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Lattice study on glueballs in radiative J/ψ decays

Ying Chen[*,†], **Long-Cheng Gui, Yi-Bo Yang**
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
E-mail: cheny@ihep.ac.cn

Chuan Liu
School of Physics, Peking University, Beijing 100871, China

Yu-Bin Liu
School of Physics, Nankai University, Tianjin 300071, China

Jian-Ping Ma
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China

Jian-Bo Zhang
Department of Physics, Zhejiang University, Hangzhou, Zhejiang 310027, China

The electromagnetic decay form factors of J/ψ to the scalar glueball are calculated in the quenched lattice QCD on anisotropic lattices. The continuum extrapolation is carried out by using two different lattice spacings. With these form factors, the partial width of J/ψ radiatively decaying into the pure gauge scalar glueball is predicted to be $0.35(8)\,\mathrm{keV}$, which corresponds to a branch ratio of $3.8(9)\times 10^{-3}$.

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1. Introduction

Glueballs, as the bound states of gluons predicted by quantum chromodynamics (QCD), are still challenging targets both for experimental and theoretical studies. A clue to the existence of a scalar glueball is that there are ten scalar mesons, such as $K^*(1430)$, $a_0(1450)$ and three isoscalars $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, which are close in mass and can be sorted into a $SU(3)$ flavor nonet and a glueball. Quenched lattice QCD studies estimate that the lowest-lying glueballs have masses ranging from 1-3 GeV [1, 2, 3] such that the radiative J/ψ decay is commonly thought to be the ideal hunting ground for glueballs for the abundance of gluons in the decay products. The width of J/ψ radiative decay to a glueball can be expressed as

$$\Gamma(J/\psi \to \gamma G) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{M_{J/\psi}^2} \frac{1}{3} \sum_{r_1,r_2,r_\gamma} \left| M_{r_1,r_2,r_\gamma} \right|^2, \tag{1.1}$$

where $\vec{q}$ is the decay momentum, $M_{J/\psi}$ the mass of J/ψ, M_{r_1,r_2,r_γ} the transition amplitude with r_1, r_2, r_γ the polarizations of J/ψ, the glueball and the photon, respectively. The major task in theory is the calculation of the transition amplitude. There have been several studies on the radiative productions rate of the scalar glueball based on the tree-level perturbative QCD approach and the dispersion relation method [4, 5, 6, 7, 8], but with theoretical uncertainties.

In constrast, lattice QCD can be a theoretically cleaner play ground. To the lowest order of QED, the amplitude M is expressed explicitly as

$$M_{r_1,r_2,r_\gamma} = \varepsilon_\mu^*(\vec{q}, r_\gamma) \langle G(\vec{p}', r_2) | j^\mu(0) | J/\psi(\vec{p}, r_1) \rangle, \tag{1.2}$$

where $\varepsilon_\mu^*(\vec{q}, r_\gamma)$ is the polarization vector of the photon, and $\langle G(\vec{p}', r_2) | j^\mu(0) | J/\psi(\vec{p}, r_1) \rangle$ the on-shell matrix elements of the electromagnetic current between the glueball and J/ψ, which can be derived directly from the lattice QCD calculation of the related three-point functions. As an exploratory study, in this work, we focus on the calculation for the scalar glueball.

2. Numerical details

We use the quenched approximation in this study. The gauge configurations are generated by the tadpole improved gauge action [1] on anisotropic lattices with the temporal lattice much finer than the spatial lattice, say, $\xi = a_s/a_t \gg 1$, where a_s and a_t are the spatial and temporal lattice spacing, respectively. Each configuration is separated by 500 heat-bath updating sweeps to avoid the autocorrelation. The much finer lattice in the temporal direction gives a high resolution to hadron correlation functions, such that masses of heavy particles can be tackled on relatively coarse lattices. The calculations are carried out on two anisotropic lattices, say, $L^3 \times T = 8^3 \times 96$ and $12^3 \times 144$. The relevant input parameters are listed in Table 1, where a_s's are determined from $r_0^{-1} = 410(20)$ MeV. For fermions we use the tadpole improved clover action for anisotropic lattices [9]. The parameters in the action are tuned carefully by requiring that the physical dispersion relations of vector and pseudoscalar mesons are correctly reproduced at each bare quark mass [10]. The bare charm quark masses at different β are determined by the physical mass of J/ψ, $m_{J/\psi} = 3.097$ GeV.

