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The 1405 MeV Lambda Resonance in Full-QCD

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The negative-parity ground state of the Λ baryon lies surprisingly low in mass. At 1405.1 MeV, it lies lower than the negative-parity ground state nucleon, even though it has a valence strange quark. Using the PACS-CS $(2+1)$-flavour full-QCD ensembles available through the ILDG, we employ a variational analysis using source and sink smearing to investigate the low-energy, odd-parity spectrum of the Λ baryon. We isolate two low-lying odd-parity states, with the lowest state approaching the $\Lambda(1405)$ in the physical limit. Moreover, for the first time we reproduce the correct level ordering with respect to the nearby multiparticle scattering thresholds.

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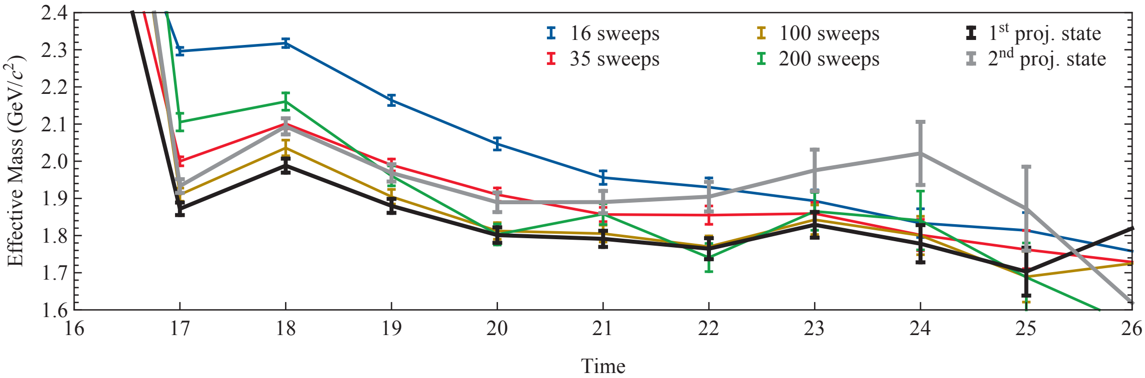

Figure 1: Comparison between unprojected and eigenstate-projected effective masses.

1. The Λ(1405)

At $1405.1^{+1.3}_{-1.0}$ MeV [1], the lowest-lying $J^P = \frac{1}{2}^-$ state of the Λ baryon lies abnormally low in mass. Even though it possesses a valence strange quark, is lies lower than the lowest-lying, odd-parity state of the nucleon, the $N(1535)$. It also lacks a nearby spin-orbit partner, with the lowest spin-$3/2^-$ state being the Λ(1520). The internal structure of this unusual resonance has mystified researchers for many years. While on one hand it is regarded as a conventional three-quark state, on the other there is much discussion of a more exotic kaon-nucleon bound state.

There have been quite a few Lattice QCD studies of the Λ baryon, however most have used the quenched approximation, and none have identified an odd-parity state low enough in energy to be identified with the Λ(1405) [2–6]. Most recently, Takahasi and Oki identified two nearly degenerate states at around 1.6 GeV [6].

Recently, the CSSM Lattice Collaboration has developed an effective technique for isolating the P_{11}(1440) Roper resonance using source and sink smearing together with a variational analysis [7, 8]. We use the same methods in this study of the odd-parity, spin-1/2 sector of the Λ baryon. We find three low-lying states that are low-enough to correspond to the physical Λ(1405), Λ(1670), and Λ(1800) S_{01} states of Nature. Moreover, we report the correct level-ordering with respect to the nearby $\pi\Sigma$ and $\overline{\text{K}}$N multiparticle scattering thresholds

2. Lattice Techniques

To isolate individual states, we use the variational method [9, 10]. This takes advantage of the information contained in the cross-correlation functions between different sources and sinks to project out correlation functions for individual energy eigenstates.

Even though the Λ(1405) is the lowest odd-parity state of the Λ baryon, such a technique is necessary as there are another two nearby low-lying states (the Λ(1670) and Λ(1800)). Since SU(3)-flavour symmetry is broken by the heavy strange quark, all three of these states will survive until the noise limit in the correlation function, and hence the usual, long-time approximation (Eq. 2.5) normally used to extract the ground state will only resolve a mixture of these states. As indicated in Fig. 1, the variational technique allows us to individually isolate the states [11].

