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Many flavor QCD with $N_f = 12$ and 16

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Information of the phase structure of many flavor SU(3) gauge theory is of great interest for finding a theory which dynamically breaks the electro-weak symmetry. We study the SU(3) gauge theory with fermions for $N_f = 12$ and 16 in fundamental representation. Both of them, through perturbation theory, reside in the conformal phase. We try to determine the phase of each theory non-perturbatively with lattice simulation and to find the characteristic behavior of the physical quantities in the phase. HISQ type staggered fermions are used to reduce the discretization error which could compromise the behavior of the physical quantity to determine the phase structure at non-zero lattice spacings. Spectral quantities such as bound state masses of meson channel and meson decay constants are investigated with careful finite volume analysis. Our data favor the conformal over chiral symmetry breaking scenario for both $N_f = 12$ and 16.

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1. Introduction

The technicolor model is one of attractive candidates of the physics beyond the standard model for the dynamical origin of the electroweak symmetry breaking. In particular the gauge theory with walking behavior near the edge of the conformal window has been considered as a realistic new physics model (walking technicolor). Therefore it is important to understand the phase structure and the low-energy dynamics of these theories for the phenomenology to be tested by the on-going LHC experiment. A concrete example of such models could be the SU(3) gauge theory with 12-flavor of massless fermions in the fundamental representation. The main purpose of this study is to understand the dynamics of this gauge theory by using the technique of the numerical simulation developed in lattice QCD, and ultimately to test whether this theory is a candidate of the walking technicolor (see [1] for more discussion and references).

A number of lattice works studied this theory, such as the ones investigating the phase diagram of Wilson fermions [2, 3], hadron spectrum [4, 5, 6], and the running coupling constant [7, 8, 9, 10]. However, whether this theory is in conformal or chiral broken phase is controversial at present.

We investigate the 12-flavor SU(3) gauge theory using a variant of the highly improved staggered quark (HISQ) action [11], eventually with multiple lattice spacings, aiming to shed light on the controversy. We present preliminary results of pseudoscalar meson mass and decay constant, through which we attempt to determine the phase of this theory, as well as the characteristic quantity associated with the phase.

Along this line we also simulate the 16-flavor SU(3) gauge theory with the same improved action. This theory resides deep in conformal window according to the perturbation theory. If this is the case, this theory could be used as a reference for the signal of conformality. Preliminary results for this study is presented here.

Another series of simulations with this set up for $N_f < 12$ have been performed to test the walking behavior of $N_f = 8$, and to have a reference for the signal of the real QCD like hadron phase at $N_f = 4$. The results are reported in these proceedings [1].

2. Simulation setup and method

We adopt the highly improved staggered quark (HISQ) action [11], but without the tadpole improvement and the mass correction term for heavy quarks. This action further suppresses the taste breaking in the QCD simulations compared to the Asqtad or stout-smeared staggered fermions [12]. We use this action for the many flavor simulation to minimize the discretization effects which potentially compromise the behavior of the physical quantities at non-zero lattice spacings.

Gauge configurations are generated through HMC algorithm with various parameter sets for the fermion mass m_f, volume and the bare coupling $\beta = 6/g^2$ for $N_f = 12$ and 16. We measure the mass and decay constant of the lowest state of the pseudoscalar channel (pion), m_π and f_π, respectively, which are tested against the chiral symmetry breaking or conformal scenario. The tree-level chiral perturbation theory (ChPT) is used for the former. The conformal hyperscaling [13] and and its finite-size version is used for the latter.

The finite-size hyperscaling [14] is derived in the conformal theory deformed by the small fermion mass and also the large, finite volume L^4. This scaling explains the fermion mass and volume dependence of the physical quantities, such as the hadron mass m_H, which is governed by the mass anomalous dimension at the infrared fixed point γ_*. According to this scaling, a physical quantity is described by a function $f(x)$ of the scaling variable $x = Lm_f^{\frac{1}{1+\gamma_*}}$ and L,

$$m_H = f(x)/L. \tag{2.1}$$

Since the finite-size hyperscaling is an extension of the hyperscaling to a finite volume, this scaling should reproduce the original hyperscaling [13] in the infinite volume limit,

$$m_H = C_H\, m_f^{\frac{1}{1+\gamma_*}}. \tag{2.2}$$

Therefore when the L is large enough, the function should become

$$f(x) = c_0 + C_H x. \tag{2.3}$$

We use this fit form in the following analysis, and check if our data present the scaling behavior or not.