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Table 1: The input parameters for the calculation. Values for the coupling β, anisotropy ξ, the lattice spacing a_s, lattice size, and the number of measurements are listed.

β	ξ	a_s(fm)	La_s(fm)	$L^3 \times T$	N_{conf}
2.4	5	0.222(2)	1.78	$8^3 \times 96$	5000
2.8	5	0.138(1)	1.66	$12^3 \times 144$	5000

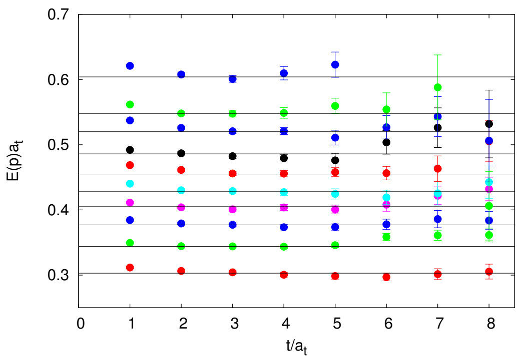

Figure 1: The effective energy plot for the A_1^{++} glueball with different spatial momenta. From top to bottom are the plateaus for momentum modes, $\vec{p} = 2\pi\vec{n}/L$, with $\vec{n} = (2,2,2)$, $(2,2,1)$, $(2,2,0)$, $(2,1,1)$, $(2,1,0)$, $(2,0,0)$, $(1,1,1)$, $(1,1,0)$, $(1,0,0)$, and $(0,0,0)$.

One of the key points is the construction of the interpolating field operator which couples dominantly to the scalar glueball. For this we adopt the variational method along with the single-link and double-link smearing schemes [2, 3]. More specifically, since the irreducible representation A_1^{++} of lattice symmetry group O gives the right quantum number $J^{PC} = 0^{++}$ in the continuum limit, we construct an A_1^{++} operator set $\{\phi_\alpha, \alpha = 1,2,\ldots,24\}$ of 24 different gluonic operators. Through the Fourier transformation,

$$\phi_\alpha(\vec{p},t) = \sum_{\vec{x}} \phi_\alpha(\vec{x},t) e^{-i\vec{p}\cdot\vec{x}}, \tag{2.1}$$

we obtain the operator set $\{\phi_\alpha(\vec{p},t), \alpha = 1,2,\ldots,24\}$ which couples to a A_1^{++} glueball state with the definite momentum $\vec{p}$. For each $\vec{p}$, by solving the generalized eigenvalue problem,

$$\tilde{C}(t_D)\mathbf{v}^{(R)} = e^{-t_D\tilde{m}(t_D)}\tilde{C}(0)\mathbf{v}^{(R)}, \tag{2.2}$$

at $t_D = 1$, where $\tilde{C}(t)$ is the correlation matrix of the operator set,

$$\tilde{C}_{\alpha\beta}(t) = \frac{1}{N_t}\sum_{\tau}\langle 0|\phi_\alpha(\vec{p},t+\tau)\phi_\beta^\dagger(\vec{p},\tau)|0\rangle, \tag{2.3}$$

we obtain an optimal combination of the set of operators, $\Phi(\vec{p},t) = \sum v_\alpha\phi_\alpha(\vec{p},t)$, which overlaps most to the ground state,

$$C(\vec{p},t) = \frac{1}{T}\sum_{\tau}\langle\Phi(\vec{p},t+\tau)\Phi^\dagger(\vec{p},\tau)\rangle \approx \frac{|\langle 0|\Phi(\vec{p},0)|S(\vec{p})\rangle|^2}{2E_S V_3} e^{-E_S t} \approx e^{-E_S t}, \tag{2.4}$$

where the normalization $C(\vec{p},0)=1$ is also used. This is actually the case that $C(t)$ can be well described by a single exponential, $C(t)=We^{-Et}$, with W's usually deviating from one by few percents. Figure 1 shows the effective energy plateaus of the A_1^{++} glueball for typical momentum modes, where one can see that the plateaus start even from the $t=1$.

