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Disconnected Contributions for Nucleon 3-pt Functions

G. Bali[a], S. Collins[a], M. Göckeler[a], R. Horsley[b], Y. Nakamura[c], A. Nobile[a], D.Pleiter[ad], P.E.L Rakow[e], A.Schäfer[∗a], G. Schierholz[f], A. Sternbeck[a], J. Zanotti[g]

[a]*Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany*
[b]*School of Physics, University of Edinburgh, Edinburgh EH9 3JZ, UK*
[c]*RIKEN Advanced Institute for Computational Science, Kobe, Hyogo 650-0047, Japan*
[d]*JSC, Research Center Jülich, 52425 Jülich, Germany*
[e]*Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK*
[f]*Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany*
[g]*CSSM, School of Chemistry & Physics, University of Adelaide, Adelaide SA 5005, Australia*
E-mail: `andreas.schaefer@physik.uni-r.de`

We determined the quark contributions to the the nucleon mass, the so-called σ-terms, as well as their contributions to the nucleon spin, i.e. Δs, Δu and Δd. Both, the connected and disconnected contributions to the respective matrix elements were computed, using the non-perturbatively improved Sheikholeslami-Wohlert Wilson Fermionic action. We simulated $n_{\rm F} = 2$ mass degenerate sea quarks with a pion mass of about 285 MeV and a lattice spacing $a \approx 0.073$ fm. We obtained the renormalized value $\sigma_{\pi{\rm N}} = (38 \pm 12)$ MeV, extrapolated to the physical mass point, and the strangeness fraction $f_{T_s} = \sigma_s/m_{\rm N} = 0.012(14)^{+10}_{-3}$ at our larger than physical sea quark mass. For the strangeness contribution to the nucleon spin we obtained $\Delta s^{\overline{MS}}(\sqrt{7.4}\,{\rm GeV}) = -0.020(10)(4)$ a value substantially smaller than expectations based two assumptions: a negligible contribution to the first moment of $g_1(x,Q^2)$ from the experimentally inaccessible small-x range and $SU_f(3)$ symmetry.

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∗Speaker.

PoS(Lattice 2011)174

1. Introduction

This conference proceeding is basically identical in content with the recent publication [1, 2] of our group.

Somewhat surprisingly the structure of even the best known hadron, the nucleon, is not really well understood. This statement holds in particular with respect to the quark sigma terms, which are crucial to understand how the mass of the nucleon is generated

$$f_{T_q} = m_q \langle N|\bar{q}q|N\rangle / m_{\rm N} = \sigma_q / m_N \quad , \tag{1.1}$$

and to the contributions of quark spin, quark angular momentum and gluon total angular momentum to the nucleon spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_G \qquad \Delta\Sigma \approx \Delta u + \Delta d + \Delta s \; . \tag{1.2}$$

The quark sigma terms parameterize the fractions of the nucleon mass $m_{\rm N}$ that are carried by quarks of flavor q and determine the coupling to scalars like the Higgs. It is thus relevant even for such exotic endeavors like the search for dark matter [3]. The combination $m_{\rm N}\sum_q f_{T_q}$, $q \in \{u,d,s\}$, appears quadratically in the cross section that is proportional to $|f_{\rm N}|^2$, where

$$f_{\rm N} = m_{\rm N}\left(\sum_{q\in\{u,d,s\}} f_{T_q}\frac{\alpha_q}{m_q} + \frac{2}{9n_h} f_{T_G} \sum_{q\in\{c,b,t,\ldots\}} \frac{\alpha_q}{m_q}\right) , \tag{1.3}$$

with the couplings $\alpha_q \propto m_q/m_W$. Here, n_h denotes the number of heavy quark flavors. Due to the trace anomaly of the energy momentum tensor the following equation holds

$$f_{T_G} = 1 - \sum_{q\in\{u,d,s\}} f_{T_q} \, , \tag{1.4}$$

such that $f_{\rm N}$ depends only weakly on heavy quark flavors [4].

