

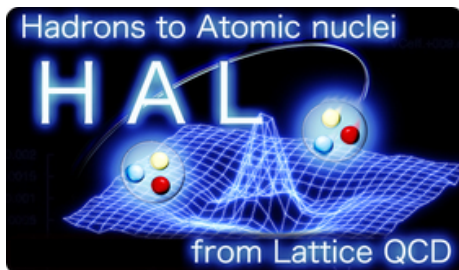
S-wave meson-baryon potentials with strangeness from Lattice QCD

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We study the s-wave $I = 2 \pi\Sigma$ and $I = 1 KN$ interactions from 2+1 flavor full lattice QCD simulation for relatively heavy quark mass corresponding to $m_\pi = 700$ MeV. The s-wave meson-baryon potentials are obtained from the Nambu-Bethe-Salpeter amplitudes. Potentials in both channels reveal short range repulsions, which suggest the importance of the Pauli blocking effect. The $I = 1 KN$ scattering phase shifts are calculated and compared with the existing experimental data.

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1. Introduction

The $\Lambda(1405)$ negative parity hyperon resonance has strangeness $S = -1$ and isospin $I = 0$, and is located just below the $\bar{K}N$ threshold. Since the $\Lambda(1405)$ is considered to be the quasi-bound state of the s-wave $\bar{K}N$, the structure of the $\Lambda(1405)$ is one of the very important issues of recent hadron physics, especially to understand \bar{K} -nucleon and \bar{K} -nucleus interactions. The $\Lambda(1405)$ also decays into the $\pi\Sigma$ continuum. Thus, for the physics of the $\Lambda(1405)$ and the \bar{K} -nucleus, dynamics of both $\pi\Sigma$ and $\bar{K}N$ is important and gives essential contributions.

The $\Lambda(1405)$ has been considered as a dynamically generated state in meson-baryon scattering and well described in a coupled-channels approach based on chiral dynamics [1]. The chiral dynamics also predicts two resonance poles for the $\Lambda(1405)$, which have different coupling nature to the $\pi\Sigma$ and $\bar{K}N$ channels [2]. A phenomenological approach also described the $\Lambda(1405)$ as a quasi-bound state of $\bar{K}N$ [3], in which an effective interaction of $\bar{K}N$ was derived so as to reproduce the $\bar{K}N$ scattering length and the mass and width of the $\Lambda(1405)$ as 1405 MeV and 40 MeV. With this approach, only one resonance pole for the $\Lambda(1405)$ is predicted, and the phenomenological model provides quantitatively stronger $\bar{K}N$ interaction than the chiral potential in the region far below the $\bar{K}N$ threshold. Thus, there are uncertainties about the theoretical extrapolation of the $\bar{K}N$ interaction below the $\bar{K}N$ threshold and the pole nature of the $\Lambda(1405)$.

Recently, threshold behavior of the $\pi\Sigma$ scattering and its impact on the binding energy of the \bar{K} -nuclei have been discussed [4]. For the physics of the $\Lambda(1405)$, it is certainly necessary to have theoretical descriptions of $\pi\Sigma$ and $\bar{K}N$ dynamics. The position of the pole singularity (bound state, virtual state, or resonance) around the $\pi\Sigma$ threshold is an important issue to investigate the \bar{K} -nuclei [3, 5]. The $\pi\Sigma$ dynamics in $I = 0$ channel can be extracted from the Λ_c baryon decay to $\pi\pi\Sigma$ states [6]. According to Ref [6], in the Λ_c decay process, it is possible to provide two constraints on the three different isospin component of the $\pi\Sigma$ scattering lengths. Therefore, the direct determination of one of $\pi\Sigma$ scattering lengths based on QCD is mandatory.

In this paper, to clarify the nature of the $\pi\Sigma$ dynamics, we calculate the $I = 2$ $\pi\Sigma$ potential on the lattice. The method to calculate the potentials from Nambu-Bethe-Salpeter wave function, which satisfies the relativistic three-dimensional Schrödinger-type equation, has been reported in Refs. [7, 8], and is developed by HAL QCD Collaboration [9, 10, 11, 12, 13, 14, 15, 16]. Further applications are also given in Refs. [17, 18, 19]. We apply this method to the $I = 2$ $\pi\Sigma$ system. We also examine the $I = 1$ KN scattering, since the $I = 1$ KN belongs to the same multiplet as the $I = 2$ $\pi\Sigma$ in flavor SU(3) limit, and there exist experimental data in this channel. Therefore, examining the $I = 1$ KN scattering is very useful to predict the $I = 2$ $\pi\Sigma$ scattering observable.

