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Hyperon vector form factors with 2+1 flavor dynamical domain-wall fermions

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We report updated results of the hyperon vector form factor f_1 for $\Xi^0 \to \Sigma^+$ and $\Sigma^- \to n$ semileptonic decays from fully-dynamical lattice QCD. The calculations are carried out with gauge configurations generated by the RBC and UKQCD Collaborations with (2+1)-flavors of dynamical domain-wall fermions and the Iwasaki gauge action at $\beta = 2.13$, corresponding to a cutoff $a^{-1} = 1.73$ GeV. Our results, which are calculated at the lightest two sea quark masses (pion mass down to approximately 330 MeV), show that a sign of the second-order correction of SU(3) breaking on hyperon vector coupling $f_1(0)$ is likely negative. The tendency of the SU(3) breaking correction observed in this work disagrees predictions of both the latest baryon chiral perturbation theory result and large N_c analysis.

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1. Introduction

The matrix element for hyperon beta decays $B \to bl\bar{\nu}$ is composed of the vector and axial-vector transitions, $\langle b(p')|V_\alpha(x)+A_\alpha(x)|B(p)\rangle$, which are described by six form factors: the vector (f_1), weak-magnetism (f_2), and induced scalar (f_3) form factors for the vector current, and the axial-vector (g_1), weak electricity (g_2), and induced pseudo-scalar (g_3) form factors for the axial current. The experimental rate of the hyperon beta decay, $B \to bl\bar{\nu}$, is given by

$$\Gamma = \frac{G_F^2}{60\pi^3}(M_B - M_b)^5(1-3\delta)|V_{us}|^2|f_1^{B\to b}(0)|^2\left[1+3\left|\frac{g_1^{B\to b}(0)}{f_1^{B\to b}(0)}\right|^2+\cdots\right], \tag{1.1}$$

where G_F and V_{us} denote the Fermi constant and an element of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix respectively [1]. Here, M_B (M_b) denotes the rest mass of the initial (final) state. The ellipsis can be expressed in terms of a power series in the small parameter $\delta = (M_B - M_b)/(M_B+M_b)$, which is regarded as a size of flavor SU(3) breaking [2]. The first linear term in δ, which should be given by $-4\delta[g_2(0)g_1(0)/f_1(0)^2]_{B\to b}$, is safely ignored as small as $\mathcal{O}(\delta^2)$ since the nonzero value of the second-class form factor g_2 [3] should be induced at first order of the δ expansion [2]. The absolute value of $g_1(0)/f_1(0)$ can be determined by measured asymmetries such as electron-neutrino correlation [1, 2]. Therefore a theoretical estimate of vector coupling $f_1(0)$ is primarily required for the precise determination of $|V_{us}|$.

The value of $f_1(0)$ should be equal to the SU(3) Clebsch-Gordan coefficients up to the second order in SU(3) breaking, thanks to the Ademollo-Gatto theorem [4]. As the mass splittings among octet baryons are typically of the order of 10-15%, an expected size of the second-order corrections is a few percent level. However, either the size, or the sign of their corrections are somewhat controversial among various theoretical studies at present as summarized in Table 1. A model independent evaluation of SU(3)-breaking corrections is highly demanded. Although recent quenched lattice studies suggest that the second-order correction on $f_1(0)$ is likely negative [12, 13], we need further confirmation from (2+1)-flavor dynamical lattice QCD near the physical point.

2. Numerical results

In this study, we use the RBC-UKQCD joint (2+1)-flavor dynamical DWF coarse ensembles on a $24^3 \times 64$ lattice [14], which are generated with the Iwasaki gauge action at $\beta = 2.13$. For the domain wall fermions with the domain-wall height of $M_5 = 1.8$, the number of sites in the fifth dimension is 16, which gives a residual mass of $am_{\rm res} \approx 0.003$. Each ensemble of configurations uses the same dynamical strange quark mass, $am_s = 0.04$. The inverse of lattice spacing is $a^{-1} = 1.73(3)$ (a=0.114(2) fm), which is determined from the Ω^- baryon mass [14]. We have already published our findings in nucleon structure from the same ensembles in three publications, Refs. [15, 16, 17].