The light quark contribution, the pion-nucleon σ-term, is defined as

$$\sigma_{\pi{\rm N}} = \sigma_u + \sigma_d = m_u \frac{\partial m_{\rm N}}{\partial m_u} + m_d \frac{\partial m_{\rm N}}{\partial m_d} \approx m_{\rm PS}^2 \frac{d m_{\rm N}}{d m_{\rm PS}^2}\bigg|_{m_{\rm PS}=m_\pi} \tag{1.5}$$

$\sigma_{\pi N}$ is a quantity which should be especially well suited to test the validity of effective descriptions on the basis of hadronic degrees of freedom. From dispersive analysis of pion-nucleon scattering data, the values [5] $\sigma_{\pi N} = 45(8)$ MeV and [6] $\sigma_{\pi{\rm N}} = 64(7)$ MeV were obtained while a recent covariant baryon chiral perturbation theory (BχPT) analysis [7] resulted in the estimate $\sigma_{\pi{\rm N}} = 59(7)$ MeV. As we will show all of these results are larger than our central lattice value implying that even one of the most basic properties of nucleons, their light quark content, might be only poorly known. Obviously, a careful estimation of the systematic uncertainties is crucial.

The importance of a careful treatment of systematic errors is also crucial for investigations of the nucleon spin structure. In this case in particular the strange quark spin contribution is very sensitive to possible violations of the SU(3) flavor symmetry, see [8]. For Δs the integral over the range in which data exists usually agrees with zero [9, 10], while global analysis tend to obtain substantial negative values [11, 12]. The latter are, however, primarily enforced by the assumed flavor symmetry for F/D. The validity of this assumption can now be checked on the lattice. The generally poor knowledge of the strangeness content of the nucleon is also illustrated by direct experimental results from HERMES [13] and by the uncertainty bands of recent fits for parton distribution functions by the NNPDF collaboration [14]. Other recent direct lattice determinations of either the sigma terms or the nucleon spin structure include Refs. [15, 16, 17, 18, 19, 20].

2. Simulation and Renormalization details

We simulated $n_\mathrm{f} = 2$ non-perturbatively improved clover fermions, using the Wilson gauge action, at $\beta = 5.29$ and $\kappa = \kappa_{ud} = 0.13632$ (corresponding to $m_{\mathrm{PS},ud} = 285(3)(7)\,\mathrm{MeV}$). Setting the scale from the chirally extrapolated nucleon mass we obtained the lattice spacing $a^{-1} = 2.71(2)(7)\,\mathrm{GeV}$, where the errors are statistical and from the extrapolation, respectively. For the valence quarks we also used two additional κ values, $\kappa_m = 0.13609$ (corresponding to $m_{\mathrm{PS},m} = 449(3)(11)\,\mathrm{MeV}$) and $\kappa_s = 0.13550$ (corresponding to $m_{\mathrm{PS},s} = 720(5)(18)\,\mathrm{MeV}$). κ_s was fixed such that the $m_{\mathrm{PS},s}$ value is close to the mass of a hypothetical strange-antistrange pseudoscalar meson: $(m^2_{K^\pm} + m^2_{K^0} - m^2_{\pi^\pm})^{1/2} \approx 686.9$ MeV. We investigated volumes of $32^3 \times 64$ and $40^3 \times 64$ lattice points.

The quark polarizations were extracted from the large-time behavior of ratios of three-point over two-point functions. We created a polarized proton at a time $t_0 = 0$, probed it with an axial current at a time t and destroyed the zero momentum proton at $t_\mathrm{f} > t > 0$. One needs to compute quark line connected and disconnected terms:

$$R^{\mathrm{con}}(t_\mathrm{f},t) = \frac{\langle \Gamma^{\alpha\beta}_{\mathrm{pol}} C^{\beta\alpha}_{3pt}(t_\mathrm{f},t)\rangle}{\langle \Gamma^{\alpha\beta}_{\mathrm{unpol}} C^{\beta\alpha}_{2pt}(t_\mathrm{f})\rangle} \quad R^{\mathrm{dis}}(t_\mathrm{f},t) = -\frac{\langle \Gamma^{\alpha\beta}_{\mathrm{pol}} C^{\beta\alpha}_{2pt}(t_\mathrm{f}) \sum_{\mathbf{x}} \mathrm{Tr}[\gamma_j\gamma_5 M^{-1}(\mathbf{x},t;\mathbf{x},t)]\rangle}{\langle \Gamma^{\alpha\beta}_{\mathrm{unpol}} C^{\beta\alpha}_{2pt}(t_\mathrm{f})\rangle} \tag{2.1}$$

Here M is the lattice Dirac operator, $\Gamma_{\mathrm{unpol}} = \frac{1}{2}(1+\gamma_4)$ a parity projector and $\Gamma_{\mathrm{pol}} = i\gamma_j\gamma_5\Gamma_{\mathrm{unpol}}$ projects out the difference between the two polarizations (in direction $\hat{j}$). We averaged over $j = 1,2,3$ to increase statistics. For the up and down quark matrix elements we computed the sum of connected and disconnected terms while only R^{dis} contributes to Δs.