This paper is organized as follows. In section 2, the formalism to extract the meson-baryon potentials from lattice QCD is briefly reviewed. Our numerical setup of the lattice QCD simulation is then shown in section 3, and the results are shown and discussed in section 4. A summary is given in section 5.

2. Formalism

2.1 Basic concept to define potentials on the lattice

Following the basic formulation to extract the nucleon-nucleon interaction [7, 8], we briefly

show our strategy to obtain meson-baryon potentials below. We start with the Schrödinger-type equation for the equal-time Nambu-Bethe-Salpeter (NBS) wave function $\Psi_{\vec{k}}(\vec{r})$:

$$(\nabla^2 + \vec{k}^2)\Psi_{\vec{k}}(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}') \Psi_{\vec{k}}(\vec{r}'), \quad (2.1)$$

where $\mu (= mM/(m+M))$ and \vec{k} denote the reduced mass of the meson (m) and the baryon (M) and the relative momentum of the meson-baryon system, respectively. The NBS wave function of the meson-baryon system is extracted from the four-point correlation functions on the lattice:

$$\begin{aligned} C_\alpha(\vec{r}, t - t_0) &= \sum_{\vec{x}, \vec{X}, \vec{Y}} \langle 0 | \phi_M(\vec{x} + \vec{r}, t) \phi_{B,\alpha}(\vec{x}, t) (P_\beta^{(s)} \phi_M(\vec{X}, t_0) \phi_{B,\beta}(\vec{Y}, t_0))^\dagger | 0 \rangle \\ &= \sum_{n, \vec{x}} A_n \langle 0 | \phi_M(\vec{x} + \vec{r}, t) \phi_{B,\alpha}(\vec{x}, t) | n \rangle e^{-E_n(t-t_0)}, \end{aligned} \quad (2.2)$$

with the spin projection operator $P^{(s)}$ and the matrix elements

$$A_n = \sum_{\vec{X}, \vec{Y}} \langle n | (P_\beta^{(s)} \phi_M(\vec{X}, t_0) \phi_{B,\beta}(\vec{Y}, t_0))^\dagger | 0 \rangle. \quad (2.3)$$

The four-point correlation function in Eq. (2.2) is dominated by the lowest energy state with total energy E_0 at large time separation ($t \gg t_0$):

$$C_\alpha(\vec{r}, t - t_0) \rightarrow A_0 \Psi_\alpha(\vec{r}; J^\pi) e^{-E_0(t-t_0)}, \quad (2.4)$$

with $E_0 = \sqrt{m^2 + \vec{k}^2} + \sqrt{M^2 + \vec{k}^2}$ being the relativistic energy of the meson-baryon system. Thus, the meson-baryon NBS wave function is defined by the spatial correlation of the four-point correlation function. In Eq. (2.4), we assume the Dirichlet boundary condition in temporal direction, so that the temporal correlation has an exponential form, $e^{-E_0(t-t_0)}$.

The NBS wave function in s-wave state is obtained under the projection onto the A_1^+ sector,

$$\Psi(\vec{r}; J^\pi = 1/2^-) = \frac{1}{24} \sum_{g \in O} P_\alpha \Psi_\alpha(g^{-1}\vec{r}; J^\pi), \quad (2.5)$$

where $g \in O$ represent 24 elements of the cubic rotational group, and the summation is taken for all these elements. Using Eq. (2.1) and Eq. (2.5), we will find the meson-baryon potential and wave function from lattice QCD.

The energy-independent and non-local potential $U(\vec{r}, \vec{r}')$ can be expanded in powers of the relative velocity $\vec{v} = -i\nabla/\mu$ at low energies,

$$\begin{aligned} U(\vec{r}, \vec{r}') &= V(\vec{r}, \vec{v}) \delta(\vec{r} - \vec{r}') \\ &= (V_{LO}(\vec{r}) + (\vec{L} \cdot \vec{\sigma}) V_{NLO}(\vec{r}) + \dots) \delta(\vec{r} - \vec{r}'), \end{aligned} \quad (2.6)$$

where the $N^{\text{th}}LO$ term is of order $O(\vec{v}^n)$, and \vec{L} and $\vec{\sigma}$ denote an orbital angular momentum of the meson-baryon system and a baryon spin, respectively.

