

Lattice study of 4d $\mathcal{N} = 1$ super Yang-Mills theory with dynamical overlap gluino

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We report on a lattice simulation result for four-dimensional $\mathcal{N} = 1$ SU(2) super Yang-Mills theory with the dynamical overlap gluino. We study the spectrum of the overlap Dirac operator at three different gluino masses $m = 0.2, 0.1$ and 0.05 with the Iwasaki action on a $8^3 \times 16$ lattice. We find that the lowest eigenvalue distributions are in good agreement with the prediction from the random matrix theory. Moreover the mass dependence of the condensate is almost constant, which gives a clean chiral limit. Our results for the gluino condensate in the chiral limit is $\langle \bar{\psi}\psi \rangle r_0^3 = 0.63(12)$, where r_0 is the Sommer scale.

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1. Introduction

The $\mathcal{N} = 1$ global supersymmetry in four dimension is phenomenologically interesting [1]. The $\mathcal{N} = 1$ super Yang-Mills theory (SYM) is a simple model containing only gluon and gluino. The model is nontrivial since it is expected to have interesting nonperturbative properties like confinement and discrete chiral symmetry breaking. An important physical quantity is the gluino condensate which is an order parameter for the spontaneous breaking of the discrete chiral symmetry. If we extend the model to include matters, the Witten index [2] can vanish implying that a dynamical supersymmetry breaking can occur with the condensate as an order parameter [3].

Lattice gauge theory could be a means to study the nonperturbative aspects of supersymmetry. However the lattice discretization explicitly breaks supersymmetry and the supersymmetry violating operators at the quantum level give rise to the fine tuning problem in general. The $\mathcal{N} = 1$ SYM is exceptional, since the mass term is the only relevant supersymmetry violating operator, which can be controlled by taking the chiral limit [4]. The model has been studied on the lattice with the Wilson fermion [5] and the domain wall fermion [6] so far. Still these approaches have difficulties in obtaining precise chiral limit due to an explicit breaking of chiral symmetry or a large residual mass. Even on the lattice, the question whether the R-symmetry of the model is spontaneously broken or not, remains unclear.

Through this work we provide a numerical evidence for nonzero gluino condensate in $\mathcal{N} = 1$ SYM by a nonperturbative lattice study. The overlap fermion [7] is a natural choice for the model because of its exact lattice chiral symmetry [8]. In this study, we use the same strategy as our previous studies in QCD [9], where clean chiral extrapolation for the chiral condensate was achieved by dynamical simulation of the overlap fermion and comparing the Dirac spectrum with the Banks-Casher relation [10] and the random matrix theory (RMT) [11]. The key benefit of this strategy is that it is free from both of the UV and IR problems when we determine the physical value of the chiral condensate, as we describe later.

2. The model and lattice formulation

The Euclidean action for $\mathcal{N} = 1$ SYM is

$$S = \int d^4x \text{Tr} \left\{ \frac{1}{4} (F_{\mu\nu})^2 + \frac{1}{2} \bar{\psi} \gamma^\mu D_\mu \psi \right\}. \quad (2.1)$$

Here $\psi = \psi^a(x) T^a$ denotes the gluino field, which is an adjoint Majorana fermion satisfying $\bar{\psi} = \psi^T C$. The action is invariant under the supersymmetry transformation

$$\delta A_\mu = -2g \bar{\psi} \gamma_\mu \varepsilon, \quad \delta \psi = -\frac{i}{g} \sigma^{\mu\nu} F_{\mu\nu} \varepsilon, \quad (2.2)$$

and the R-symmetry

$$\psi \rightarrow e^{-i\alpha\gamma_5} \psi, \quad \bar{\psi} \rightarrow \bar{\psi} e^{-i\alpha\gamma_5}, \quad (2.3)$$

which coincides with the $U(1)$ axial symmetry. This $U(1)_A$ is anomalous and the axial current $J_\mu = \bar{\psi} \gamma_\mu \gamma_5 \psi$ satisfies

$$\partial^\mu J_\mu = \frac{N_c g^2}{16\pi^2} F^{a\mu\nu} \tilde{F}_{\mu\nu}^a. \quad (2.4)$$

The anomaly leaves discrete subgroup Z_{2N_c} invariant at quantum level but it can be spontaneously broken down to Z_2 . An order parameter is the gluino condensate $\langle \bar{\psi}\psi \rangle$. No Nambu-Goldstone boson appears since Z_{2N_c} is a discrete group.