The Δq were obtained in the limit $t_\mathrm{f} \gg t \gg 0$. For the disconnected contribution we fixed $t = 4a \approx 0.29$ fm and vary t_f. Using the sink and source smearing described in [2], we find the asymptotic limit to be effectively reached for $t_\mathrm{f} \simeq 6a$–$7a$ and fitted the ratios to a constant for $t_\mathrm{f} \geq 8a \approx 0.58$ fm, see Fig. 1 for an example. The connected part, for which the statistical accuracy is less of an issue, was obtained at the larger, fixed value $t_\mathrm{f} = 15a$, building upon previous experience [21], varying t.

The disconnected contribution was computed with the stochastic estimator methods described in [22, 23], employing time partitioning, a second order hopping parameter expansion and the truncated solver method. We computed the Green functions for four equidistant source times on each gauge configuration. We also constructed backwardly propagating nucleons, replacing the positive parity projector $\frac{1}{2}(1+\gamma_4)$ by $\frac{1}{2}(1-\gamma_4)$, seeding the noise vectors on eight (four times two) time slices. In addition to the 48 (four times spin-color) solves for smeared conventional sources, that are necessary to construct the two-point functions, we ran the Conjugate Gradient (CG) algorithm on $N_1 = 730$ complex Z_2 noise sources for $n_\mathrm{t} = 40$ iterations. The bias from this

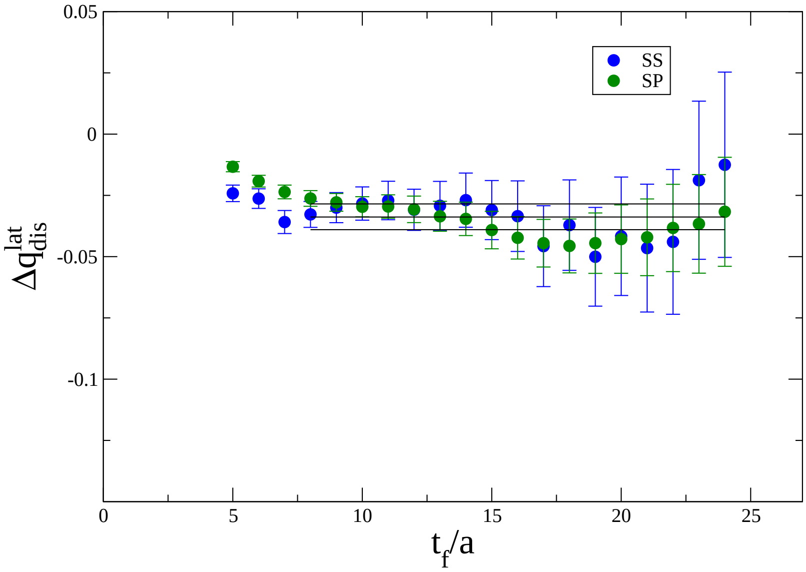

Figure 1: The disconnected ratio $R^{\rm dis}$ versus $t_{\rm f}$ on the $40^3 \times 64$ volume at $\kappa_{\rm val} = \kappa_{\rm cur} = \kappa_s$ for smeared/smeared (SS) and smeared/point (SP) source/sink combinations.

truncation is corrected for [22] by $N_2 = 50$ BiCGstab solves that were run to convergence. We analyzed a total of 2024 thermalized trajectories on each of the two volumes where we binned the data to eliminate autocorrelations.

The non-singlet axial current renormalization factor is taken from [24]: $Z_A^{ns} = 0.76485(64)(73)$.

