

Scalar mass corrections from compact extra dimensions on the lattice

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We explore the phase diagram of the $SU(2)$ Yang–Mills theory in 5 dimensions by numerical simulations. The lattice system shows a dimensionally–reduced phase where the extra dimension is small compared to the four-dimensional correlation length. In this phase, at low energies, this system behaves like a four–dimensional gauge theory coupled to an adjoint scalar field.

By tuning the bare parameters of the lattice model, we identify lines of constant physics, and analyse the behaviour of the adjoint scalar mass as a function of the compactification and the cut–off scales.

The perturbative prediction that the effective theory contains a light particle with a mass that is independent of the cut–off is tested against non–perturbative results.

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1. Introduction

Five-dimensional Yang–Mills theories with an extra dimension compactified on a circle predict the existence of a scalar low-energy mode, whose mass renormalisation is suppressed by the remnant of the higher-dimensional gauge symmetry. It is well known that quantum corrections yield divergences in the mass of a scalar particles: the scalar mass receives contributions proportional to the square of the ultra-violet (UV) cut-off. However, the mass of the scalar particle coming from the compactification of a higher-dimensional gauge field remains finite, as suggested by one-loop and two-loop calculations [1].

Since the extra dimensional gauge theory is non-renormalisable, it can only be defined as a regulated theory with an ultra-violet cut-off Λ_{UV} . In this theory it is rather surprising to find quantum corrections to the scalar mass independent of Λ_{UV} and taking the form

$$m_5^2 \equiv \delta m^2 = \frac{9g_5^2 N_c}{32\pi^3 R^3} \zeta(3), \quad (1.1)$$

where ζ is the Riemann Zeta-function, N_c is the number of colours, R is the radius of the extra dimension and g_5^2 is the coupling constant of the five-dimensional Yang–Mills theory. However, Eq. (1.1) is valid only in the regime where there is a scale separation between the compactification scale $\Lambda_R \sim R^{-1}$ and the cutoff Λ_{UV} , because in this case the details of the regularisation can be neglected.

The aforementioned result makes the compactification mechanism a very interesting and promising scenario to protect the mass of scalar particles from cut-off effects. In this proceeding, we report on our recent work [2] where we studied a simple extra dimensional model (cfr. Sec. 2) regularised on the lattice with periodic boundary conditions. Using numerical simulations in the region of phase space where there is a hierarchy of scales $\Lambda_{UV} \gg \Lambda_R$, we are able to study the parametric dependence of the adjoint scalar mass m_5 on the cut-off Λ_{UV} and on the compactification scale Λ_R . This allows us to clarify the status of Eq. (1.1) in the non-perturbative regime.

2. Scales separation in the lattice model

In recent years, there have been a number of studies on the simplest of these extra dimensional theories on the lattice, namely a SU(2) pure gauge theory on a five-dimensional torus with anisotropic couplings [3, 4, 5].

We discretise the pure gauge action using the following *anisotropic* lattice Wilson action:

$$\mathcal{S}_W = \beta_4 \sum_{x; 1 \leq \mu < \nu \leq 4} \left[1 - \frac{1}{2} \text{Re Tr } P_{\mu\nu}(x) \right] + \beta_5 \sum_{x; 1 \leq \mu \leq 4} \left[1 - \frac{1}{2} \text{Re Tr } P_{\mu 5}(x) \right], \quad (2.1)$$

where the two bare coupling constants β_4 and β_5 can be tuned independently. This describes a lattice system with two independent lattice spacings a_4 and a_5 , corresponding respectively to the lattice spacing in the four-dimensional subspace, and in the extra fifth direction; the bigger of the two defines the inverse of the cut-off Λ_{UV} .

The anisotropy of the couplings $\gamma = \sqrt{\frac{\beta_5}{\beta_4}}$ is related to the ratio of the lattice spacings $\xi = a_4/a_5$.

At tree-level $\gamma = \xi$, but quantum corrections make ξ deviate from this value. The relation between ξ and γ for this action has already been studied in bare parameter space and can be found interpolating the data of Ref. [3]. In the following we restrict ourselves to study $\xi \geq 1$ and the cut-off is given by $\Lambda_{UV} \sim a_4^{-1}$.