In comparison to the Roper resonance, where significant finite-size effects appear in the form of avoided level crossings from interactions between the baryon and multiparticle scattering states, the low-energy, odd-parity $I = 0$ sector is quite clean. The $\Lambda(1405)$ is relatively independent of the box size, and the two nearby multiparticle scattering thresholds, $\pi\Sigma$ and $\overline{\mathrm{K}}\mathrm{N}$, lie, respectively, below and above the $\Lambda(1405)$ and do not cross for the lattice volumes depicted in Fig. 2. As such, finite-size effects are benign in this analysis.

Consider a set of N operators $\chi_i(\mathbf{x},t)$ that couple to our baryon of interest, and construct the $N \times N$ correlation matrix or parity-projected correlation functions

$$G^{\pm}_{ij}(\mathbf{p},t) = \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \operatorname{tr}\left(\Gamma_{\pm}(\mathbf{p}) \langle \Omega | \chi_i(\mathbf{x},t) \overline{\chi}_j(\mathbf{0},0) | \Omega \rangle\right), \tag{2.1}$$

where $\Gamma_{\pm}(\mathbf{p})$ are the parity projection operators at momentum $\mathbf{p}$ used to project into definite positive- or negative-parity, and the trace is taken over the (implicit) spinor indices. Working at zero momentum, these projectors are $\Gamma_{\pm}(\mathbf{0}) := (\gamma_0 \pm 1)/2$, and then

$$G^{\pm}_{ij}(\mathbf{0},t) = \sum_{\mathbf{x}} \operatorname{tr}\left(\frac{\gamma_0 \pm 1}{2} \langle \Omega | \chi_i(\mathbf{x},t) \overline{\chi}_j(\mathbf{0},0) | \Omega \rangle\right) = \sum_{\alpha} \lambda^{\alpha\pm}_i \overline{\lambda}^{\alpha\pm}_j \mathrm{e}^{-m^{\pm}_{\alpha} t}. \tag{2.2}$$

The α index enumerates the energy eigenstates of parity $\pm$ with mass $m^{\pm}_{\alpha}$, and $\lambda^{\alpha\pm}_i$ and $\overline{\lambda}^{\alpha\pm}_j$ are the couplings of the operators χ_i and $\overline{\chi}_j$ to these eigenstates.

The t dependence is only in the exponential terms, and so we look for a linear combination of operators $\phi^{\alpha} = \sum_i v^{\alpha}_i \chi_i$ and $\overline{\phi}^{\alpha} = \sum_i u^{\alpha}_i \overline{\chi}_i$ such that

$$G^{\pm}(t_0 + \Delta t)\mathbf{u}^{\alpha} = \mathrm{e}^{-m^{\pm}_{\alpha}\Delta t} G^{\pm}(t_0)\mathbf{u}^{\alpha} \quad \text{and} \quad \mathbf{v}^{\alpha\mathrm{T}} G^{\pm}(t_0 + \Delta t) = \mathrm{e}^{-m^{\pm}_{\alpha}\Delta t} \mathbf{v}^{\alpha\mathrm{T}} G^{\pm}(t_0) \tag{2.3}$$

for sufficiently large t_0 and Δt. That is, $\mathbf{u}^{\alpha}$ and $\mathbf{v}^{\alpha}$ are the generalised eigenvectors of the matrices $G^{\pm}(t_0)$ and $G^{\pm}(t_0 + \Delta t)$. These eigenvectors will diagonalise the correlation matrix at times t_0 and $t_0 + \Delta t$ through $\mathbf{v}^{\alpha\mathrm{T}} G^{\pm} \mathbf{u}^{\beta} \propto \delta^{\alpha\beta} \mathrm{e}^{-m_{\alpha} t}$, and this lets us define eigenstate-projected correlation functions

$$G^{\pm}_{\alpha}(t) := \mathbf{v}^{\alpha\mathrm{T}} G^{\pm} \mathbf{u}^{\alpha}. \tag{2.4}$$

These new correlation functions should contain isolated energy eigenstates (that is, individual baryon states), and these can now be analysed using standard effective mass techniques,

$$m^{\pm}_{\alpha}(t) = \ln \frac{G^{\pm}_{\alpha}(t)}{G^{\pm}_{\alpha}(t+1)}. \tag{2.5}$$

More details can be found in [7, 12].