On the lattice the overlap Dirac operator has no sign problem, since it possesses Ginsparg-Wilson relation, γ_5 hermiticity, and two-fold degeneracy from the adjoint representation. Therefore we can write the fermionic action as

$$S_f = -\frac{1}{4} \log \det(D_{\text{ov}}^\dagger D_{\text{ov}}), \quad (2.5)$$

where the overlap Dirac operator D_{ov} in the adjoint representation is constructed from the Wilson Dirac operator D_W as

$$D_{\text{ov}} = M_0 \left(1 + D_W / \sqrt{D_W^\dagger D_W} \right), \quad (2.6)$$

$$(D_W)_{yx}^{ba} = \delta_{yx} \delta^{ba} - K \sum_{\mu=1}^4 [\delta_{y,x+\hat{\mu}} V_{x\mu}^{ba} + \delta_{y+\hat{\mu},x} (V_{y\mu}^{ba})^T]. \quad (2.7)$$

$V_{x\mu}^{ba}$ is the adjoint link variable defined as $V_{x\mu}^{ba} = 2\text{Tr}(U_{x\mu}^\dagger T^b U_{x\mu} T^a)$ and the hopping parameter is set to $K = \frac{1}{8-2M_0}$. We introduce an explicit mass m for gluino in order for a comparison with the random matrix models.

3. Banks-Casher relation and random matrix theory

The Banks-Casher relation [10] relates the Dirac spectrum to the chiral condensate. Moreover the details of low-lying Dirac eigenvalue distributions are described by the chiral random matrix theory (ChRMT) [11] including finite volume corrections to the Banks-Casher formula.

There are two advantages in extracting the condensate via ChRMT. One is that the Dirac spectrum is free from UV power divergences, which we can explicitly see in the extended Banks-Casher formula :

$$\pi\rho(\lambda) = \lim_{\varepsilon \rightarrow 0} \frac{1}{2} \left(-\langle \bar{\psi}\psi \rangle|_{m=i\lambda+\varepsilon} + \langle \bar{\psi}\psi \rangle|_{m=i\lambda-\varepsilon} \right). \quad (3.1)$$

Another advantage is that the finite volume effects and finite mass effects can be corrected by ChRMT. In RMT, the low-lying eigenvalue distributions are characterized by the dimensionless parameter $\mu = m\Sigma V$. For any given volume V and mass m , the condensate Σ is the only free parameter that is defined in the infinite volume and massless limits.

The gluino is an adjoint representation fermion and the corresponding Dirac operator satisfies $[D, C^{-1}K] = 0$, where C is the charge conjugation and K is the complex conjugation operator. Consequently the universality class for $\mathcal{N} = 1$ SYM is different from QCD, and it belongs to the chiral symplectic ensemble (ChGSE). The partition function is the same as that for chiral unitary ensemble, which describes QCD, except that the matrix elements are real quaternion instead of complex number. This is equivalently described by partially quenched chiral perturbation theory.

For later convenience, we present the first eigenvalue distribution of ChGSE for zero topological charge,

$$p_1(\zeta, \mu) = c e^{-\zeta^2/2} \zeta \tilde{\mu}^4 \frac{\int_0^1 dt \tilde{\mu}^{-3} I_3(2t\tilde{\mu})}{\int_0^1 dt \mu I_{-1}(2t\mu)}, \quad (3.2)$$

m	$d\tau \times N_\tau$	N_{trj}	accept	$\langle e^{-\Delta H} \rangle$	$\langle P \rangle$	r_0/a
0.20	0.25×4	396	0.700(29)	0.977(58)	0.68747(21)	4.34(17)
0.10	0.20×5	372	0.850(15)	0.971(18)	0.68752(19)	5.25(30)
0.05	0.25×4	382	0.658(26)	0.996(42)	0.68795(16)	5.44(33)

Table 1: Summary of the simulation. m is the gluino mass, N_{trj} is the number of thermalized trajectory, P is the plaquette, and r_0 is the Sommer scale.

where $\zeta = \lambda \Sigma V$, $\mu = m \Sigma V$, $\tilde{\mu} = \sqrt{\zeta^2 + \mu^2}$, c is a normalization constant, and λ is the eigenvalue. The general formula for the higher eigenvalues is given in [12]. We note that the quenched limit can be obtained by simply taking the limit $\mu \rightarrow \infty$.

4. Results for Dirac spectrum and gluino condensate

Our dynamical simulation for $\mathcal{N} = 1$ SYM with the overlap fermion is performed on a $8^3 \times 16$ lattice with $SU(2)$ gauge link fields. For this small lattice, we need to carefully pick up a gauge action and the parameters β, M_0 to avoid the Aoki phase. The β is chosen in such a way that the lattice has a sufficiently large physical volume, and the M_0 is chosen to guarantee the locality. The hermitian Wilson-Dirac spectrum for the quenched theory suggests that the region for an implementation of the overlap Dirac operator inside two fingers of the Aoki phase is narrower in the present case of the adjoint representation than in the case of the fundamental representation. Based on a Dirac spectrum survey in the quenched theory, we use the Iwasaki gauge action with $\beta = 1.05$ and $M_0 = 1.9$.