Two more parameters in the lattice model can be adjusted in order to realise the desired separation of scales; they are N_4 , the number of lattice sites in the usual four directions, and N_5 , the number of lattice sites in the extra dimension. Together with the corresponding lattice spacings, they determine the *physical* size of the system: $L_4 = a_4 N_4$ in four dimensions and $L_5 = 2\pi R = a_5 N_5$ in the fifth dimension.

Using the model described above, we would like to find a region of its parameters space where we observe the following:

- A separation between the compactification scale and the cut-off

$$\Lambda_{UV} \gg \Lambda_R. \quad (2.2)$$

This translates into the following relation for the lattice model parameters

$$\frac{a_5 N_5}{a_4} = \frac{N_5}{\xi} \gg 1, \quad (2.3)$$

and allows us to rely on the results of the five-dimensional theory when describing the low-energy physics.

- A separation between the four-dimensional physics and the cut-off

$$\sqrt{\sigma} \ll \Lambda_{UV}; \quad m_5 \ll \Lambda_{UV}. \quad (2.4)$$

The above relations translate into

$$a_4 \sqrt{\sigma} \ll 1; \quad a_4 m_5 \ll 1. \quad (2.5)$$

If the above relations are true, we expect the long distance physics to be independent of the regularisation details of the theory.

- A separation between the four-dimensional physics and the compactification scale

$$\sqrt{\sigma} \ll \Lambda_R; \quad m_5 \ll \Lambda_R. \quad (2.6)$$

In terms of the lattice model we have

$$\frac{N_5}{\xi} a_4 \sqrt{\sigma} \ll 1; \quad \frac{N_5}{\xi} a_4 m_5 \ll 1. \quad (2.7)$$

If this holds, higher Kaluza–Klein modes do not enter the relevant dynamics for the low-energy physics.

- A scalar mass m_5 in *physical* units independent of the cut-off

$$\frac{m_5^2}{\sigma} \propto \Lambda_R^2, \quad (2.8)$$

as expected from Eq. (1.1).

3. Phase diagram and results from lattice simulations

Since it is crucial for our purposes to simulate the theory in the correct phase, we briefly discuss the current understanding of the phase diagram of the SU(2) pure gauge theory in five dimensions described by the action in Eq. (2.1). Results are available both at $\gamma > 1$ [3, 4] and $\gamma \leq 1$ [5]. The isotropic model, where the lattice spacings are the same $a_4 = a_5$, has a bulk phase transition when all the dimensions are equal and large. The bulk line separates a confined phase ($\sigma > 0$) that is connected to the strong coupling regime from a Coulomb-like phase ($\sigma = 0$) connected to the weak coupling. This bulk transition disappears when the lattice size in any one dimension is decreased below a critical size, L_c , which is the critical length of the Polyakov loop in that direction. Below L_c center symmetry is broken. In this case the phase transition becomes a second order one in the same universality class of the four-dimensional Ising model. We take L_5 to be our compactified length and in Fig. 1 we show the pattern of phase transition for $L_5 = 4a_5$ and $L_5 = 6a_5$ in the region $\gamma > 1$. The main feature is that at fixed lattice geometry the nature of the phase transition line strongly depends on the anisotropy: the second order phase transition related to centre breaking merges into the bulk phase transition when $\gamma \lesssim \gamma_c$. The emerging physical picture tells us that the disappearance of the bulk phase transition happens as soon as the five-dimensional system compactifies; in other words, γ_c defines a critical lattice spacing in the extra dimension a_{5c} that makes $2\pi R = a_5 N_5$ smaller than the critical $L_{5c} = a_{5c} N_5$. The interesting region for our purposes, is at $\gamma > \gamma_c$ and above the line of second order phase transition, where the extra dimension is smaller than its critical value $L_5 < L_{5c}$. We refer to this phase as the *dimensionally reduced* phase, following Ref. [4]. Since this is the first time that this particular region of the phase space is explored with lattice

