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Nucleon scalar matrix elements with $N_f = 2+1+1$ twisted mass fermions

Simon Dinter[a], **Vincent Drach**[*a], **Karl Jansen**[a]
[a]*NIC, DESY Zeuthen, Platanenallee 6, D-15738 Zeuthen, Germany*

E-mail: `vincent.drach@desy.de`

We investigate scalar matrix elements of the nucleon using $N_f = 2+1+1$ flavors of maximally twisted mass fermions at a fixed value of the lattice spacing of $a \approx 0.078$fm. We compute disconnected contributions to the relevant three-point functions using an efficient noise reduction technique. Using these methods together with an only multiplicative renormalization applicable for twisted mass fermions, allows us to obtain accurate results in the light and strange sector.

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*Speaker.

1. Introduction

The various evidences of the existence of dark matter have led to the development of experiments dedicated to detect dark matter directly. The detection relies on the measurements of the recoil of atoms hit by a dark matter candidate. One popular class of dark matter models involve an interaction between a WIMP and a Nucleon mediated by a Higgs exchange. Therefore, the scalar content of the nucleon is a fundamental ingredient in the WIMP-Nucleon cross section. In this way, the uncertainties of the scalar content translates directly into the accuracy of the constraints on beyond the standard model physics. Since the coupling of the Higgs to quarks is proportional to the quark masses, it is important to know how large scalar matrix elements of the nucleon are, in particular for the strange and charm quarks.

One common way to write the parameters entering the relevant cross section are the so-called sigma-terms of the nucleon:

$$\sigma_{\pi N} \equiv m\langle N|\bar{u}u+\bar{d}d|N\rangle \quad \text{and} \quad \sigma_0 \equiv m\langle N|\bar{u}u+\bar{d}d-2\bar{s}s|N\rangle\ , \tag{1.1}$$

where m denotes the light quark mass. Quantifying the scalar strangeness content of the nucleon a parameter y_N is introduced,

$$y_N \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+dd|N\rangle}, \tag{1.2}$$

which can be also related to the sigma terms of the nucleon in eq. (1.1).

The direct computation of the above matrix elements is known to be challenging on the lattice for several reasons. First, it involves the computation of "singlet" or "disconnected" diagrams that are very noisy on the lattice. Second, discretisations that break chiral symmetry generally suffer from a mixing under renormalization between the light and strange sector, which is difficult to treat in a fully non-perturbative way. However, twisted mass fermions offer two advantages here: they provide both an efficient variance noise reduction for disconnected diagrams and a convenient way to avoid the chirally violating contributions that are responsible for the mixing under renormalization.

2. Lattice Setup

In our simulations we use the mass-degenerate twisted mass action in the light sector and the mass non-degenerate twisted mass action in the strange and charm sector. The quark masses of the heavy quark doublet have been tuned such that the Kaon and D-mesons masses assume approximately their physical value. The reader interested in more details about aspects of this setup is referred to [1, 2]. The twisted mass action in the light sector reads

$$\begin{aligned} S[\chi,\overline{\chi},U] \ =& \sum_x \overline{\chi}_q(x) D_{\rm tm}[U]\chi_q(x) = \sum_x \Big\{ \overline{\chi}_q(x)(\frac{1}{2\kappa}+ia\mu_q\gamma_5\tau^3)\chi_q(x) \\ &- \frac{1}{2}\overline{\chi}_q(x)\sum_{\mu=0}^{3}\Big[U_\mu(x)(r+\gamma_\mu)\chi_q(x+a\hat{\mu})+U_\mu^\dagger(x-a\hat{\mu})(r-\gamma_\mu)\chi_q(x-a\hat{\mu})\Big]\Big\}. \end{aligned} \tag{2.1}$$

where the hopping parameter $\kappa = (2am_0 + 8r)^{-1}$ is defined in terms of am_0, the bare Wilson mass, r is the Wilson parameter and μ_q is the bare twisted mass parameter. The Wilson parameter is fixed to $|r| = 1$ is all our simulations. When κ is tuned to its critical value a situation called maximal twist is achieved which guarantees $O(a)$ improvement of physical observables.

For further needs we also introduce the operators $D_{q,\pm}$ denoting the upper and lower components of the Wilson twisted mass operator in flavour space (also referred to as the Osterwalder-Seiler Dirac operator):

$$D_{q,\pm}[U] = \mathbf{tr}\, \frac{1 \pm \tau_3}{2} D_{\rm tm}[U], \tag{2.2}$$

where **tr** denotes the trace in flavour space. $D_{q,\pm}[U]$ then corresponds to 1-flavour twisted mass operators with Wilson parameter $r = \pm 1$.

