states. This seems plausible, since the noise-to-signal ratio in baryonic correlation functions is much worse than for mesons. Thus, one cannot firmly rule out the possibility that excited state contributions are still present within the relatively short Euclidean time interval before the signal is lost.

The theoretical foundations of baryonic ChPT, which is used to constrain the chiral behaviour of g_A, are unfortunately on a weaker footing compared to the mesonic sector. Since the mass gap between the nucleon and the nearest resonance, i.e. the Δ, is much smaller than the mass scale defined by the nucleon itself, it is difficult to define a consistent chiral counting scheme. The established formalisms include Heavy Baryon ChPT [38], the infrared regularisation of loop integrals [39] and the related extended on-mass shell regularisation [40], some of which have been carried to high orders in the expansion. Another approach is the so-called small-scale expansion (SSE) [41], in which the nucleon-Δ splitting is treated as a small parameter and included in the chiral power counting in the framework of Heavy Baryon ChPT. One severe drawback for the interpretation of lattice data is the large number of coupling terms, each of which carries a low-energy constant. Some of these LECs can be constrained from phenomenology, but unless one has access to extremely detailed information from lattice simulations deeply in the chiral regime, it seems impossible to determine the full set.

The bare value of g_A can be extracted from a suitable ratio of two- and three-point functions. In the simplest case, i.e. when the same operators are used to create and annihilate the nucleon, the expression reads

$$R_{\rm A}(t,t_s) = \frac{C_3^{\rm A}(t,t_s)}{C_2(t_s)} \xrightarrow{t,(t_s-t)\gg 0} g_{\rm A}^{\rm bare} + {\rm O}({\rm e}^{-\Delta_{\rm N}t}) + {\rm O}({\rm e}^{-\Delta_{\rm N}(t_s-t)}). \tag{4.1}$$

Here t_s denotes the Euclidean time separation between the initial and final nucleons, while the axial current is inserted at time t with $0 \leq t \leq t_s$. Due to the rapidly increasing statistical noise, typical values of t_s are of order 1 fm, and thus the correlation functions must reach their asymptotic behaviour for separations $t, (t_s - t) \lesssim 0.5$ fm. Since the gap $\Delta_{\rm N}$ between the nucleon and its first excitation is expected to scale like $\Delta_{\rm N} \sim 2m_\pi$ in the chiral regime, it is clear from eq. (4.1) that corrections from excited states become increasingly important as the physical pion mass is approached.

Several collaborations have investigated the issue of excited state contaminations recently. Using the CLS configurations generated with $N_{\rm f} = 2$ flavours of O(a)-improved Wilson fermions, the Mainz group has calculated baryonic three-point functions for several different source-sink separations t_s [42]. After computing the so-called "summed insertions" [43] according to

$$S_{\rm A}(t_s) = \sum_{t=0}^{t_s} R_{\rm A}(t,t_s) \xrightarrow{t_s\gg 0} \text{const.} + g_{\rm A}^{\rm bare}t_s + {\rm O}(t_s{\rm e}^{-\Delta_{\rm N}t_s}), \tag{4.2}$$

they determine the axial charge from the linear slope of $S_{\rm A}$ in the source-sink separation t_s. Since $t_s > t, (t_s - t)$ by construction, it is clear that the corrections due to excited state contamination in
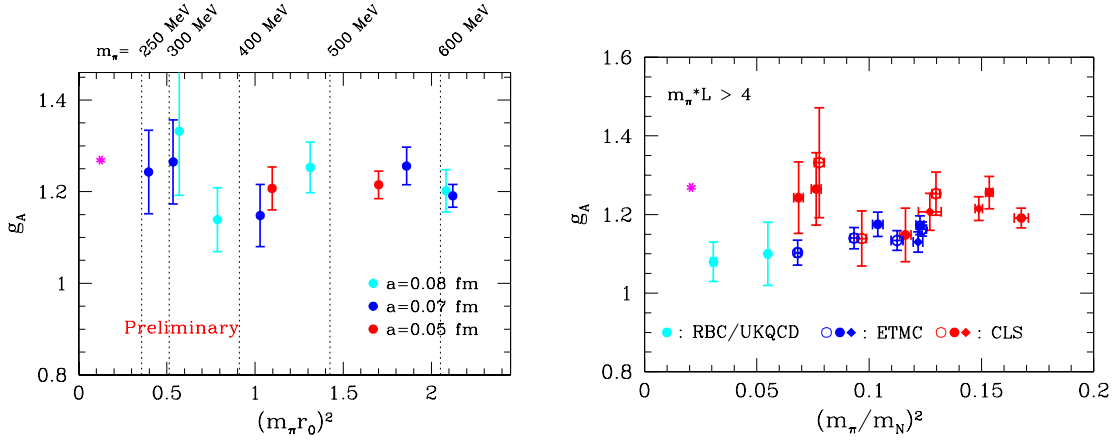
Figure 7: Left: preliminary results for g_A computed on the CLS configurations using the technique of summed insertions [42]. Right: compilation of recent results for g_A from refs. [42,46,47]. The experimental result for g_A is represented by the pink asterisk.