In this study, we calculate the vector coupling $f_1(0)$ for two different hyperon beta-decays, $\Xi^0 \to \Sigma^+ l\bar{\nu}$ and $\Sigma^- \to nl\bar{\nu}$, where $f_1^{\Xi\to\Sigma}(0) = +1$ and $f_1^{\Sigma\to n}(0) = -1$ in the exact SU(3) limit. We will present our results for $am_{ud} = 0.005$ and 0.01, which correspond to about 330 MeV and 420 MeV pion masses [1]. We use 4780 (2380) trajectories separated by 20 trajectories for $am_{ud} = 0.005$

[1] Preliminary results obtained at $am_{ud} = 0.005$ were first reported in Ref. [18].

Type of result (reference)	$\Lambda \to p$	$\Sigma^- \to n$	$\Xi^- \to \Lambda$	$\Xi^0 \to \Sigma^+$
Bag model [5]	0.97	0.97	0.97	0.97
Quark model [6]	0.987	0.987	0.987	0.987
Quark model [7]	0.976	0.975	0.976	0.976
$1/N_c$ expansion [8]	1.02(2)	1.04(2)	1.10(4)	1.12(5)
Full $\mathcal{O}(p^4)$ HBChPT [9]	1.027	1.041	1.043	1.009
Full $\mathcal{O}(p^4)$ + partial $\mathcal{O}(p^5)$ HBChPT [10]	1.066(32)	1.064(6)	1.053(22)	1.044(26)
Full $\mathcal{O}(p^4)$ IRChPT [11]	0.943(21)	1.028(02)	0.989(17)	0.944(16)
Full $\mathcal{O}(p^4)$ IRChPT + Decuplet [11]	1.001(13)	1.087(42)	1.040(28)	1.017(22)
Quenched lattice QCD [12, 13]	N/A	0.988(29)	N/A	0.987(19)

Table 1: Theoretical uncertainties of $\tilde{f}_1 = |f_1/f_1^{\mathrm{SU(3)}}|$ for various hyperon beta-decays. HBChPT and IRChPT stand for heavy baryon chiral perturbation theory and the infrared version of baryon chiral perturbation theory.

(0.01). [14]. The total number of configurations is 240 for $am_{ud} = 0.005$ and 120 for $am_{ud} = 0.01$ as summarized in Table 2. We make two measurements on each configuration using two locations of the source time slice, $t_{\mathrm{src}} = 0$ and 32. Details of our calculation of the quark propagators are described in Ref [16].

For convenience in numerical calculations, instead of the vector form factor $f_1(q^2)$, we consider the so-called scalar form factor

$$f_S^{B\to b}(q^2) = f_1^{B\to b}(q^2) + \frac{q^2}{M_B^2 - M_b^2} f_3^{B\to b}(q^2), \tag{2.1}$$

where f_3 represents the second-class form factor, which are identically zero in the exact SU(3) limit [3]. The value of $f_S(q^2)$ at $q^2_{\mathrm{max}} = -(M_B - M_b)^2$ can be precisely evaluated by the double ratio method proposed in Ref. [12], where all relevant three-point functions are determined at zero three-momentum transfer $|\mathbf{q}| = 0$.

Here we note that the absolute value of the renormalized $f_S(q^2_{\mathrm{max}})$ is exactly unity in the flavor SU(3) symmetric limit, where $f_S(q^2_{\mathrm{max}})$ becomes $f_1(0)$, for the hyperon decays considered here. Thus, the deviation from unity in $f_S(q^2_{\mathrm{max}})$ is attributed to three types of the SU(3) breaking effect: (1) the recoil correction ($q^2_{\mathrm{max}} \neq 0$) stemming from the mass difference of B and b states, (2) the presence of the second-class form factor $f_3(q^2)$, and (3) the deviation from unity in the renormalized $f_1(0)$. Taking the limit of zero four-momentum transfer of $f_S(q^2)$ can separate the third effect from the others, since the scalar form factor at $q^2 = 0$, $f_S(0)$, is identical to $f_1(0)$. Indeed, our main target is to measure the third one.