The other key point of this work is the large statistics. 5000 configurations are generated for both lattice systems. In order to increase the statistics more, for each configuration, we calculate T charm quark propagators $S_F(\vec{x},t;\vec{0},\tau)$ by setting point source on each time-slice τ, which permit us to average over the temporal direction when calculating the three-point functions. Therefore, the three-point functions we calculate in this work are

$$\begin{aligned}
\Gamma^{(3)}_{\mu,j}(\vec{p}_f,\vec{q};t_f,t) &= \frac{1}{T}\sum_{\tau=0}^{T-1}\sum_{\vec{y}} e^{i\vec{q}\cdot\vec{y}}\langle\Phi(\vec{p}_f,t_f+\tau)J_\mu(\vec{y},t+\tau)O_{V,j}(\vec{0},\tau)\rangle \\
&= \frac{1}{T}\sum_{\tau=0}^{T-1}\sum_{\vec{y}} e^{i\vec{q}\cdot\vec{y}}\langle\Phi(\vec{p}_f,t_f+\tau)\mathrm{Tr}\left[\gamma_\mu S_F(\vec{y},t+\tau;\vec{0},\tau)\gamma_j\gamma_5 S_F^\dagger(\vec{y},t+\tau;\vec{0},\tau)\gamma_5\right]\rangle \\
&= \sum_{S,V,r}\frac{e^{-E_S(t_f-t)}e^{-E_V t}}{2E_S(\vec{p})V_3 2E_V}\langle 0|\Phi(\vec{p}_f,0)|S(\vec{p}_f)\rangle\langle S(\vec{p}_f)|J_\mu(0)|V(\vec{p}_i,r)\rangle\langle V(\vec{p}_i,r)|O^\dagger_{V,j}(0)|0\rangle,
\end{aligned} \tag{2.5}$$

where $J_\mu(x)=\bar{c}(x)\gamma_\mu c(x)$ is the electro-magnetic current, $O_{V,i}=\bar{c}\gamma_i c$ the conventional interpolation field for the vector charmonium, and the summation in the last equality is over all the possible states and vector polarizations, $\vec{p}_i$ is the spatial momentum of the initial vector charmonium and satisfies the relation $\vec{p}_i=\vec{p}_f-\vec{q}$. Obviously the conserved vector current $J_\mu(x)$ in the continuum limit is not conserved any more on the lattice due to the broken Lorentz symmetry, and should be renormalized properly. In this work, we adopt the non-perturbative strategy proposed by Ref. [11] to define the renormalization constant,

$$Z_V^\mu(t)=\frac{p^\mu}{2E(p)}\frac{\Gamma^{(2)}_{\eta_c\eta_c}(\vec{p},t_f=T/2)}{\Gamma^{(3)}_{\eta_c\gamma_\mu\eta_c}(\vec{p}_f=\vec{p}_i=\vec{p},t_f=T/2,t)}, \tag{2.6}$$

where $\Gamma^{(2)}_{\eta_c\eta_c}$ is the two-point function of the pseudoscalar charmonium η_c, $\Gamma^{(3)}_{\eta_c\gamma_\mu\eta_c}$ the corresponding three point function with the vector current insertion on one of the quark lines. It should be remarked that, the possible disconnected diagrams due to the charm and anti-charm quark annihilation are neglected in this work in the calculation of all the relevant two-point and three-point functions.

The parameters E_S, E_V, the matrix elements $\langle 0|\Phi(\vec{p}_f,0)|S(\vec{p}_f)\rangle$ and $\langle 0|O_{V,j}|V(\vec{p}_i,r)\rangle$ can be derived from the relevant two-point functions of glueballs and J/ψ. Specifically, from Eq. 2.4 we have

$$\langle 0|\Phi(\vec{p}_f,0)|S(\vec{p}_f)\rangle\approx\sqrt{2E_S(\vec{p}_f)V_3}, \tag{2.7}$$

and by definition, we also have the relation,

$$\langle 0|O_{V,j}(0)|V(\vec{p},r)\rangle=f_V\varepsilon_j(\vec{p},r), \tag{2.8}$$

where f_V is a parameter independent of $\vec{p}$, and $\varepsilon_j(\vec{p},r)$ the polarization vector of the vector meson, whose concrete expression depends on reference frames and is irrelevant to the calculation in this work. By using the multi-pole decomposition, the matrix elements $\langle S(\vec{p}_f)|J_\mu(0)|V(\vec{p}_i,r)\rangle$ can be written as [11],