As the Λ baryon lies in the centre of approximate SU(3)-flavour, we can consider operators of different flavour-symmetry structures in addition to the usual various Dirac structures. Initially, we consider just the “comon” interpolating operators [13]

$$\begin{aligned} \chi^{\mathrm{c}}_1(x) &:= \frac{1}{2}\varepsilon^{abc}\left((u^{a\mathrm{T}}(x) C\gamma_5 s^b(x)) d^c(x) - (d^{a\mathrm{T}}(x) C\gamma_5 s^b(x)) u^c(x)\right) \text{ and} \\ \chi^{\mathrm{c}}_2(x) &:= \frac{1}{2}\varepsilon^{abc}\left((u^{a\mathrm{T}}(x) C s^b(x)) \gamma_5 d^c(x) - (d^{a\mathrm{T}}(x) C s^b(x)) \gamma_5 u^c(x)\right). \end{aligned} \tag{2.6}$$

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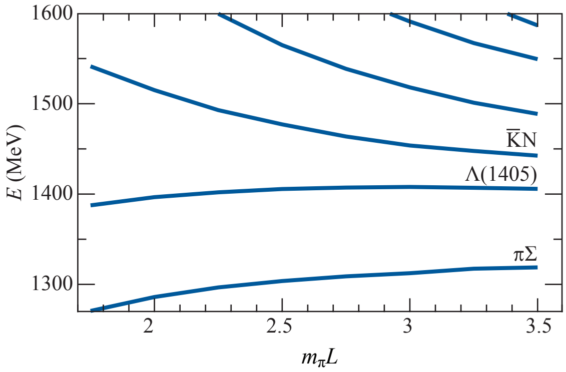

Figure 2: Energy spectrum of the $I = 0$, $S = 1$, $J^P = 1/2^-$ meson-baryon sector obtained from the $\overline{\mathrm{K}}$N Jülich model of hadron exchange [17], showing small finite-size effects for the $\Lambda(1405)$. Adapted from [17].

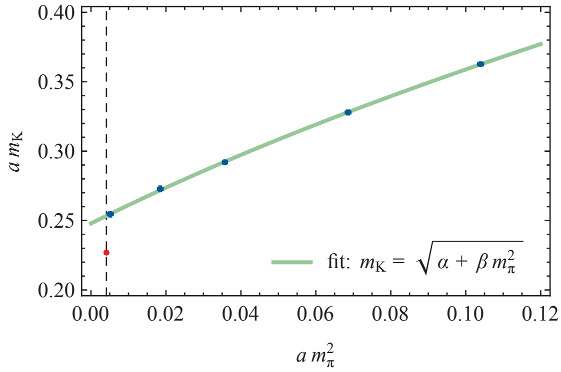

Figure 3: The kaon mass plotted against m_π^2 in lattice units on the PACS-CS ensembles, as quoted in [15]. Extrapolating to the physical limit using $m_K^2 = \alpha + \beta m_\pi^2$, the kaon is approximately 60 MeV higher than the physical kaon.

These isospin-0 operators make no assumptions about the flavour-symmetry structure of the baryon, and couple to all states of the Λ.

To round out our operator basis, similarly to the method for isolating the Roper resonance we employ gauge-invariant Gaussian smearing [14] in the spatial dimensions at both the source and sink. This is an iterative procedure; with $\psi_0(\mathbf{x},t)$ as the original field, for the i-th sweep, we take

$$\psi_i(\mathbf{x},t) = \sum_{\mathbf{x}'} F(\mathbf{x},\mathbf{x}')\psi_{i-1}(\mathbf{x}',t), \tag{2.7}$$

where the smearing function is

$$F(\mathbf{x},\mathbf{x}') = (1-\alpha)\delta_{\mathbf{x},\mathbf{x}'} + \frac{\alpha}{6}\sum_{\mu=1}^{3}\left(U_\mu(\mathbf{x},t)\delta_{\mathbf{x}',\mathbf{x}+\hat{\mu}} + U_\mu^\dagger(\mathbf{x}-\hat{\mu},t)\delta_{\mathbf{x}',\mathbf{x}-\hat{\mu}}\right). \tag{2.8}$$

We use $\alpha = 0.7$ in our calculations.

