PROCEEDINGS OF SCIENCE

Magnetic Properties of the Nucleon

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We present preliminary results from an investigation into the magnetic properties of the nucleon using the background field method of lattice QCD. The method involves introducing a uniform magnetic field across the lattice and measuring the resulting energy shift. The investigation is performed on a quenched $32^3 \times 40$ lattice with periodic boundary conditions in the spatial dimensions and a uniform quantised field in the $x-y$ plane. We discuss the Landau levels of a charged particle in a magnetic field and the difficulty this contribution causes when attempting to perform background field calculations with the proton.

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1. Introduction

The magnetic moment and magnetic polarisability are fundamental properties of a particle that describe its response to an applied magnetic field. The background field method is a well-known technique for examining these properties via lattice QCD[1 – 8]. In this method a uniform background magnetic field is applied to the lattice in the form of a phase factor on the usual QCD gauge field. This causes a shift in the calculated ground-state energy of the particle which is combined with an energy relation to extract the moment and polarisability values.

We also discuss the Landau energy, which is an additional contribution that affects charged particles in a magnetic field. This contribution can not be isolated from the polarisability and makes getting proton polarisabilities extremely difficult. Even in the moment case where the Landau energy can be removed, its presence affects the validity of our momentum projection and therefore our proton magnetic moment calculations. The magnetic moment and polarisability for the (zero charge) neutron avoids these problems.

2. The Background Field Method

In order to put a magnetic field on the lattice we modify the covariant derivative with the addition of a minimal electromagnetic coupling

$$D_\mu = \partial_\mu + gG_\mu + qA_\mu, \tag{2.1}$$

where A_μ is the electromagnetic four-potential and q is the charge on the fermion field. On the lattice this is equivalent to multiplying the usual gauge links by a simple phase factor

$$U_\mu^{(B)}(x) = \exp(iaqA_\mu(x)). \tag{2.2}$$

To obtain a uniform magnetic field along the z-axis we note that $\vec{B} = \vec{\nabla} \times \vec{A}$, and hence

$$B_z = \partial_x A_y - \partial_y A_x. \tag{2.3}$$

Note that this equation does not specify the gauge potential uniquely. There are multiple valid choices of A_μ that give rise to the same field, however they are equivalent up to a gauge transformation. We choose $A_x(x,y) = By$ to produce a constant magnetic field of magnitude B in the z direction. Examining a single plaquette in the $(\mu,\nu) = (x,y)$ plane shows that this gives the desired field for a general point on the lattice.

However on a finite lattice $(0 \le x \le N_x - 1), (0 \le y \le N_y - 1)$ there is a discontinuity at the boundary due to the periodic boundary conditions. In order to fix this problem we make use of the $\partial_x A_y$ term from equation (2.3), giving A_y the following values,

$$A_y(x,y) = \begin{cases} 0 & \text{for } y < N_y - 1 \\ -N_y Bx & \text{for } y = N_y - 1, \end{cases} \tag{2.4}$$

such that we now get the required value at the $y = N_y - 1$ boundary.

This still leaves the double boundary, $x = N_x - 1$ and $y = N_y - 1$, where the plaquette only has the required value under the condition $\exp(-ia^2 qBN_xN_y) = 1$. Therefore we have an important quantisation condition which limits the choices of magnetic field strength based on the lattice size

$$a^2 qB = \frac{2\pi n}{N_x N_y}, \tag{2.5}$$

where $n = 0, \pm 1, \pm 2, \ldots$ is an integer specifying the field strength in multiples of the minimum field strength quantum.

A consequence of the quantisation condition is that imposing a magnetic field with a small field strength requires the use of a large lattice. This is needed to ensure that the energy shift due to the magnetic field is small when compared to the unperturbed mass of the particle, enabling us to disregard higher terms when it comes to the perturbative expansion of $E(B)$. It is possible to avoid the quantisation condition by using Dirichlet boundary conditions rather than periodic and by replacing the exponential phase factor with a linearised version. This allows for arbitrary choices of field strength, however it complicates the formalism and leads to an increase in finite volume effects. Doing so also particularly affects the polarisability due to the difference in the exponential and linear forms changing the action at order B^2, the same order at which the polarisability comes in [8].