2.2 Strategy to extract potentials on the lattice

In lattice simulations, one may suffer from possible contaminations from higher energy states in Eq. (2.4). To extract the reliable potentials on the lattice [20] even with the presence of such higher energy states, we consider the time evolution of normalized four-point correlation functions (R-correlators) [20]:

$$\begin{aligned} -\frac{\partial}{\partial t} R_\alpha(\vec{r}, t - t_0) &= \sum_n A_n \Delta E(\vec{k}_n) e^{-\Delta E(\vec{k}_n)(t-t_0)} \Psi_{\alpha, \vec{k}_n}(\vec{r}) \\ &\simeq \sum_n A_n \left(\frac{\vec{k}_n^2}{2\mu} + U \right) e^{-\Delta E(\vec{k}_n)(t-t_0)} \Psi_{\alpha, \vec{k}_n}(\vec{r}), \end{aligned} \quad (2.7)$$

with

$$R_\alpha(\vec{r}, t - t_0) \equiv e^{(m+M)(t-t_0)} C_\alpha(\vec{r}, t - t_0), \quad (2.8)$$

$$\Delta E(\vec{k}_n) = \sqrt{m^2 + \vec{k}_n^2} + \sqrt{M^2 + \vec{k}_n^2} - (m + M). \quad (2.9)$$

In Eq. (2.7), we assume the non-relativistic energy levels. Since the potential $U(\vec{r}, \vec{r}')$ is non-local but energy-independent by construction [7, 8], one can define the potentials even with the higher energy state [20]. At the leading order of the velocity expansion in Eq. (2.6) and after the s-wave projection through Eq. (2.5), Eq. (2.7) reads

$$-\frac{\partial}{\partial t} R(\vec{r}, t - t_0; J^\pi = 1/2^-) = \left(-\frac{\nabla^2}{2\mu} + V_C^{\text{eff}}(\vec{r}) \right) R(\vec{r}, t - t_0; J^\pi = 1/2^-). \quad (2.10)$$

Eq. (2.10) is the time-dependent Schrödinger-type equation for the R-correlator, from which the s-wave meson-baryon effective central potentials are extracted.

3. Numerical setup

In order to calculate the meson-baryon potentials in 2+1 flavor full QCD, we have utilized the gauge configurations of JLDG(Japan Lattice Data Grid)/ ILDG(International Lattice Data Grid) generated by PACS-CS Collaboration on a $32^3 \times 64$ lattice [21]. The renormalization group improved Iwasaki gauge action and non-perturbatively $O(a)$ improved Wilson quark action are used at $\beta = 1.90$, which corresponds to the lattice spacing $a = 0.09$ fm determined from π , K and Ω masses. The physical size of the lattice is about $(2.9 \text{ fm})^3$ and the hopping parameters are taken to be $\kappa_u = \kappa_d = 0.1370$ and $\kappa_s = 0.1364$.

In the present simulation, we adopt the spatial wall source located at t_0 with the Dirichlet boundary condition at time slice $t = t_0 + 32$ in the temporal direction and the periodic boundary condition in each spatial direction. The Coulomb gauge fixing is employed at $t = t_0$. The number of gauge configurations used in the simulation is 399. With this setup, we obtain $m_\pi = 705(2)$, $m_K = 793(2)$, $M_N = 1590(8)$ and $M_\Sigma = 1665(7)$ MeV.

4. Numerical results and discussion

By using Eq. (2.10), we obtain the s-wave $I = 2 \pi\Sigma$ and $I = 1 KN$ effective central potentials, which are shown in Figs. 1(a) and (b) at time slice $t - t_0 = 13$. We observe the repulsive core at short distance ($r < 0.5$ fm) in both channels. The strong repulsions near origin can be expected by the quark Pauli blocking effects. This was first pointed out by Machida and Namiki [22] for the meson-baryon systems. In the $I = 1 KN$ ($K^+ p$) state whose configuration is

$$K^+ p \sim (u\bar{s}) (uud), \quad (4.1)$$

one of u-quarks can not be in the S-state. The strong repulsion at short distance in both channels found in our simulations suggest a manifestation of the quark Pauli blocking effects.

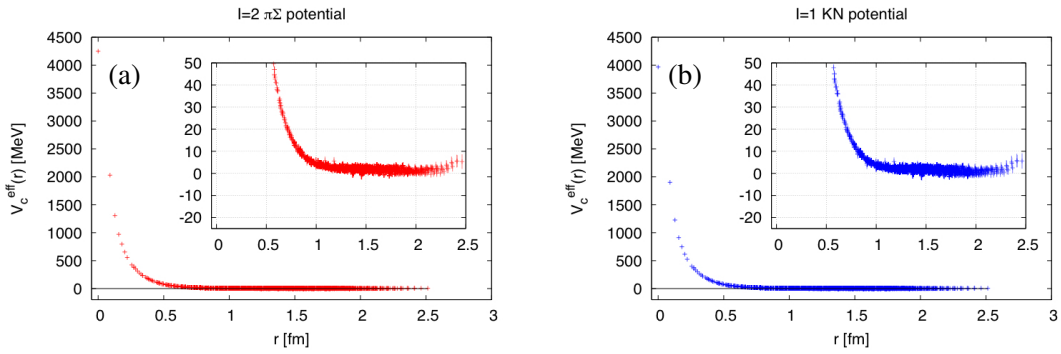


Figure 1: The s-wave potential of (a) $I = 2 \pi\Sigma$ and (b) $I = 1 KN$ states.