$$\sum_r \langle S(\vec{p}_f)|J_\mu(0)|V(\vec{p}_i,r)\rangle \varepsilon_j(\vec{p}_i,r) = \alpha_{\mu j}E_1(Q^2) + \beta_{\mu j}C_1(Q^2), \tag{2.9}$$

where $\alpha_{\mu i}$ and $\beta_{\mu i}$ are known functions of p_f and p_i (their explicit expressions are neglected here), $E_1(Q^2)$ and $C_1(Q^2)$ are the two form factors which depend only on $Q^2 = -(p_f - p_i)^2$. The form factor $E_1(Q^2)$ will enter the formula of the the radiative decay width of J/ψ to the scalar glueball as follows,

$$\Gamma(J/\psi \to \gamma G_{0^{++}}) = \frac{4}{27}\alpha \frac{|\vec{p}_\gamma|}{M^2_{J/\psi}} |E_1(0)|^2, \tag{2.10}$$

where α is the fine structure constant, p_γ the photon momentum with $|\vec{p}_\gamma| = (M^2_{J/\psi} - M^2_G)/(2M_{J/\psi})$. Therefore we only focus the the extraction of $E_1(Q^2)$ in this work.

In practice, we let the J/ψ to have momentum $\vec{p}_i = \vec{0}$ and $|\vec{p}_i| = 2\pi/La_s$, and make the scalar glueball moving with a momentum $\vec{p}_f = 2\pi\vec{n}/La_s$ with $\vec{n}$ ranging from $(0,0,0)$ to $(2,2,2)$. Among all the combinations of the vector current index μ, the polarization index j, the glueball momentum p_f and the J/ψ momentum p_i, it is found that there are specific combinations which gives $\alpha_{\mu i}(p_f,p_i) = 1$ and $\beta_{\mu i}(p_f,p_i) = 0$, and are thereafter taken into consideration in the practical data analysis. An additional benefit of this selection is that, for the specific polarization j and J/ψ momentum $\vec{p}_i$, one has, $\sum_r \varepsilon^*_j(\vec{p}_i,r)\varepsilon_j(\vec{p}_i,r) = 1$.

With these prescriptions, the form factor $E_1(Q^2)$ can be derived as,

$$\tilde{E}_1(Q^2,t_f,t) \approx \frac{Z_V^{(s)}\Gamma^{(3)}(\vec{p}_f,\vec{p}_i;t_f,t)}{C(\vec{p}_f,t_f-t)\Gamma^{(2)}(\vec{p}_i,t)} f_V \sqrt{2E_S(\vec{p}_f)V_3} \ \ (t, t_f - t \gg 0), \tag{2.11}$$

where Q^2 can be given by $\vec{p}_i$ and $\vec{p}_f$, the indices of the three-point function $\Gamma^{(3)}$ and the related two-point functions $\Gamma^{(2)}$ are omitted here, $Z_V^{(s)}$ is the renormalization constant of the spatial components of the vector current. In practice, the symmetric indices and momentum combinations which give the same Q^2 are averaged to increase the statistics. Traditionally, the time separation t and $t_f - t$ should be kept large enough for the ground states to contribute dominantly to the three point function. Even with this large statistics, we find that the signal of the glueball damps rapidly with respect to $t_f - t$. However, this is not a real disaster since the optimal glueball operators we use couple almost exclusively to the ground state, as is mentioned before. So we fix $t_f - t = 1$ with t varying and extract $E_1(Q^2)$ from the plateaus of $\tilde{E}_1(Q^2,t_f,t)$. With the very high statistics in this work, the hadron parameters, such as the energies of the glueball and J/ψ, the constant f_V in Eq. 2.8 and the matrix elements $\langle 0|\Phi(\vec{p},0)|S(\vec{p})\rangle$ can be determined very precisely and are treated as known parameters.