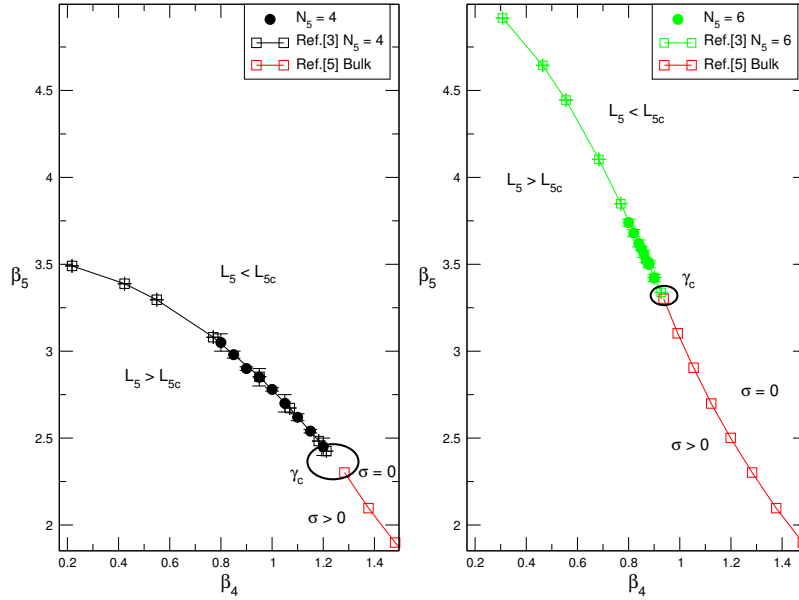


Figure 1: Phase diagram of the five dimensional SU(2) pure gauge lattice model in the (β_4, β_5) plane for different values of N_5 . The bulk phase transition separating a confinement from a Coulomb-like phase disappears for $\gamma > \gamma_c$ into a physical thermal-like phase transition. The location of this transition changes in the parameter space as we change N_5 .

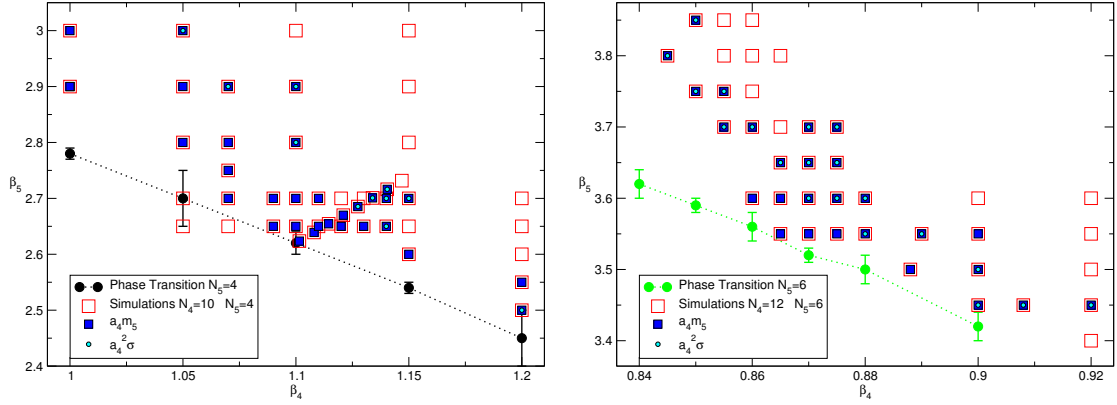


Figure 2: The plots show the region of the phase diagram that we explored with numerical simulations, both for $N_5 = 4$ (left) and $N_5 = 6$ (right). The location of the second order phase transition is also shown. The blue squares are points where the scalar mass $a_4 m_5$ was reliably extracted, whereas the green circles represent points where we were able to measure the string tension $a_4 \sqrt{\sigma}$.

simulations, we performed a broad scan, aiming primarily at identifying the interesting region. As a consequence, there are lattices for which we were unable to measure precisely both the string tension $a_4 \sqrt{\sigma}$ and the scalar mass $a_4 m_5$. In Fig. 2, we show all the simulations' points and at the same time we identify the ones where either $a_4 m_5$ or $a_4 \sqrt{\sigma}$ could not be extracted satisfactorily from correlation functions of suitable gauge invariant operators.