3. Results of $N_f = 12$

After a pilot study with several bare gauge couplings $\beta \equiv 6/g^2$ for the $N_f = 12$, we determined to carry out the production run for $\beta = 3.7$. We generate the gauge ensembles at three volumes, $L^3 \times T$, $(L,T) = (12,24)$, $(18,24)$, $(24,32)$ [1], and the various fermion masses, $0.04 \le m_f \le 0.2$. We accumulate typically 500–1000 trajectories at each parameter set of L and m_f, and measure the m_π and f_π on these configurations.

3.1 ChPT vs. hyperscaling fit

Figure 1 shows the result of the m_π at each L and m_f. In this analysis we use the result obtained from only the largest volume, $L = 24$, because in this volume we expect that finite volume effects of the m_π is negligible in our fermion mass range.

We first attempt to analyze pion mass with a fit form motivated by the chiral perturbation theory (ChPT) given by

$$m_\pi^2 = c_0 + c_1 m_f + c_2 m_f^2. \tag{3.1}$$

As the m_π should vanish at the chiral limit, c_0 must be zero within the error if the chiral symmetry is broken, thus, ChPT describes our data. The values of the χ^2/d.o.f. are 4.1 and 28 for the fit form with and without the c_0 in eq. (3.1), respectively. The former gives better χ^2/d.o.f, while the c_0 is non-zero $c_0 = -0.0287(26)$. On the other hand, the hyperscaling fit eq. (2.2), gives $\chi^2/\text{d.o.f.} = 3.6$ and $\gamma_* \sim 0.46$. This means that the hyperscaling, where the fit gives the smallest χ^2/d.o.f. with $m_\pi = 0$ at the chiral limit, is favored by our data.

3.2 Analysis with finite-size hyperscaling

Figure 2 shows the Lm_π at three lattice sizes as a function of the scaling variable $x = Lm_f^{\frac{1}{1+\gamma}}$, with γ being different in each panel. In $\gamma = 0$, the data are scattered, but as the γ increases, the

[1] Ideally, the ratio L/T should be fixed, which is not the case for $L = 12$. However, we find the spectrum is not sensitive to L/T when $m_\pi L \gg 1$, and we only use the data in such a region in this analysis.

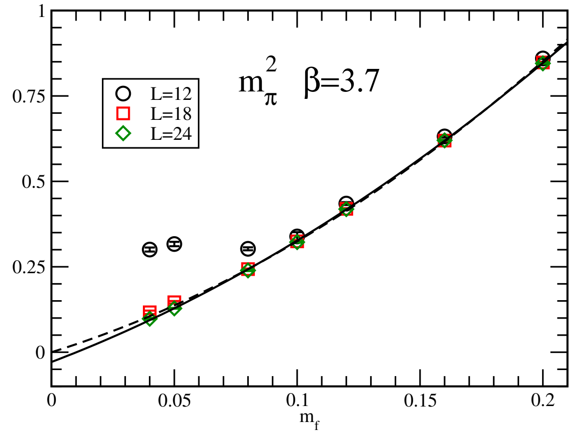

Figure 1: m_π^2 in the 12 flavors as a function of the bare fermion mass m_f. The different symbols denote the data at the different volumes. The solid and dashed lines represent the ChPT fit with and without the constant term c_0 in eq. (3.1), respectively, using the largest volume data.

data in the different volumes tend to align. The data with $\gamma = 0.4$ show a good alignment, which is taken as the signal of the scaling, eq. (2.1). The scaling again disappears beyond $\gamma = 0.4$. Thus, the optimal γ is around $\gamma = 0.4$. This analysis is based on the method in Ref. [15] for an estimate of the γ_*.