The scalar form factor $f_S(q^2)$ at $q^2 > 0$ [2] is also calculable with non-zero three-momentum transfer ($|\mathbf{q}| \neq 0$) [13]. We use four lowest non-zero momenta: $\mathbf{q} = 2\pi/L \times (1,0,0)$, $(1,1,0)$, $(1,1,1)$ and $(2,0,0)$, corresponding to a q^2 range from about 0.2 to 0.8 GeV^2. We then can make the q^2 interpolation of $f_S(q^2)$ to $q^2 = 0$ by the values of $f_S(q^2)$ at $q^2 > 0$ together with the precisely measured value of $f_S(q^2)$ at $q^2 = q^2_{\mathrm{max}} < 0$ from the double ratio. In Fig. 1, we plot the absolute

[2] We note that q^2 quoted here is defined in the Euclidean metric convention.

am_{ud}	$N_{\rm conf}$	MD range	$N_{\rm sep}$	$N_{\rm meas}$	m_π[GeV]	m_N[GeV]	m_Σ[GeV]	m_Ξ[GeV]
0.005	240	940-5720	20	2	0.331(1)	1.13(1)	1.33(1)	1.43(1)
0.01	120	5060-7440	20	2	0.419(1)	1.23(2)	1.39(1)	1.47(1)

Table 2: Summary of simulation parameters: the number of gauge configurations, the range, where measurements were made, in molecular-dynamics (MD) time, the number of trajectory separation between each measured configuration, and the number of measurements on each configurations. The table also lists the pion, nucleon, Σ and Ξ masses.

value of the renormalized $f_S(q^2)$ as a function of q^2 for $\Xi^0 \to \Sigma^+$ (left) and $\Sigma^- \to n$ (right) at $am_{ud} = 0.005$ (upper panels) and 0.01 (lower panels). Open circles are $f_S(q^2)$ at the simulated q^2. The solid (dashed) curve is the fitting result with all nine data points by using the monopole (quadratic) interpolation form [13], while the open diamond (square) represents the interpolated value to $q^2 = 0$. As shown in Fig. 1, two determinations to evaluate $f_S(0) = f_1(0)$ from measured points are indeed consistent with each other. Thus, this observation indicates that the choice of the interpolation form does not affect the interpolated value $f_1(0)$ significantly.

We finally quote the values obtained from the monopole fit as our final values. The values of the renormalized $f_1(0)$ divided by the SU(3) symmetric value are obtained for the $\Xi^0 \to \Sigma^+$ decay,

$$
\begin{aligned}
[f_1(0)/f_1^{\rm SU(3)}]_{\Xi\to\Sigma} &= 0.982(10) \text{ at } m_\pi = 330\,\text{MeV}, \\
[f_1(0)/f_1^{\rm SU(3)}]_{\Xi\to\Sigma} &= 0.976(7) \quad \text{at } m_\pi = 420\,\text{MeV},
\end{aligned}
$$

and for the $\Sigma^- \to n$ decay,

$$
\begin{aligned}
[f_1(0)/f_1^{\rm SU(3)}]_{\Sigma\to n} &= 0.966(14) \text{ at } m_\pi = 330\,\text{MeV}, \\
[f_1(0)/f_1^{\rm SU(3)}]_{\Sigma\to n} &= 0.982(8) \quad \text{at } m_\pi = 420\,\text{MeV},
\end{aligned}
$$

which are consistent with the sign of the second-order corrections on $f_1(0)$ reported in earlier quenched lattice studies [12, 13] and preliminary results from mixed action calculation [19] and $n_f = 2+1$ dynamical improved Wilson fermion calculations [20]. However, we recall that the tendency of the SU(3) breaking correction observed here disagrees predictions of both the latest baryon ChPT result [11] and large N_c analysis [8].

3. Summary

We have presented results of the flavor SU(3) breaking effects on hyperon vector coupling $f_1(0)$ for the $\Xi^0 \to \Sigma^+$ and $\Sigma^- \to n$ decays in (2+1)-flavor QCD using domain wall quarks. We have observed that the second-order correction on $f_1(0)$ is still *negative* for both decays at much smaller pion mass, $m_\pi = 330$ and 420 MeV, than in the previous quenched simulations. The size of the second-order corrections observed here is also comparable to what was observed in our DWF calculations of K_{l3} decays [21]. To extrapolate the value of $f_1(0)$ to the physical point, our simulations at two heavier sea quark masses ($am_{ud} = 0.02$ and 0.03) are now in progress.