The singlet current has an anomalous dimension. To first non-trivial order this reads [25, 26] $\gamma_A^s(\alpha_s) = -6C_{\rm F}n_{\rm f}[\alpha_s/(4\pi)]^2$. Z_A^s deviates from Z_A^{ns} starting at $\mathscr{O}(\alpha_s^2)$ in perturbation theory. Both factors have been calculated to this order, with the following result for the conversion into the $\overline{MS}$ scheme at a scale μ [27]

$$z(\mu,a) = Z_A^s(\mu,a) - Z_A^{ns}(a) = C_{\rm F}n_{\rm f}\left[15.8380(8) - 6\ln(a^2\mu^2)\right]\left(\frac{\alpha_s}{4\pi}\right)^2, \tag{2.2}$$

where we have set $c_{\rm SW} = 1$ to be consistent to this order in perturbation theory. We used $\alpha_s = -3\ln\langle U_\Box\rangle/(4\pi) = 0.14278(5)$, with the chirally extrapolated value[28] $\langle U_\Box\rangle = 0.54988(11)$. $\mathscr{O}(a)$ improvement implies [29, 30] $Z_A^{ns} \mapsto Z_A^{ns}(1+b_A am)$ and $Z_A^s \mapsto Z_A^s(1+b_A^s am)$ with $b_A = b_A^s + \mathscr{O}(\alpha_s^2) \approx 1 + 18.02539 C_{\rm F}\frac{\alpha_s}{4\pi}$. For $n_{\rm f} = 2$ we got $z(\sqrt{7.4}\,\mathrm{GeV}) = 0.0055(1)(27)$ at the renormalization scale $\mu^2 = 7.4\,\mathrm{GeV}^2 = 1.01(5)\,a^{-2}$.

For $n_{\rm f} = 2$ sea quarks the singlet current is $\Delta u + \Delta d$ rather than the $\Delta\Sigma$ of Eq. (1.2). This modifies the renormalization pattern to:

$$\begin{pmatrix}\Delta u(\mu)\\ \Delta d(\mu)\\ \Delta s(\mu)\end{pmatrix}^{\overline{MS}} = \begin{pmatrix} Z_A^{ns}(a) + \frac{z(\mu,a)}{2} & \frac{z(\mu,a)}{2} & 0\\ \frac{z(\mu,a)}{2} & Z_A^{ns}(a) + \frac{z(\mu,a)}{2} & 0\\ \frac{z(\mu,a)}{2} & \frac{z(\mu,a)}{2} & Z_A^{ns}(a)\end{pmatrix}\begin{pmatrix}\Delta u(a)\\ \Delta d(a)\\ \Delta s(a)\end{pmatrix}^{\rm lat}. \tag{2.3}$$

3. Results

In Fig. 2 we display the volume and (light) valence quark mass dependence of our unrenormalized $\Delta s^{\rm lat}$. No statistically significant size and valence quark mass dependencies were observed.

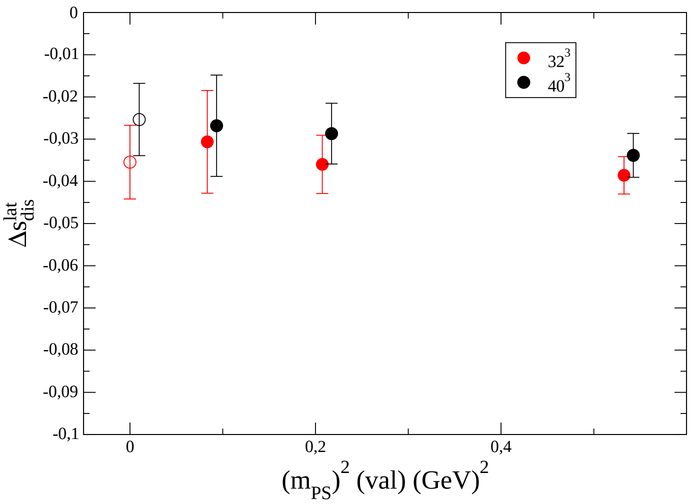

Figure 2: Volume and valence quark mass dependence of the unrenormalized $\Delta s^{\rm lat}$.