We use the PACS-CS $(2+1)$-flavour full-QCD ensembles [15], available through the ILDG [16]. They have a lattice extent of $32^3 \times 64$ with $\beta = 1.90$, giving a lattice spacing of $a = 0.0907(13)$ fm. There are 5 light quark masses available with hopping parameters $\kappa_{\mathrm{u,d}} = 0.13700$, 0.13727, 0.13754, 0.13770, and 0.13781, corresponding to pion masses ranging from 702 MeV down to 156 MeV. The strange quark mass is the same for all light quark masses, with hopping parameter $\kappa_s = 0.13640$. However, this is slightly too high to reproduce the physical kaon mass. Plotting the kaon mass data provided in [15] against m_π^2, as in Fig. 3, and extrapolating to the physical limit, we see that the kaon lies approximately 60 MeV too high.

3. Results

The first step in our analysis is to determine the optimal parameters t_0 and Δt, and operators χ_i to use in the variational analysis. We compare the masses extracted from both the eigenvalues ($\mathrm{e}^{-m_\alpha^\pm \Delta t}$) and the fits to the projected effective masses (Eq. 2.5) across parameters ranges of $t_0 \in \{17,\ldots,21\}$ and $\Delta t \in \{1,\ldots,5\}$ (with the source located at $t = 16$), as depicted in Fig. 4, and

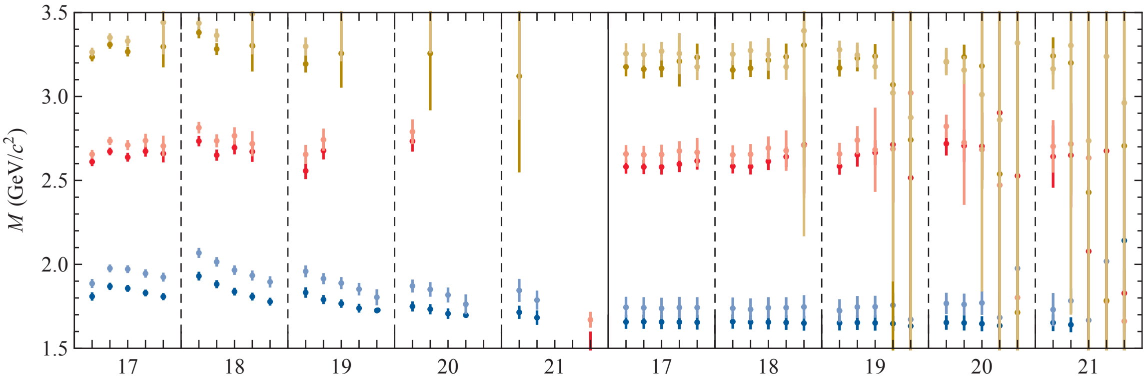

Figure 4: Comparison of the mass as extracted from the eigenvalues (left) and from fitting the projected effective mass (right) with $\kappa_{\mathrm{u,d}} = 0.13770$ for $t_0 \in \{17, \ldots, 21\}$ and $\Delta t \in \{1, \ldots, 5\}$. Numbers of the abscissa indicate t_0 with Δt increasing within each t_0. Complex eigenvalues are not displayed.