Since $E_1(Q^2)$ for different Q^2 are extracted from the same configuration ensemble and are therefore highly correlated, in the data analysis we fit them through the correlated data fitting. For each lattice system, the 5000 configurations are divided into 100 bins with 50 configurations in each bin, and the measurements in each bin are averaged to be taken as an independent measurement. After that, all the $E_1(Q^2)$ are extracted simultaneously through the jackknife method. In order

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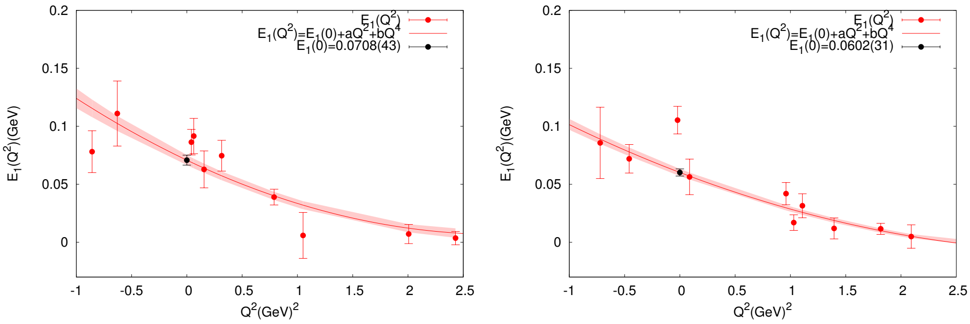

Figure 2: The extracted form factors $E_1(Q^2)$ in the physical units. The left panel is for $\beta = 2.4$ and the right one for $\beta = 2.8$. The curves with error bands show the polynomial fit with $E_1(Q^2) = E_1(0) + aQ^2 + bQ^4$, as the black dot is the interpolated value $E_1(0)$ at $Q^2 = 0$.

Table 2: Listed in the table are the A_1^{++} glueball masses M_G, the renormalization constants $Z_V^{(s)}(a)$ of the spatial component of the vector current, and the form factors $E_1(Q^2 = 0, a)$ calculated on the two lattices with $\beta = 2.4$ and 2.8, respectively. Also shown are the continuum extrapolation of $E_1(0)$ and the resultant partial width Γ.

β	M_G(GeV)	$Z_V^{(s)}(a)$	$E_1(0,a)$ (GeV)	Γ(keV)
2.4	1.360(9)	1.39(2)	0.0708(43)	-
2.8	1.537(7)	1.15(1)	0.0602(31)	-
∞	1.710(90) [3]	-	0.0536(57)	0.35(8)

to get the form factor at $Q^2 = 0$, we carry out a correlated polynomial fit to the $E_1(Q^2)$ from $Q^2 = -1\,\text{GeV}^2$ to $2.5\,\text{GeV}^2$,

$$E_1(Q^2) = E_1(0) + aQ^2 + bQ^4. \tag{2.12}$$

Figure 2 shows the final results of $E_1(Q^2)$ for $\beta = 2.4$ (left panel) and $\beta = 2.8$ (right panel), where the red points are the calculated value with jackknife errors, and the red curves are the polynomial fit with jackknife error bands, the black points label the interpolated $E_1(0,a)$.

The last step is the continuum extrapolation using the two lattice systems. Since we have only two different lattice spacings, we only do the linear extrapolation in a_s^2. The continuum limit of $E_1(0,a)$ is determined to be $E_1(0) = 0.0536(57)\,\text{GeV}$. In order to get the continuum decay width, the continuum limit of scalar glueball mass should be also extrapolated, For which we quote the value $M_G = 1.710(90)$ [3]. Thus, according to Eq. 2.10, we finally get the decay width $\Gamma(J/\psi \to \gamma G_{0^{++}}) = 0.35(8)\,\text{keV}$. Using the reported total width of J/ψ, $\Gamma_{\text{tot}} = 92.9(2.8)\,\text{keV}$, the corresponding branch ratio is

$$\Gamma(J/\psi \to \gamma G_{0^{++}})/\Gamma_{\text{tot}} = 3.8(9) \times 10^{-3}. \tag{2.13}$$