When we discuss the 2-point and 3-point functions necessary for this work, we will use the so-called physical basis of quark fields denoted as ψ_q. This field basis is related to the twisted quark field basis, χ_q by the following field rotation

$$\psi_q \equiv e^{i\frac{\omega_l}{2}\gamma_5\tau^3}\chi_q \quad \text{and} \quad \overline{\psi}_q \equiv \overline{\chi}_q e^{i\frac{\omega_l}{2}\gamma_5\tau^3}, \tag{2.3}$$

where the twist angle $\omega_l = \pi/2$ at maximal twist.

In order to compute matrix elements involving strange quarks, we will work within a mixed action setup. For reasons that will become clear later, we choose to introduce in the valence sector an additional doublet of degenerate twisted mass quark of mass μ_q . The mass μ_q can then be tuned to reproduce the Kaon and D-mesons mass in the unitary setup. Preliminary estimates of the matching masses gives $a\mu_s = 0.0185$ in the strange sector and $a\mu_c = 0.2514$ in the charm sector and we will approximately use these values for μ_q further on. In this contribution, we work at a fixed lattice spacing corresponding to $a \approx 0.078$ fm with $m_{\rm PS}L \geq 4$ and pion masses ranging approximately from 300 to 500 MeV.

3. Matrix elements

In the following, ψ_q with index l, s, d will denote the quark fields of the light (l), strange (s) or charm (c) quarks in the physical basis. In order to be self-contained, we describe in this section the relevant correlation functions used in this work. The nucleon two-point function reads :

$$C^{\pm}_{N,\rm 2pts}(t - t_{\rm src}, \vec{x}_{\rm src}) = \sum_{\vec{x}} \mathbf{tr}\, \Gamma^{\pm} \langle \mathscr{J}_N(x) \overline{\mathscr{J}_N}(x_{\rm src}) \rangle, \tag{3.1}$$

where $x_{\rm src} \equiv (t_{\rm src}, \vec{x}_{\rm src})$ is the source point and the subscript N refers to the proton or to the neutron states for which the interpolating fields are given by:

$$\mathscr{J}_p = \varepsilon^{abc}\left(u^{a,T}\mathscr{C}\gamma_5 d^b\right)u^c \quad \text{and} \quad \mathscr{J}_n = \varepsilon^{abc}\left(d^{a,T}\mathscr{C}\gamma_5 u^b\right)d^c.$$

The projectors used are $\Gamma^{\pm} = \frac{1\pm\gamma_0}{2}$, and $\mathscr{C}$ is the charge conjugation matrix.

The nucleon three-point functions is

$$C^{\pm,O_q}_{N,\rm 3pts}(t_s, \Delta t_{\rm op}, \vec{x}_{\rm src}) = \sum_{\vec{x},\vec{x}_{\rm op}} \mathbf{tr}\, \Gamma^{\pm} \langle \mathscr{J}_N(x) O_q(x_{\rm op}) \overline{\mathscr{J}_N}(x_{\rm src}) \rangle, \tag{3.2}$$

where O_q is an operator having scalar quantum numbers, e.g. $O_q = \bar{q}q$, $\Delta t_{\rm op} = t_{\rm op} - t_{\rm src}$ is the time of insertion of the operator, and $t_s = t - t_{\rm src}$ gives the so-called source-sink separation. Note that in the twisted basis the scalar operators read

$$\widetilde{O}_q = i\overline{\chi}_q \gamma_5 \tau^3 \chi_q, \quad \text{where} \quad q = l, s, c \ , \tag{3.3}$$

and are hence given by the pseudo scalar density. The flavour structure of the operators in the twisted basis will be crucial in the following.