The repulsive interactions in the s-wave $I = 1 KN$ state can be expected much stronger than that of the $I = 0 KN$ state due to the quark Pauli blocking effects. To investigate whether this expectation is correct or not, we also calculate the s-wave $I = 0 KN$ potential. As shown in Fig. 2, the repulsion at short distance for the KN potential becomes significantly smaller in the $I = 0$ channel than $I = 1$ channel. This again confirms the expectation from the quark Pauli blocking effects. In addition, we observe the attractive well in the mid range ($0.4 < r < 1.2$ fm) in the $I = 0 KN$ channel. In the constituent quark model of hadrons [23], similar short range repulsion in KN system has been predicted, while the attraction has not been found.

By using the potentials which fit the lattice data in Fig. 1(a) and (b), we can calculate observable such as the scattering phase shifts. Fig. 3 shows such phase shifts of the $I = 2 \pi\Sigma$ and $I = 1 KN$ scatterings together with the experimental data as functions of the laboratory momentum of the mesons. Although the hadron masses are heavy in the present simulation, qualitative behavior of the phase shifts in $I = 1 KN$ channel is consistent with the experimental data. Simulations along this line with lighter quark masses will eventually lead to a definite conclusion of the $I = 2 \pi\Sigma$ scattering.

5. Summary

We have performed the 2+1 flavor full lattice QCD simulation to investigate the $I = 2 \pi\Sigma$ interaction, which is relevant to determine the structure of the $\Lambda(1405)$. The s-wave $I = 2 \pi\Sigma$

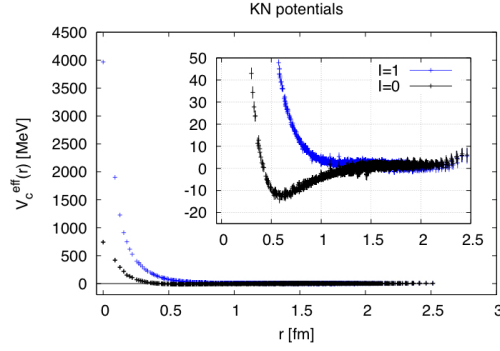


Figure 2: The $I = 1$ (blue) and $I = 0$ (black) KN potentials. Short range repulsion in $I = 1$ is much stronger than that in $I = 0$ as predicted by Pauli blocking effects.

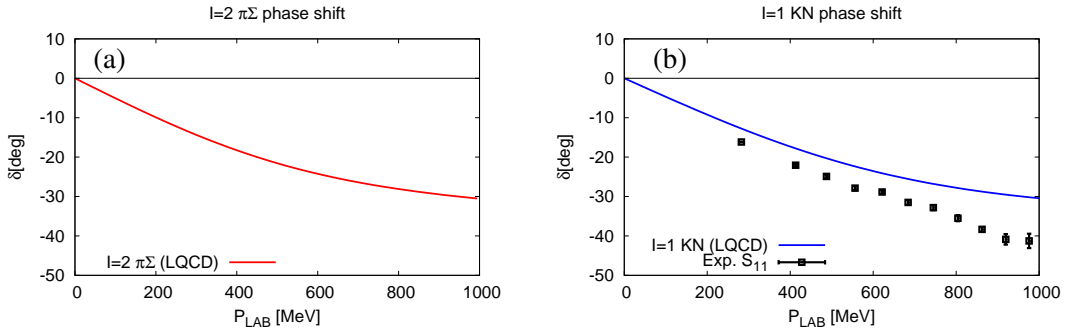


Figure 3: The s-wave phase shifts of (a) the $I = 2 \pi\Sigma$ and (b) the $I = 1 KN$ scatterings together with the experimental data [24].

and $I = 1 KN$ potentials are extracted from the NBS wave functions for relatively heavy quark mass corresponding to $m_\pi = 700$ MeV. Potentials in both channels reveal short range repulsions. From these potentials, the s-wave scattering phase shifts are calculated and compared with the existing experimental data in $I = 1 KN$ channel. Although the quark mass is heavy in the present simulation, the results indicate that our method is promising for future quantitative studies of the $\pi\Sigma$ interactions at lighter quark masses.

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