The overlap fermion has exact zero modes which are related to the topological charge of the gauge fields by the Atiyah-Singer index theorem. The topology plays a significant role on the small volume for a comparison to ChRMT. In our simulation, the global topological charge is fixed by introducing extra Wilson fermions to produce a Boltzman weight $\det \frac{H_W^2}{H_W^2 + m_{tm}^2}$ where $H_W = \gamma_5 D_W$ is the hermitian Wilson-Dirac operator with $K = \frac{1}{8-2M_0}$. This term suppresses the near zero modes of H_W and generates a gap, which supports the exponential locality of the overlap Dirac operator and reduces the numerical cost for the sign function in the overlap Dirac operator [13]. The inclusion of $\det \frac{H_W^2}{H_W^2 + m_{tm}^2}$ should not affect the low energy physics. We select zero topological charge and small twisted mass of $m_{tm} = 0.01$. The choice of the twisted mass does not affect the low-lying Dirac spectrum in the quenched theory.

With the Hybrid Monte Carlo algorithm, we generated ensembles with three different gluino mass as shown in the table 1. For the low-lying eigenvalue distribution of the overlap Dirac operator, 28 lowest eigenvalues are measured at every other trajectory by implicitly restarted Lanczos method. Note that the adjoint representation gives two-fold degeneracy, and the γ_5 hermiticity enforces eigenvalues to form complex conjugate pairs. Therefore there are essentially only 7 different eigenvalues among 28. Our main interest is in the lowest eigenvalue, for which the data shows clear complex conjugate pairing with two-fold degeneracy.

The integrated autocorrelation time is estimated by the history of the lowest Dirac eigenvalue, which is smaller than 20 trajectories for all three input gluino masses. All statistical errors are estimated by the jackknife method with a binsize of 20 trajectories.

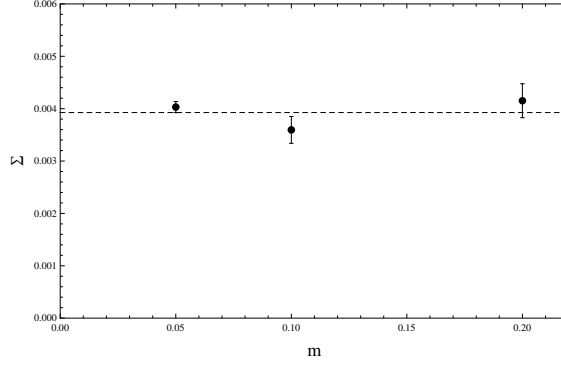


Figure 1: The condensate determined by (4.1) is plotted for each gluino mass. The dashed line shows the constant fit, which is in good agreement with the data points.

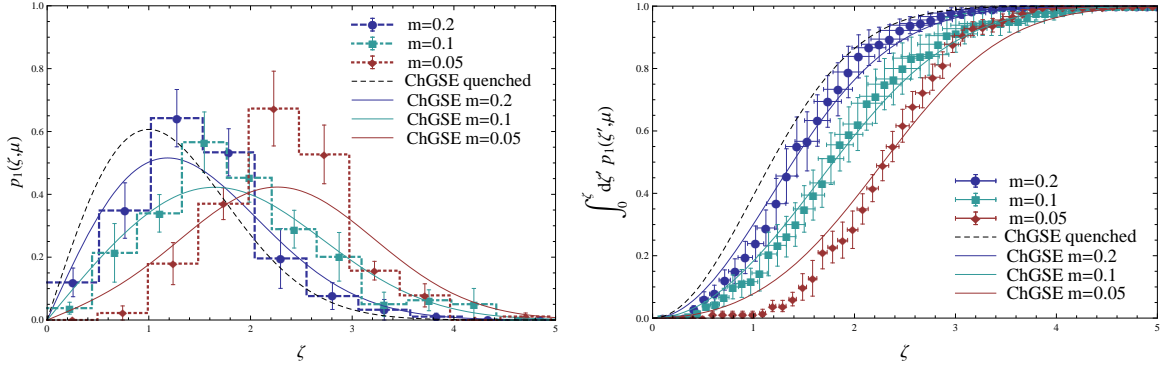


Figure 2: The lowest eigenvalue distribution is compared with ChGSE. Curves are the predictions from the ChGSE for various masses. The condensate determined from (4.1) for each mass was used as an input. The horizontal errorbar (right) reflects the uncertainty of the condensate.