To extract the magnetic moment from an effective mass calculated in the presence of a background field, we use the following energy relation,

$$E(B) = M_N + \frac{e|B|}{2M_N} + \mu \cdot \mathbf{B} - \frac{4\pi}{2}\beta B^2 + \mathscr{O}(B^3), \tag{2.6}$$

where M is the bare mass of the particle, e is its charge, μ is the magnetic moment and β is the magnetic polarisability.

To isolate the magnetic moment term we make use of the fact that it has the same magnitude but opposite sign depending on whether the particle is spin-up or spin-down, where we take up to mean aligned with the magnetic field and down to mean anti-aligned with the magnetic field. We can therefore remove the quadratic polarisability term and the bare mass term by subtracting the spin-up energy from the spin-down energy. This also removes the term describing the Landau energy, $e|B|/2M_N$, however this will be zero for the neutron anyway because it has no charge.

As the energy appears in the exponent of a correlation function, energy sums and differences can be obtained by taking products and ratios of correlation functions. There is some choice in the order of combining and fitting these correlation functions. The form we found to give the best results is

$$\delta E(B) = \frac{1}{2}\left(\ln\left(\frac{G_\uparrow(B,t)}{G_\uparrow(0,t)}\frac{G_{2\downarrow}(0,t)}{G_\downarrow(B,t)}\right)\right)_{\text{fit}}. \tag{2.7}$$

By fitting after taking the difference of the spins and explicitly dividing out the zero field correlation functions we allow as many background field and spin dependent fluctuations to cancel as possible. Once these effective mass shifts are calculated they are fitted against the field (eB) in order to get a value for the magnetic moment μ.

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To calculate magnetic polarisabilities in the background field method we take the average of the spin up and spin down energies rather than the difference,

$$\delta E(B) = \frac{1}{2}\left(\ln\left(\frac{G_\uparrow(B,t)}{G_\uparrow(0,t)}\frac{G_\downarrow(B,t)}{G_\downarrow(O,t)}\right)\right)_{\text{fit}}. \tag{2.8}$$

Doing so eliminates the magnetic moment term, leaving the quadratic polarisability term but also the Landau energy term $e|B|/2M_N$. Fortunately this term is always zero for the neutron because it is a neutral particle, therefore we can do a fit to a quadratic in order to find the polarisability of the neutron.

3. Landau Levels

In the energy-field relation (2.6) we have the Landau energy term $e|B|/2M_N$. This describes only the lowest Landau level, which is always non-zero for a charged particle in a magnetic field. The Landau levels arise due to the quantisation of orbital angular momentum and can be derived by considering the Dirac equation for a charged particle in an external magnetic field.

The polarisability's $\mathcal{O}(B^2)$ contribution to the energy in Eq. (2.6) is small compared to the ground state Landau energy, making it very difficult to determine the value of the polarisability. This is especially true because the creation operator creates not just states in the lowest Landau level but also a tower of Landau states, potentially contaminating the results.

These higher Landau levels can be seen in Figure 1, which shows an effective mass plot for the proton where all but the Landau energy, polarisability and higher order contributions (which should be negligible) have been removed by spin averaging. The plot shows two distinct plateaus at different energy levels, with a difference between them of 26 MeV. We can compare this with the calculated value for the expected difference between two Landau levels at this field strength and proton mass, setting $L_x = N_x a,\ L_y = N_y a$ to obtain

$$\begin{aligned}\frac{e|B|}{2M_N}(3-1) &= \frac{e|B|}{M_N}\\ &= \frac{-3\cdot 2\pi}{L_x L_y}\frac{1}{M_N}\\ &= 26\ \text{MeV},\end{aligned}$$

which is in clear agreement with the fitted plateaus. The calculated value for the lowest Landau level is also inside the (relatively large) margin for error on the lower plateau, which means that any contribution from the polarisability or higher order terms is impossible to isolate. Although this clear double plateau is not visible in our results at lighter quark masses or higher field strengths it shows that we cannot be confident that what we do see is in fact the ground state energy of the proton.