Since we consider an operator with a non-vanishing vacuum expectation value, we also define

$$C^{\pm, O_q, \rm vev}_{N,3\rm pts}(t_s, \Delta t_{\rm op}, \vec{x}_{\rm src}) = C^{\pm, O_q}_{N,3\rm pts}(t_s, \Delta t_{\rm op}, \vec{x}_{\rm src}) - C^{\pm}_{N,2\rm pts}(t, \vec{x}_{\rm src}) \sum_{\vec{x}_{\rm op}} \langle O_q(x_{\rm op}) \rangle \ . \tag{3.4}$$

The desired scalar matrix elements can then be computed using the asymptotic behaviour of the ratio of a three and two-point functions:

$$R_{O_q}(t_s, \Delta t_{\rm op}) = \frac{C^{+, O_q, \rm vev}_{N,3\rm pts}(t_s, t_{\rm op})}{C^{+}_{N,2\rm pts}(t, x_{\rm src})} = \langle N | \bar{q}q | N \rangle + \mathscr{O}(e^{-\Delta t_{\rm op}}) + \mathscr{O}(e^{-\Delta (t_s - t_{\rm op})}) \ . \tag{3.5}$$

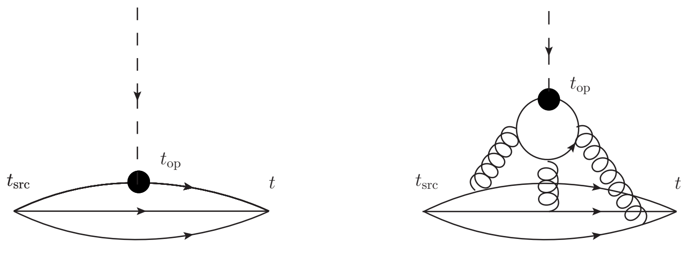
The general form of the 3-point functions in Eq. (3.2) lead to both, connected ($\widetilde{C}$, illustrated in fig. 1a) and disconnected ($\mathscr{D}$, illustrated in 1b) contributions,

$$C^{\pm, O_q}_{N,3\rm pts}(t_s, \Delta t_{\rm op}) = \widetilde{C}^{\pm, O_q}_{N,3\rm pts}(t_s, \Delta t_{\rm op}) + \mathscr{D}^{\pm, O_q}_{N}(t_s, t_{\rm op}, x_{\rm src}) \tag{3.6}$$

In the following we will denote by $R_{\rm conn.}$ (resp. $R_{\rm disc.}$) the contribution of $\widetilde{C}^{\pm, O_q}$ (resp. $\mathscr{D}^{O_q}$) to the ratio defined in Eq. (3.5). The sum of the connected and disconnected contribution to the ratio will be denoted $R_{\rm full}$.

In order to improve the signal over noise ratio, we have averaged the disconnected part over forward and backward propagating proton and neutron states. In addition, we have used up to 4 randomly chosen source points per configuration for the 2-point function computation to enhance our statistics.

As will be detailed in a forthcoming publication [3], the operators $\widetilde{O}_q$ do not mix under renormalization and hence have a straightforward renormalization pattern very similar to chirally invariant overlap fermions. We consider this fact as a major advantage of our twisted mass approach for computing the scalar quark contents of the nucleon.



(a) (b)

Figure 1: We illustrate the connected (left) and the disconnected (right) graphs that arise from the contractions leading to the 3-point function discussed in the text.

4. Variance reduction

The main challenge to compute the 3-point functions of Eq. (3.2) is to evaluate $\mathscr{D}^{\pm,O_q}$. Our strategy is based on a variance reduction technique for twisted mass fermions introduced in [4] and used to study the η' meson in [6]. It relies on the fact that in the twisted basis the disconnected contribution that has to be evaluated is related to the difference of $1/D_{q,-} - 1/D_{q,+}$.

Here, we will employ the one-end-trick [4] to compute the disconnected distribution stochastically evaluating

$$2ia\mu_q \sum_{\vec{x}} \left[\phi^*_{[r]}(x)\gamma_5\Gamma\phi_{[r]}(x)\right]_R = \sum_{\vec{x}} \mathbf{tr}\,\Gamma\left(\frac{1}{D_{q,-}} - \frac{1}{D_{q,+}}\right)(x,x) + \mathscr{O}\left(R^{-1/2}\right), \tag{4.1}$$

where

$$\phi_{[r]} = (1/D_{q,+})\xi_{[r]} \quad \text{and} \quad \phi^*_{[r]} = \xi^*_{[r]}(1/D_{q,+})^\dagger\,, \tag{4.2}$$

where we have introduce N_R independent random volume sources denoted $\xi_{[r]}$. For the generation of the random sources we have used a $\mathbf{Z}_2$ noise setting all field components randomly from the set $\{1,-1\}$.