The emergent pattern seems to suggest that the lattice spacing a_4 changes dramatically in these regions of the phase space. For example, looking at Fig. 2, we notice that the string tension $a_4 \sqrt{\sigma}$ can only be measured in a small subset of points; the points closer to the line of second order phase transition are characterised by spatial Polyakov loops whose mass is too high for a signal to be extracted reliably. Since the mass of the loops is given by $N_4 a_4 \sigma$, in this region the lattice spacing a_4 is getting larger in units of the string tension. Following our discussion in Sec. 2, we regard the region close to the line of phase transition as the one characterised by a small cut-off Λ_{UV} . In this region, there is not a clear separation between the low-energy physics and the cut-off, and we expect to observe large discretisation errors. To make things even more interesting, we find the scalar mass $a_4 m_5$ to be small in this same region, where a_4 is large. In fact, it turns out to be very difficult to find points in the phase diagram where both $\sqrt{\sigma}$ and m_5 are separated from the cut-off scale at the same time. This results in a scalar mass $m_5 \sim \sqrt{\sigma}$ for all the points on which we were able to reliably measure the string tension. On the other hand, a *light* scalar does exist very close to the second order transition line, where $a_4 m_5$ is small and $a_4 \sqrt{\sigma}$ is large.

A more quantitative statement can be made by looking at the measured observables as functions of the bare parameters. For example, our data allow us to study the behaviour of $a_4 \sqrt{\sigma}$ at fixed value of β_4 as we change β_5 , and viceversa. The same can be done with $a_4 m_5$. In the left panel of Fig. 3 we see that the scalar mass approaches the cut-off scale $a_4 m_5 \gtrsim 1$ as we move away from the line of second order phase transition at fixed β_4 . While the scalar mass becomes smaller as we approach the critical line, the opposite happens to the string tension, as it is shown in the right panel of Fig. 3. Each point we have simulated on the phase diagram correspond to a precise location in the space given by the three energy scales Λ_{UV} , Λ_R and m_5 . We can therefore translate our results at $N_5 = 4$

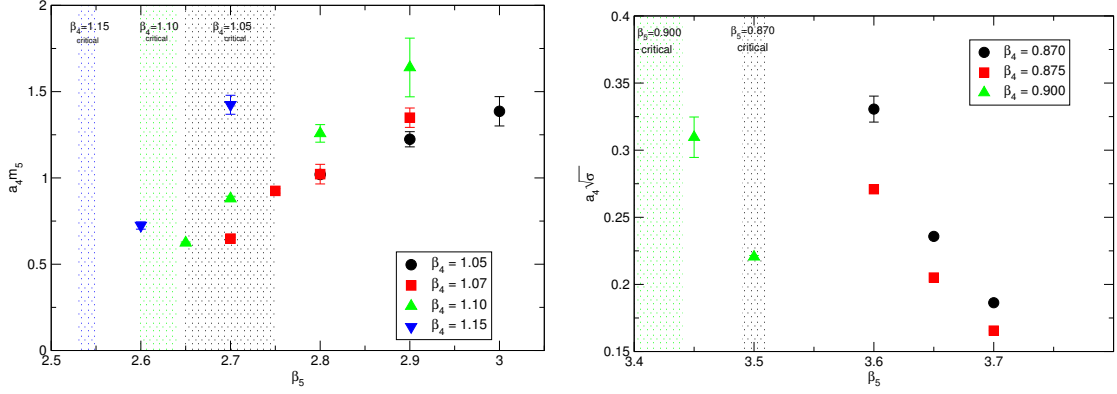


Figure 3: (left) The scalar mass in units of the lattice spacing $a_4 m_5$ as a function of β_5 and for four different values of β_4 at $N_5 = 4$. (right) The string tension in units of the lattice spacing $a_4 \sqrt{\sigma}$ as a function of β_5 and for three different values of β_4 at $N_5 = 6$. In both panels the approximate location of the critical region is shown.