4. Simulation Details

Our calculations were performed using a FLIC fermion action on a $32^3 \times 40$ lattice, with 192 quenched configurations, at $\beta = 4.52$ and lattice spacing $a = 0.1275$ fm. We used 7 kappa

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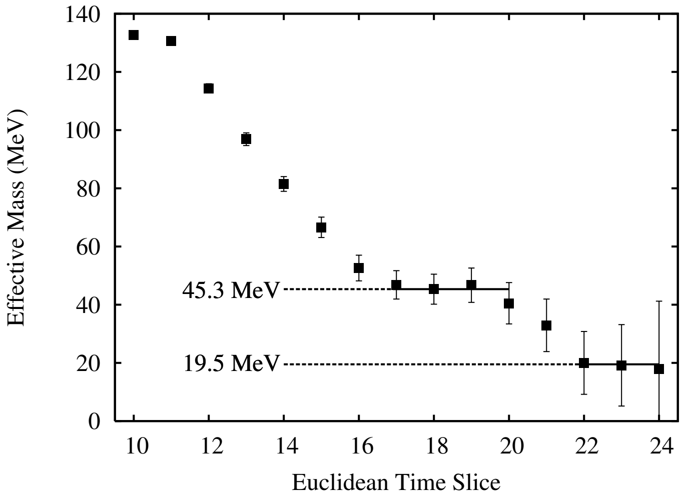

Figure 1: Spin-averaged effective mass for the proton at the smallest magnetic field and heaviest quark mass

values corresponding to pion masses $m_\pi = 0.8400, 0.7745, 0.6929, 0.6261, 0.5399, 0.4353.\ 0.2751$ GeV. The four non-zero magnetic field strengths used were those given by $n =$ 1, -2, 4, -8 in the quantisation condition (2.5). This is the field on the down quark, with the field felt by the whole baryon being -3 multiplied by these values. Having each field related to the one before it by a factor of -2 serves to reduce the computation requirements since some can serve double duty as down and up quarks. Energies were extracted from the correlation functions by applying an automated fitting routine to our effective mass plots. In order to try and minimise the errors we allowed the start and end of the fit window for the different field strength results to move a small amount, so long as they all overlapped on a certain range.

5. Results

Figure 2 shows a selection of effective mass plots from the heaviest quark mass. These show how the plateaus get shorter and later while the errors get larger as the field strength goes up, demonstrating another reason to use as small a field as possible. Figure 3 gives plots for the spin difference mass shift vs. field strength at each kappa. We found that in order to fit the largest field strength, and to a lesser extend the second largest, we were required to include a cubic term in our fit. This is due to the fact that we have small errors and were forced to go to large fields by following the quantisation condition (2.5). To ensure the higher order terms were not unduly affecting the result we also did a purely linear fit to just the two smallest field strengths, which agreed well within errors.

6. Conclusion

We have performed the first investigation of the magnetic properties of the nucleon using periodic boundary conditions and a quantised uniform background field. We have demonstrated the contaminating effect of Landau levels on background field calculations for charged particles such as the proton and are developing techniques to overcome this issue. Results for the neutron magnetic moment and polarisability are forthcoming.