3. Summary and Discussion

We carry out the first lattice study on the E_1 decay amplitude of J/ψ radiatively decaying into the pure gauge scalar glueball $G_{0^{++}}$ in the quenched approximation, where glueballs are well defined objects. With two different lattice spacings, the amplitude, which is derived from the matrix elements of the electromagnetic current of the charm quark between the scalar glueball and J/ψ, is extrapolated to the continuum limit with a value $E_1(Q^2=0)=0.0536(57)\,\text{GeV}$. Thus the partial decay width $\Gamma(J/\psi\to\gamma G_{0^{++}})$ is predicted to be $0.35(8)\,\text{keV}$, which gives the branch ratio $\Gamma/\Gamma_{\text{tot}}=3.8(9)\times10^{-3}$. This result can be compared with the production rates of $f_0(1710)$, $f_0(1500)$ and $f_0(1370)$ in the radiative decays of J/ψ. From PDG2010 [12], the branch ratios of the observed radiative decay modes of J/ψ to $f_0(1710)$ are $Br(J/\psi\to\gamma f_0(1710)\to\gamma K\bar{K})=8.5^{+1.2}_{-0.9}\times10^{-4}$, $Br(J/\psi\to\gamma f_0(1710)\to\gamma\pi\pi)=(4.0\pm1.0)\times10^{-4}$, $Br(J/\psi\to\gamma f_0(1710)\to\gamma\omega\omega)=(3.1\pm1.0)\times10^{-4}$, which add up to 1.5×10^{-3}. If one goes further to take the branch ratio $Br(f_0(1710)\to K\bar{K})=0.36\pm0.12$ [13], and the ratio $\Gamma(f_0(1710)\to\pi\pi)/\Gamma(f_0(1710)\to K\bar{K})=0.41^{+0.11}_{-0.17}$ [14], one can estimate the production rate of $f_0(1710)$ to be $(2.4\pm0.8)\times10^{-3}$ or $(2.7\pm1.3)\times10^{-3}$ [15], which is compatible with our result. As for the $f_0(1500)$, BESII reported a branch ratio $Br(J/\psi\to\gamma f_0(1500))=(1.01\pm0.32)\times10^{-4}$ [12, 14], which is much smaller than our prediction. It should be mentioned that a glueball-$q\bar{q}$ meson mixing model study [16] claims that $f_0(1710)$ is composed primarily of a scalar glueball, and expects that $\Gamma(J/\psi\to\gamma f_0(1710))\gg\Gamma(J/\psi\to\gamma f_0(1500)$. At last, there is no evidence of the production of $f_0(1370)$ in the J/ψ radiative decays.

To summarize, even though the systematic uncertainties owing to the quenched approximation are not under control in this work, our results are helpful to the identification of the scalar glueball both from the phenomenology and experiments.

References

[1] C.J. Morningstar and M. Peardon, Phys. Rev. D **56**, 4043 (1997).

[2] C.J. Morningstar and M. Peardon, Phys. Rev. D **60**, 034509 (1999).

[3] Y. Chen *et al.*, Phys. Rev. D **73**, 014516 (2006).

[4] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Vainshtein, Nucl. Phys. B **165**, 55 (1980).

[5] B.A. Li and Q.X. Shen, Phys. Lett. B**126**, 416 (1981).

[6] J.F. Donoghue, K. Johnson and B.A. Li, Phys. Lett. B**99**, 125 (1983).

[7] Y. Liang, K.F. Liu *et al.*, Phys. Lett. B**307**, 375 (1993).

[8] H.B. Meyer, JHEP**01**, 071 (2009).

[9] Chuan Liu, Junhuan Zhang, Ying Chen, J.P. Ma, Nucl. Phys. B **624**, 360 (2002).

[10] S. Su, L. Liu, X. Li, and C. Liu, Int. J. Mod. Phys. A **21**, 1015 (2006), Chin. Phys. Lett. **22**, 2198 (2005).

[11] J.J. Dudek, R.G. Edwards, and D.G. Richards, Phys. Rev. D **73**, 074507 (2006).

[12] K. Nakamura *et al.*, J. Phys. D **37**, 075021 (2010)

[13] M. Albaladejo *et al.*[ZEUS Collaboration], Phys. Rev. Lett. **101**, 52002 (2008).

[14] M. Ablikim *et al.*[BES Collaboration], Phys. Lett. B **642**, 441 (2006).

[15] H.Y. Cheng, private communication.

[16] H.Y. Cheng, C.K. Chua, and K.F. Liu, Phys. Rev. D **74**, 094005 (2006).

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