We attempt to determine the γ_* by fitting the data to the finite-size hyperscaling with the linear assumption, eq. (2.3). Since this assumption is valid in large L region, we restrict ourselves to use the data in the larger x region. From the fit shown by the dashed line in Fig. 3 we obtain $\gamma_* \sim 0.44$ with $\chi^2/\text{d.o.f.} = 4.0$. The value of γ_* is reasonably consistent with the value estimated above and also that obtained from the hyperscaling fit in Sect. 3.1. We note that our γ_* roughly agrees with the result [5] and the results with similar analysis [16, 17] using the data in Ref. [5].

These results are encouraging as preliminary results, while at present we have not evaluated errors of the γ_*. There are possibly several systematic errors in this analysis, such as the one due to the assumption of the asymptotic form of $f(x)$, corrections of the finite-size hyperscaling from large m_f and small L effects [18]. If the effect of these corrections is large, we cannot obtain the correct γ_* even if the data shows the scaling behavior. In order to get reliable estimate of the systematic errors, we need to expand our simulation towards lighter mass and larger volume. Detailed analysis using those data are ongoing.

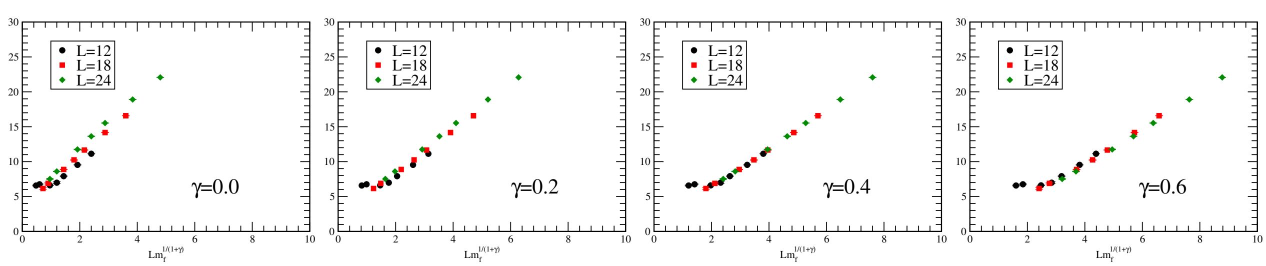

Figure 2: The scaling behavior of Lm_π in the 12 flavors as a function of $x = Lm_f^{\frac{1}{1+\gamma}}$ where the values of γ are taken as γ =0, 0.2, 0.4, 0.6 from left to right. The different symbols denote the data at the different volumes.

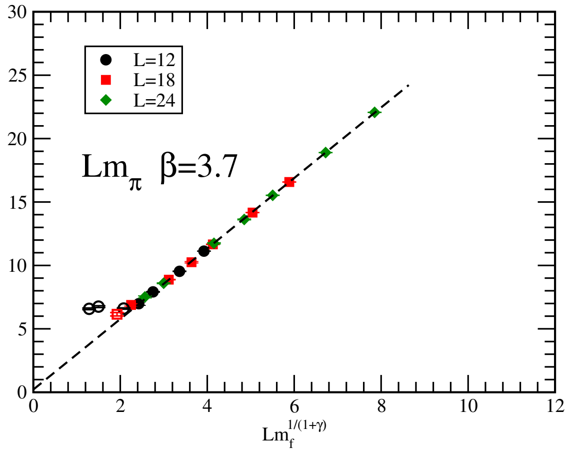

Figure 3: Lm_π in the 12 flavors and the fit result of the finite-size hyperscaling denoted by the dashed line. The different symbols denote the data at the different volumes. The filled and open symbols represent the data included in the fit and omitted from the fit, respectively.