The relationship between ChRMT and the chiral perturbation theory (ChPT) suggests an identification $\zeta = \lambda \Sigma V$, from which we can extract the condensate. The condensate at each gluino mass is determined by comparing the average eigenvalues of lattice data and that of ChGSE.

$$\langle \zeta \rangle_{\text{RMT}} = \int_0^\infty d\zeta \zeta p_1(\zeta, m\Sigma V) = \langle \lambda \rangle_{\text{lat}} \Sigma V. \quad (4.1)$$

Figure 1 shows the gluino mass dependence of Σ . The comparison with ChGSE gives almost the same values of the gluino condensate for different input gluino masses. The mass dependence is weaker than that in QCD, which can be explained by the absence of Nambu-Goldstone modes, or the absence of chiral logarithms. The constant fit gives $\Sigma = 0.00393(18)$. With the Sommer scale obtained from the lightest mass ($m = 0.05$), we obtain $\langle \bar{\psi}\psi \rangle r_0^3 = 0.63(12)$ in the chiral limit.¹

Let us look into more details of the eigenvalue distribution. Figure 2 shows the lowest eigenvalue distribution $p_1(\zeta, \mu)$ (left) and the cumulative distribution $\int_0^\zeta d\zeta' p_1(\zeta', \mu)$ (right) for each mass, where we used the determined condensate value as an input ($\mu = m\Sigma V$). The distributions

¹The value $\Sigma^{1/3} r_0 = 0.86(5)$ is slightly larger than that for QCD in our previous study : $\Sigma^{1/3} r_0 \approx 0.6$.

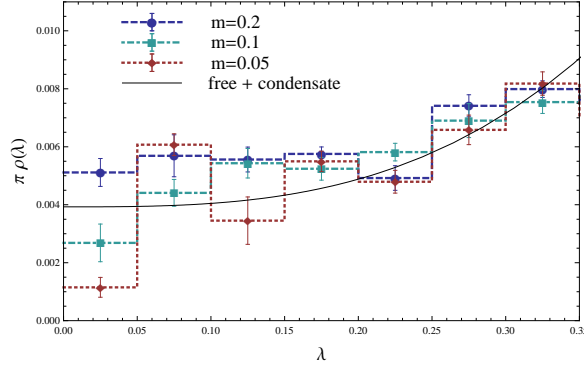


Figure 3: The low-lying eigenvalue distribution is shown for each mass. Black curve shows $\Sigma + \frac{3}{8\pi^2}\lambda^3$, where the second term comes from the free theory in 4-dimension.

show clear mass dependence, and the data at $m = 0.1$ and 0.2 agree well with the prediction from the ChGSE. There is a common tendency that the distribution is narrower than the predicted curves, and this tendency becomes clearest for the lightest mass data. Since ChGOE or ChGUE has wider distribution compared to ChGSE, ChGSE fits best to the data anyway. This deviation is most likely from the higher order corrections of ChPT, which is not considered in RMT.

Figure 3 shows the low-lying eigenvalue distribution, which can be roughly divided into three regions : the lowest edge near $\lambda \approx 0.05$, a plateau region between $0.05 < \lambda < 0.25$, and an increasing region for $\lambda > 0.25$. The overall shape is consistent with the behavior $\frac{3}{8\pi^2}\lambda^3$ of the free theory plus the condensate $0.00393(18)$, which is shown as the black curve in the figure. Note that the plateau values, which are without leading volume corrections, do not depend on masses very much. Since the difference from these values to the obtained condensate value in the chiral limit is around 20%, we expect that the systematic error from the higher order effects are less than 20%.

5. Summary

We have performed a dynamical simulation of 4d $\mathcal{N} = 1$ SYM with the overlap fermion for the gluino on a $8^3 \times 16$ lattice. The exact chiral symmetry of the overlap fermion and the prediction from ChRMT allow us to extract the gluino condensate $\langle \bar{\psi}\psi \rangle r_0^3 = 0.63(12)$ in the chiral limit. The lowest Dirac eigenvalue distribution for two heavy masses ($m = 0.2, 0.1$) nicely matches with ChGSE. For the lightest gluino mass ($m = 0.05$) data, the distribution shows a deviation from the prediction possibly due to corrections from the next order finite volume effect. The expectation value of the lowest Dirac eigenvalue is used to determine the condensate, and the obtained condensate for each mass does not much depend on the mass. This confirms that our strategy works very well in the $\mathcal{N} = 1$ SYM, since there is no Nambu-Goldstone boson nor chiral log effect. The low-lying eigenvalue distribution is consistent with the simple sum of the constant (condensate) and the free theory result.

The only relevant supersymmetry violating operator in the model is the mass term, which can be controlled by taking the chiral limit. Therefore it is interesting to see the violation of the supersymmetry Ward identity and its dependence on the input gluino mass.

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