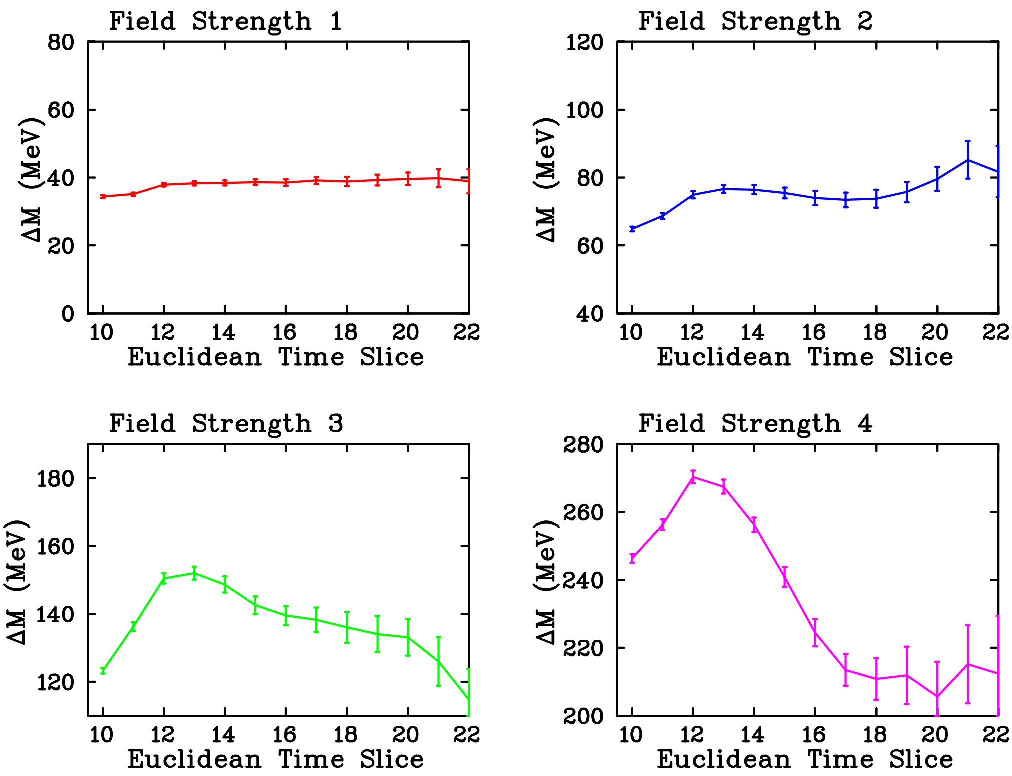

Figure 2: The spin-difference effective mass plots for the heaviest quark mass at the four non-zero field strengths.

References

[1] C. W. Bernard, T. Draper, K. Olynyk, M. Rushton, *Lattice QCD Calculation Of Some Baryon Magnetic Moments*, Phys. Rev. Lett. **49**, 1076 (1982).

[2] G. Martinelli, G. Parisi, R. Petronzio, F. Rapuano, *The Proton And Neutron Magnetic Moments In Lattice Qcd*, Phys. Lett. **B116**, 434 (1982).

[3] H. R. Rubinstein, S. Solomon, T. Wittlich, *Dependence of lattice hadron masses on external magnetic fields*, Nucl. Phys. **B457**, 577-593 (1995). [hep-lat/9501001].

[4] F. X. Lee, R. Kelly, L. Zhou, W. Wilcox, *Baryon magnetic moments in the background field method*, Phys. Lett. **B627**, 71-76 (2005). [hep-lat/0509067].

[5] F. X. Lee, S. Moerschbacher, W. Wilcox, *Magnetic moments of vector, axial, and tensor mesons in lattice QCD*, Phys. Rev. **D78**, 094502 (2008). [arXiv:0807.4150 [hep-lat]].

[6] F. X. Lee, L. Zhou, W. Wilcox, J. C. Christensen, *Magnetic polarizability of hadrons from lattice QCD in the background field method*, Phys. Rev. **D73**, 034503 (2006). [hep-lat/0509065].

[7] M. Burkardt, D. B. Leinweber, X. -m. Jin, *Background field formalism in quantum systems*, Phys. Lett. **B385**, 52-56 (1996). [hep-ph/9604450].