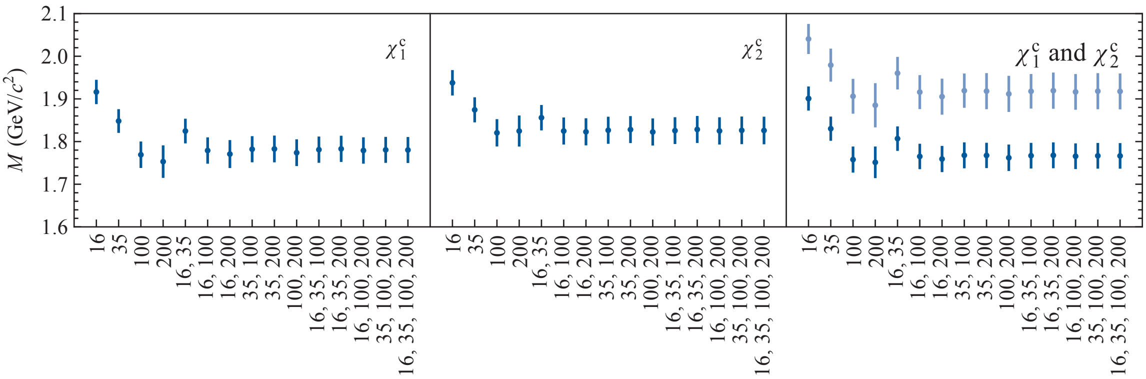

Figure 5: Comparison of the lowest-lying masses extracted from fitting the projected effective mass for $\kappa_{\mathrm{u,d}} = 0.13727$ over bases formed from combinations of either or both ”common“ operators χ_1^c and χ_2^c and all smearing levels.

find that while the eigenvalues show a significant dependence on parameters, the fits to the projected effective masses are stable for sufficiently large t_0 and Δt. We select $t_0 = 18$ and $\Delta t = 2$ as representative.

In Fig. 5, we investigate the dependence of the spectrum on the operator basis used to form the correlation matrix. We find that we require both common interpolating operators χ_1^c and χ_2^c to isolate the lowest state. Using either of these operators individually results in only a mixed state. We cannot resolve the three low-lying states that are present in the physical spectrum with these operators. If we include sufficiently large amounts of smearing, there is little to separate the bases. We select the dimension-6 basis formed from χ_1^c and χ_2^c at 16, 100, and 200 sweeps of smearing.

Using the variational parameters and basis selected above, we calculate the spectrum of the odd-parity, spin-1/2 sector of the Λ baryon for all available quark masses. Plotted in Fig. 6, we see that we do have low-lying states that are trending towards the physical values, however the lowest-lying state sits too high to approach the $\Lambda(1405)$, especially given that finite-size effects are expected to be negligible. However, the slightly-too-heavy strange quark interferes with a comparison with the physical values, and so we calculate the nearby $\pi\Sigma$ and $\overline{\mathrm{K}}\mathrm{N}$ multiparticle scattering thresholds (plotted on the same figure). Our lowest-lying state lies between these thresholds, as in

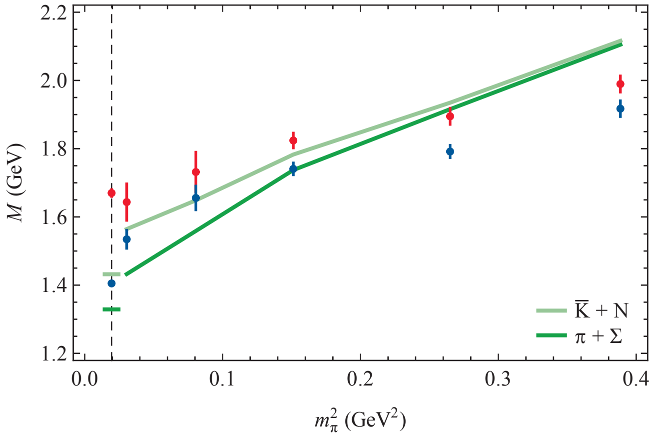

Figure 6: The lowest-lying eigenstate-projected masses plotted against m_π^2, along with the nearby multiparticle scattering thresholds. A correlated error analysis indicates the lowest-lying odd-parity state lies more than one standard deviation below the $\overline{\mathrm{K}}$N scattering threshold at the lightest quark mass. The ordering of the states is in accordance with Nature.

Nature, indicating that we have isolated the Λ(1405). To gain insight into the dependence of the spectrum on the strange quark mass, future work will repeat our analysis using a partially quenched strange quark on the lightest two up and down quark masses.

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