In our tests for the signal to noise ratio (SNR) we first investigated how the SNR depends on the number of stochastic sources N_R used. We found that for $N_R \gtrsim 7$ there is no significant improvement of the SNR. Nevertheless, we have used 12 stochastic sources per configurations in all our results. In Fig. 4 we compare the SNR of the twisted mass specific variance reduction technique to a more standard method based on the the hopping parameter expansion[5]. We show the ratio R_{O_q} of eq. (3.5), for a fixed value of $\Delta t_{\rm op} = t_s/2 = 6$ as a function of the number of configurations N. We conclude that the SNR is increased by a factor ~ 3 with our improved noise reduction technique which allows to obtain a result at the 5σ significance level with only a moderate statistics.

5. Light $\sigma-$term

In Fig. 3 we show the results obtained for the bare ratio $R_{\rm full}$ introduced in Eq. (3.5) from which $\sigma_{\pi N}$ can be computed. The connected part, $R_{\rm conn.}$, for a source-sink separation of $t_s = 12a$, is shown by the black filled circles. It has been computed using the standard method of "sequential inversions through the sink". As can be seen, $R_{\rm conn.}$ shows a time dependence indicating excited state contributions, a systematic effect that has not been taken into account yet in this work. The disconnected part, $R_{\rm disc.}$, is represented by blue triangles in Fig. 3. The disconnected part is significantly smaller than the connected part $R_{\rm conn.}$ and contributes at the $\sim 10\%$ level to the full ratio $R_{\rm full}$ represented by the red diamonds.

We have computed $\sigma_{\pi N}$ so far for three values of the pion mass, see table 1. Our present data do not allow for a reliable extrapolation to the physical point for which additional data at more pion masses would be necessary.

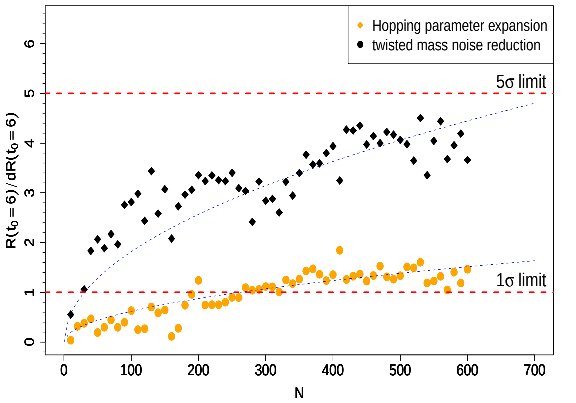


Figure 2: Signal to noise ration (SNR) of the ratio R_{O_q} for a fixed time $\Delta t_{\rm op} = t_s/2 = 6$ as a function of the number of gauge field configurations N. R_{O_q} is evaluated here in the strange quark regime. We compare our noise reduction technique, with the hopping parameter expansion technique. The dashed lines indicate the 1σ and 5σ significance levels and the short dotted lines are only shown to guide the eyes.

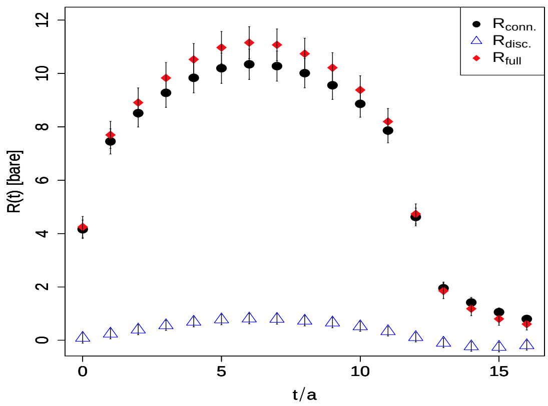


Figure 3: Plot of the two contributions $R_{\rm disc.}$ and $R_{\rm conn.}$ to the ratio R_{O_q} relevant for the extraction of $\sigma_{\pi N}$. $R_{\rm full}$ is the sum of both contributions.

$m_{\rm PS}$ (MeV)	$\sigma_{\pi N}$ (MeV)
318	99(6)
392	152(9)
455	228(15)

Table 1: Fit results for $\sigma_{\pi N}$ as a function of the pion mass. Only statistical error are estimated.