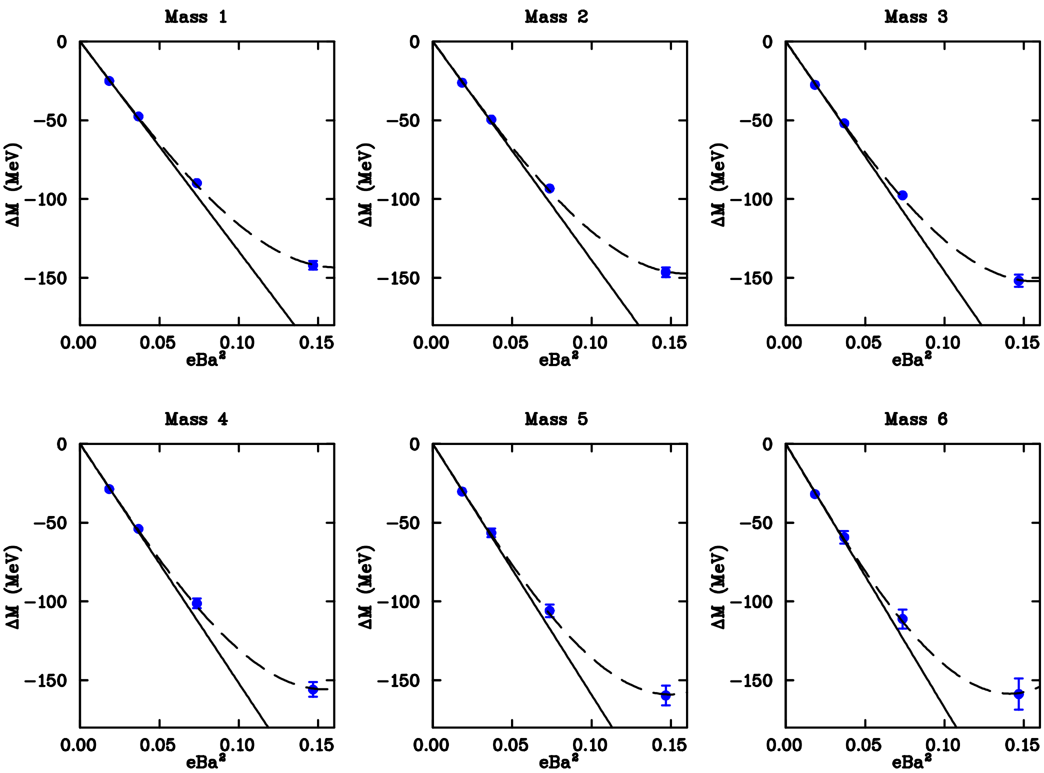

Figure 3: The spin-difference mass shifts vs field strength with line from a linear plus cubic least chi-squares fit.

[8] A. Alexandru, F. X. Lee, *The Background field method on the lattice*, PoS **LATTICE2008**, 145 (2008). [arXiv:0810.2833 [hep-lat]].

[9] D. B. Leinweber, R. M. Woloshyn, T. Draper, *On the electromagnetic properties of the baryon octet*, Nucl. Phys. Proc. Suppl. **20**, 463-466 (1991).

[10] S. Boinepalli, D. B. Leinweber, A. G. Williams, J. M. Zanotti, J. B. Zhang, *Precision electromagnetic structure of octet baryons in the chiral regime*, Phys. Rev. **D74**, 093005 (2006). [hep-lat/0604022].

[11] M. N. Butler, M. J. Savage, *Electromagnetic polarizability of the nucleon in chiral perturbation theory*, Phys. Lett. **B294**, 369-374 (1992). [hep-ph/9209204].

[12] V. Bernard, N. Kaiser, A. Schmidt, U. G. Meissner, *Consistent calculation of the nucleon electromagnetic polarizabilities in chiral perturbation theory beyond next-to-leading order*, Phys. Lett. **B319**, 269-275 (1993). [hep-ph/9309211].

[13] V. Bernard, N. Kaiser, U. GMeissner, A. Schmidt, *Aspects of nucleon Compton scattering*, Z. Phys. **A348**, 317 (1994). [hep-ph/9311354].

[14] A. W. Thomas, W. Weise, *The Structure of the Nucleon*, Berlin, Germany: Wiley-VCH (2001) 389 p.

[15] K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg/lbl.gov).