Table 1: The connected and disconnected contributions to $\Delta q^{\rm lat}$ as well as the renormalized spin content at a scale $\mu = \sqrt{7.4}$ GeV. The first error is statistical, the second is from the renormalization.

q	V, L	$\Delta q^{\rm lat}_{\rm con}$	$\Delta q^{\rm lat}_{\rm dis}$	$\Delta q^{\overline{\rm MS}}(\mu)$
u		1.065(22)	-0.034(16)	0.794(21)(2)
d		-0.344(14)	-0.034(16)	-0.289(16)(1)
s	$V = 32^3 64$	0	-0.031(12)	-0.023(10)(1)
T_3	$L \approx 2.33\,{\rm fm}$	1.409(24)	0	1.082(18)(2)
T_8		0.721(26)	-0.006(18)	0.550(24)(1)
Σ		0.721(26)	-0.098(42)	0.482(38)(2)
u		1.071(15)	-0.049(17)	0.787(18)(2)
d		-0.369(9)	-0.049(17)	-0.319(15)(1)
s	$V = 40^3 64$	0	-0.027(12)	-0.020(10)(1)
T_3	$L \approx 2.91\,{\rm fm}$	1.439(17)	0	1.105(13)(2)
T_8		0.702(18)	-0.044(19)	0.507(20)(1)
Σ		0.702(18)	-0.124(44)	0.448(37)(2)

Our results for the first moments of the polarized quark distributions in the $\overline{MS}$ scheme are given in Table 1

$$\Delta q^{\overline{MS}}(\mu) = Z_A^{ns}(1 + b_A a m_q)\Delta q^{\rm lat} + \frac{z(\mu)}{2}(\Delta u + \Delta d)^{\rm lat} \quad . \tag{3.1}$$

Our most important result with respect to the nucleon spin structure is that Δs comes out very small.

Also for the sigma terms renormalization is non-trivial. As the Wilson action explicitly breaks chiral symmetry, singlet and non-singlet flavor combinations renormalize differently. Consequently, the renormalized strangeness matrix element receives large subtractions from light quark contributions.

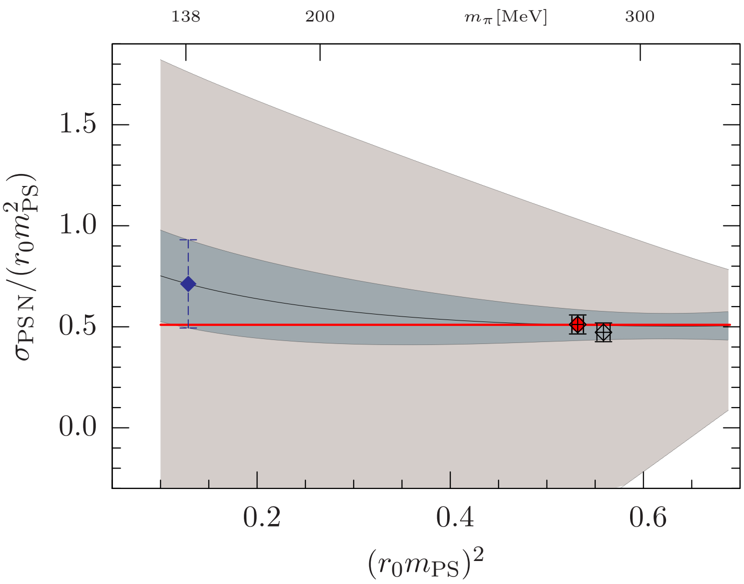

Figure 3: Extrapolation of $\sigma_{\mathrm{PS\,N}}/m_{\mathrm{PS}}^2$ to the physical point using covariant BχPT for the $40^3\times 64$ volume (solid symbol). The broad error band is obtained from nucleon mass data alone. The horizontal line is the leading order expectation and the open symbol our result for the $32^3\times 64$ volume.

Again we find no significant finite size effects and obtain $\sigma_{\mathrm{PS\,N}}(m_{\mathrm{PS}} \approx 285\,\mathrm{MeV}) = 106(11)(3)$ MeV. Using additional nucleon mass data we extrapolate our value to the physical point and obtain [2]

$$\sigma_{\pi\mathrm{N}}^{\mathrm{phys}} = (38 \pm 12)\,\mathrm{MeV}\,, \tag{3.2}$$

where the dominant error is from the chiral extrapolation, see Fig. 3.

For the strangeness and gluon contributions to the nucleon mass we got

$$f_{T_s} = 0.012(14)^{+10}_{-3}\,, \quad f_{T_G} = 0.951^{+20}_{-27}\,. \tag{3.3}$$

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PoS(Lattice 2011)174