6. Strangeness of the nucleon

In Fig. 4, we show $R_{\rm disc.}$ for a quark mass of $a\mu_q = 0.018$ corresponding approximately to the strange quark mass. The in principle freely selectable source-sink separation has been fixed to 12 lattice units. The ratio R_{O_q} of eq. (3.5) shows a time dependence indicating that also in the case of the strange quark excited states may be important. We nevertheless extract a plateau value as indicated in Fig. 4 which is clearly different from zero. Combining this value with the result for the scalar matrix element obtained in the light quark sector discussed above, allows us finally to compute y_N. We have performed such a computation at four values of the pion mass at fixed value of the lattice spacing and the results are summarized in Fig. 5. Performing a simple linear extrapolation we obtain, as shown by a red star, $y_N = 0.069(27)$ in the chiral limit, where only the statistical errors have been taken into account. Although an investigation of systematic effects are still missing, we checked that by varying the bare strange quark mass to $a\mu_s = 0.016$ does not change significantly the value of y_N .

As a final remark we mention that we have also computed the charm quark content of the nucleon. Unfortunately, here the SNR is of order one and hence no clear signal can be extracted. From our present data we can only provide a qualitative estimate that $\langle N|O_c|N\rangle \lesssim \langle N|O_s|N\rangle$.

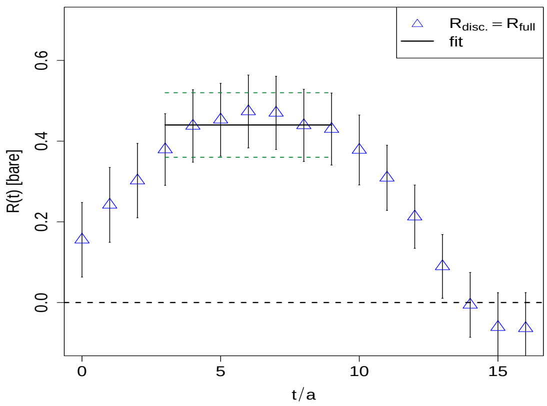


Figure 4: Time dependence of $R_{\text{disc.}}$ in the strange quark mass regime ($a\mu_q = 0.018$). The source-sink separation has been fixed to $12a$ and 842 configurations have been.

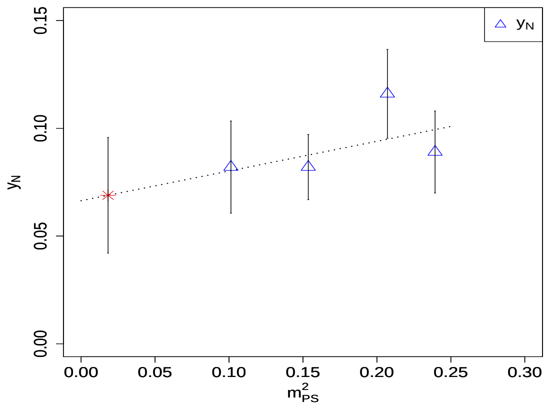


Figure 5: Our data for y_N as a function of the pion mass. The data are extrapolated to the chiral limit using a simple linear extrapolation.

7. Conclusion

In this proceedings contribution we have shown that with twisted mass fermions it is possible address the disconnected contribution to the scalar matrix elements of the nucleon. This becomes especially important when the strange and the charm content are computed since there only disconnected graphs appear. In addition, with twisted mass fermions the renormalization pattern of such matrix elements is the same as for chirally invariant discretizations. As a result we were able to obtain accurate results in the light and strange sector at fixed lattice spacing and for several quark masses. Our main result is a value $y_N = 0.069(27)$. The still missing systematic uncertainties on this result will be addressed in future simulations.

Acknowledgments

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References

[1] R. Baron, P. .Boucaud, J. Carbonell, A. Deuzeman, V. Drach, F. Farchioni, V. Gimenez, G. Herdoiza *et al.*, JHEP **1006**, 111 (2010). [arXiv:1004.5284 [hep-lat]].

[2] R. Baron *et al.* [European Twisted Mass Collaboration], Comput. Phys. Commun. **182**, 299-316 (2011). [arXiv:1005.2042 [hep-lat]].

[3] S. Dinter, et.al., In preparation.

[4] C. Michael *et al.* [ETM Collaboration], PoS **LAT2007**, 122 (2007). [arXiv:0709.4564 [hep-lat]].

[5] C. McNeile *et al.* [UKQCD Collaboration], Phys. Rev. **D63**, 114503 (2001). [hep-lat/0010019].

[6] K. Jansen *et al.* [ETM Collaboration], Eur. Phys. J. **C58**, 261-269 (2008). [arXiv:0804.3871 [hep-lat]].

[7] V. Drach, K. Jansen, J. Carbonell, M. Papinutto, C. Alexandrou, PoS **LATTICE2010**, 101 (2010). [arXiv:1012.3861 [hep-lat]].