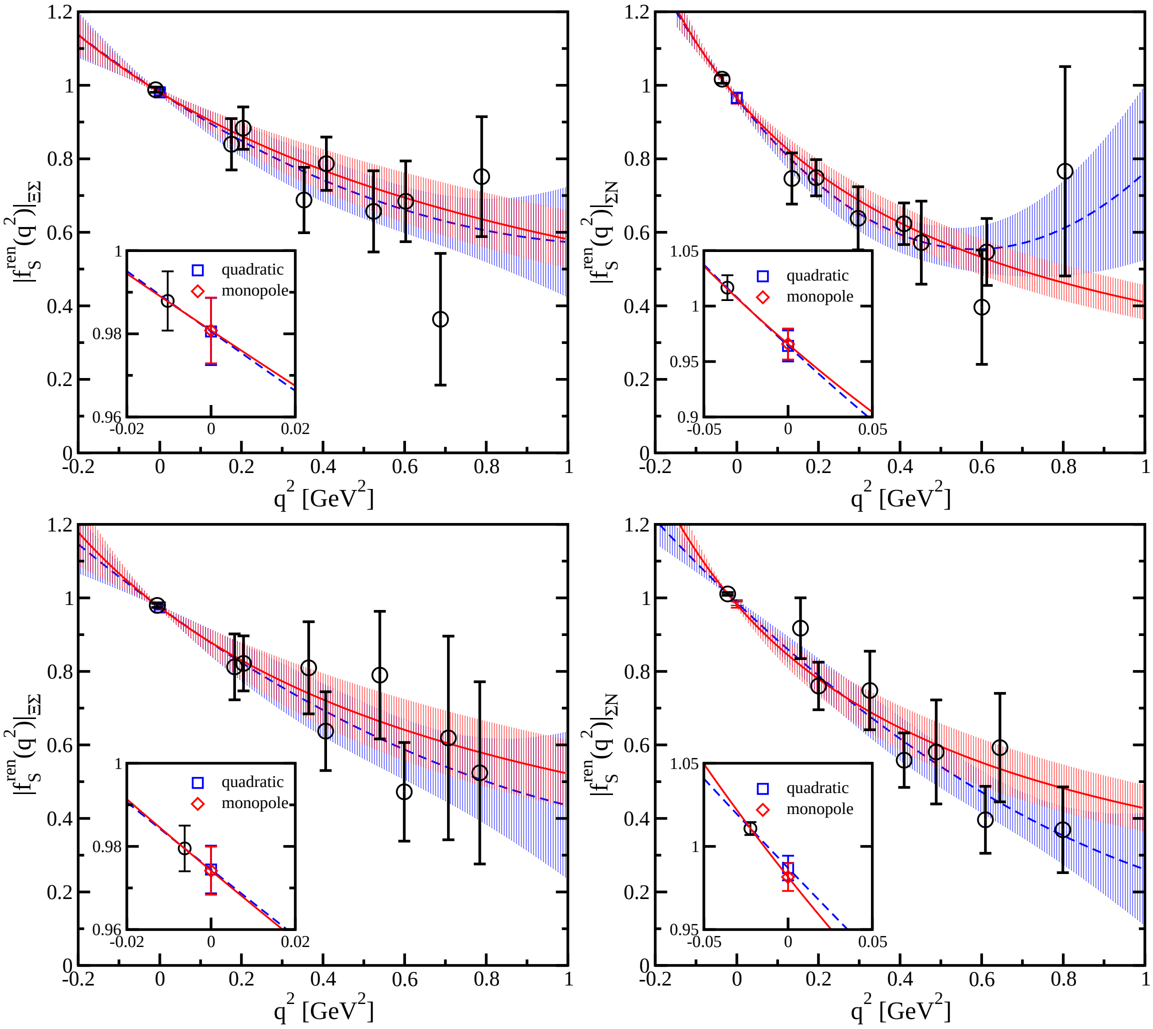

Figure 1: Interpolation of $|f_S(q^2)|$ to $q^2 = 0$ for $\Xi^0 \to \Sigma^+$ (left figures) and $\Sigma^- \to n$ (right figures). Upper (lower) panels are for $am_{ud} = 0.005$ (0.01).

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