and $N_5 = 6$ into a common set of points $(\Lambda_{UV}, \Lambda_R, m_5)$. This approach allows us to study m_5 as a function of the other two energy scales, instead of the bare parameters. From now on we express the energies Λ_{UV} and Λ_R using their length counterpart, $a_4 \sqrt{\sigma}$ and $R \sqrt{\sigma}$ respectively. This two length scales are related to each other by Eq. (2.3) and they are both measured non-perturbatively: the first is directly measured, whereas the second relies on our interpolation of ξ from Ref. [3]. With our available data, we can explore the behaviour of the scalar mass m_5 in the following region of lattice spacings a_4 and compactification radii R

$$0.15 < a_4 \sqrt{\sigma} < 0.40, \quad 0.05 < R \sqrt{\sigma} < 0.12. \quad (3.1)$$

The major advantage of interpreting the data in this new physical space is that we can disentangle compactification effects from cut-off effects. Since our values for the lattice spacing usually correspond to different compactification radii, we look at the combination $m_5 R$. This is expected to be independent on R , while retaining any dependence on the cut-off. If Eq. (1.1) holds, then $m_5 R$ should be independent of R and a_4 at leading order. In Fig. 4 we plot $m_5 / \sqrt{\sigma}$ as a function of $a_4 \sqrt{\sigma}$ in the left panel, and $m_5 R$ in the right panel. The observed range for the scalar mass in units of the string tension is

$$2 < m_5 / \sqrt{\sigma} < 10, \quad (3.2)$$

whereas the scalar mass in units of the compactification radius is in the range

$$0.2 < m_5 R < 0.5. \quad (3.3)$$

The latter range is smaller by almost one order of magnitude for same interval of lattice spacings. This evidence support the observation that the dependence on $a_4 \sqrt{\sigma}$ is mild.

4. Conclusions

In this work we presented a non-perturbative study of pure SU(2) gauge theory in five dimensions. If the scales of the theory are properly separated, we expect the low-energy dynamics of this

theory to describe a four-dimensional gauge theory coupled to a scalar field. We have measured the mass of the scalar particle in a specific region of the bare parameters space, where we expect to find the desired separation between physical scales. We have also determined numerically the four-dimensional lattice spacing in units of the string tension.

The final picture seems to confirm the possibility of effectively describing a four-dimensional Yang–Mills theory with a scalar adjoint particle in the continuum limit. Even though the search for a light scalar requires fine tuning in this simple model, we have shown that its mass is only very mildly affected by the ultra-violet cut-off, whereas it strongly depends on the radius of the compactified extra dimension. This is entirely compatible with the perturbative result of Eq. (1.1) and it is the first non-perturbative evidence that the mass of scalar particles coming from a compactification mechanism does not have a quadratic dependence on the cut-off.

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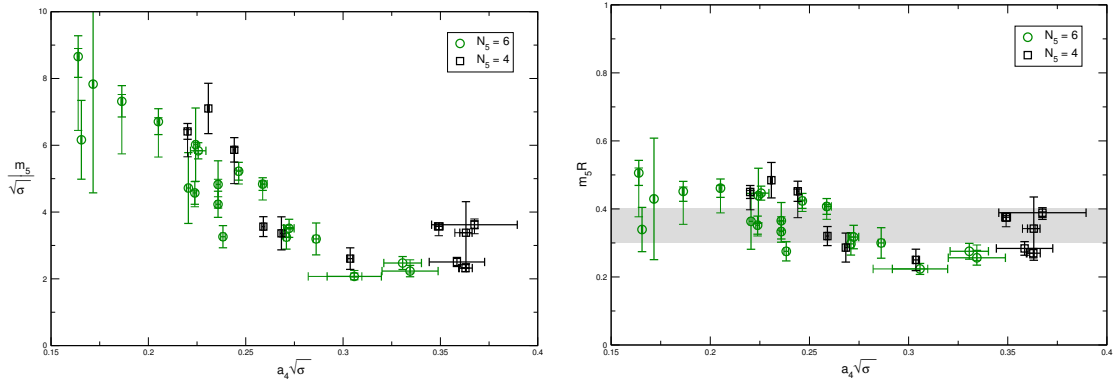


Figure 4: In this plot we show the dependence on the cut-off length given by the lattice spacing $a_4 \sqrt{\sigma}$ for two observables: (left) the scalar mass in units on the string tension and (right) the scalar mass in units of the compactification radius. The grey band includes all points within two standard deviations. When present, systematic errors are shown with thinner error bars, whereas the thicker ones represent standard deviations.