eq. (4.2) are parametrically more strongly suppressed than for the simple ratio $R_A(t, t_s)$. Other ways to address excited state contamination include the use of multi-exponential fits [44] and systematic studies of the dependence of g_A on the source-sink separation for t_s as large as 1.9 fm [45]. All these efforts do not allow for a firm conclusion at this stage. The preliminary results by the Mainz group (left panel of Fig. 7) suggest that summed insertions lead to a better agreement with the experimental result for g_A. ETMC [45] report the absence of a bias in g_A at $m_\pi = 380$ MeV but see some evidence for a distortion in the case of $\langle x \rangle_{u-d}$. For the latter quantity, LHPC [44] can confirm that multi-exponential fits lead to a better agreement with experiment as the pion mass is lowered, albeit with a larger statistical error.

New results on g_A by the RBC/UKQCD Collaborations were presented at this conference [46], computed on the set of gauge configurations which included the recently added lighter pion masses discussed in section 3. They report a stable gap between their preliminary results and the experimental value of g_A across the entire mass range. The favoured explanation offered by RBC/UKQCD is that the discrepancy is a result of finite-volume effects rather than excited state contamination. A compilation of recent results for g_A [42, 46, 47] is shown in the right panel of Fig. 7, where the chiral behaviour is compared among different groups after applying the cut $m_\pi L > 4$. Despite the very different systematics concerning the discretisation of the quark action, the values of the lattice spacing and the numerical procedures to extract g_A from the measured correlation functions, the results are broadly consistent with each other. However, it would appear that those calculations which address the issue of excited state contamination compare more favourably with the experimental value.

5. Summary and conclusions

Chiral extrapolations have been a persistent source of systematic errors in lattice calculations. In this review I have tried to assess the reliability of chiral extrapolations in order to investigate

whether the claimed accuracy of lattice results for several phenomenologically interesting quantities is justified. The emergence of simulation data around the physical pion mass was crucial, since it allowed for a systematic study into the effects of replacing the chiral extrapolation by an interpolation.

Pion masses in the range of $250 - 400\,\text{MeV}$ appear to be sufficient to guarantee that lattice estimates for the light quark masses can be obtained with overall uncertainties at the level of a few percent. Similarly, the chiral behaviour of the ratio f_K/f_π is under good control. The latter allows for a precise determination of the ratio $|V_{us}/V_{ud}|$ and for a test of first-row unitarity with permille accuracy, based on lattice results and experiment alone. For individual decay constants, however, small inconsistencies among different calculations remain and must be resolved. The separation of lattice artefacts from systematic effects associated with the description of the chiral behaviour must be improved not only for f_K and f_π but also for quantities such as r_0.

In spite of these successes, one finds that lattice calculations for the axial charge are still in an unsatisfactory state, since the chiral behaviour of g_A is clearly obscured by systematic effects. With the presently available data it is difficult to decide whether one single cause is chiefly responsible or whether it is a convolution of finite-volume effects, excited state contamination and lattice artefacts. Due to the unfavourable signal-to-noise ratio of baryonic correlation functions it is likely that this can only be resolved via an enormous increase in statistics.

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