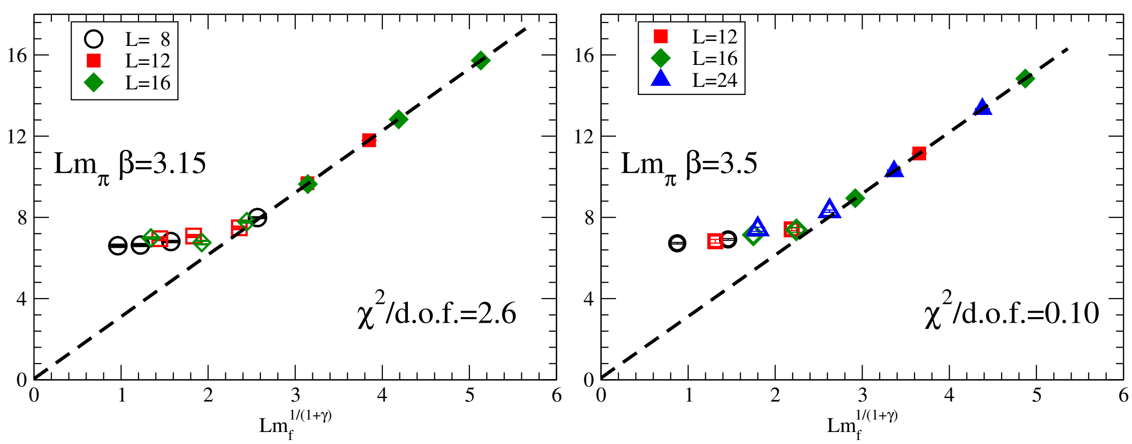

Figure 4: Lm_π in the 16 flavors at $\beta = 3.15$(left) and $\beta = 3.5$(right). The different symbols denote the data at the different volumes. The dashed line denotes the fit result of the finite-size hyperscaling. The filled and open symbols represent the data included in the fit and omitted from the fit, respectively.

4. Results of $N_f = 16$

In the $N_f = 16$ SU(3) gauge theory we perform the simulations at $\beta \equiv 6/g^2 = 3.15$ and 3.5. We choose three spatial volumes $L = 8, 12, 16$ at $\beta = 3.15$, and add one more volume $L = 24$ at $\beta = 3.5$. The range of the fermion mass is $0.03 \le m_f \le 0.2$. The typical length of the trajectory is roughly 1000.

We analyzed the results of the m_π and f_π in the $N_f = 16$ case using the same procedure as in the $N_f = 12$ case discussed in the previous section. The ChPT fit failed as it did for $N_f = 12$, which is expected as the $N_f = 16$ theory is much deeper in conformal window if $N_f = 12$ was.

Using the finite-size hyperscaling analysis, we attempt to determine the value of the γ_* at two β's. Figure 4 shows the fit result of the Lm_π with the finite-size hyperscaling assuming the asymptotic form of $f(x)$, eq. (2.3). The both panels show good scaling behavior of the Lm_π, and the sizes of the χ^2/d.o.f. are reasonable. From the fits we obtain $\gamma_* \sim 0.42$ and 0.35 at $\beta = 3.15$ and 3.5, respectively. We also analyze the data of f_π and obtain γ_* which is roughly consistent with those with m_π. Again we have not yet estimated errors of the obtained γ_* discussed in the above section.

These observations lead to a converged picture of the conformal theory with decreasing γ_* towards weaker bare coupling. One question arises as to the size of the γ_* being much bigger than the result with 2-loop perturbation theory $\gamma_*^{\mathrm{pert}} \sim 0.025$. One possible scenario is that the γ_* further decreases towards much weaker coupling (continuum limit) and eventually gets compatible to the perturbation theory. Another possible scenario is our bare gauge coupling is too large to investigate the property of this theory in the continuum limit. To investigate further, more detailed study of the β dependence of the γ_* is necessary.

5. Summary and outlook

We have studied the SU(3) gauge theories with the fundamental 12 and 16 fermions using a HISQ type staggered fermion action, and presented preliminary results in this report. For the 12-flavor case, we attempt to determine the phase of this theory through the analysis of pion mass. Our present data favors the conformal hyperscaling over the ChPT. The mass anomalous dimension, γ_* at the infrared fixed point was estimated though the (finite-size) hyperscaling analysis. So far the size of γ_* is not as big as $\gamma_* \sim 1$ for the theory to be close to the realistic technicolor theory. We have not yet estimated the errors of the γ_*, so that we need to estimate it to reach a definite conclusion. More detailed analyses with more data at larger volume and lighter mass are underway. For the 16-flavor case, our data shows the conformal signal, while the obtained value of the γ is much higher than the perturbative result. We plan to investigate this by studying the β dependence of this value.

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