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Chiral transition temperature and aspects of deconfinement in 2+1 flavor QCD with the HISQ/tree action

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We present results on the chiral transition temperature T_c in 2+1 flavor QCD extrapolated to the continuum limit and the physical light quark mass. The extrapolations are based on the data from simulations on lattices with temporal extent $N_\tau = 6$, 8 and 12 with the HISQ/tree and $N_\tau = 8$ and 12 with the asqtad action. The chiral transition is analyzed in terms of universal $O(N)$ scaling functions. After performing simultaneous asqtad and HISQ/tree continuum extrapolation the chiral transition temperature is $T_c = 154 \pm 9$ MeV. We also discuss the deconfinement aspects of the transition in terms of the renormalized Polyakov loop, fluctuations and correlations of several conserved charges and the trace anomaly.

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1. Introduction

In these proceedings we follow up on the line of work on 2+1 flavor QCD thermodynamics with improved staggered fermions by the HotQCD collaboration. The setup of simulations and some preliminary results have been reported earlier, e.g. [1, 2, 3, 4] and the continuum extrapolation for chiral T_c at the physical quark masses is presented in [5]. Full data set includes several lines of constant physics down to the light quark mass $m_l = m_s/20$ for asqtad and $m_l = m_s/40$ for HISQ/tree[1]. The lattice spacings cover the range of temperatures $T = 130-440$ MeV with $N_\tau = 6$, 8 and 12 for HISQ/tree and $T = 148-304$ MeV with $N_\tau = 8$ and 12 for asqtad.

Performing the continuum limit requires control over cutoff effects. In improved staggered discretization schemes the leading $O(a^2)$ errors at low temperature (coarse lattices) originate from violations of taste symmetry, that distort the hadron spectrum. For this reason it is important to perform simulations on fine enough lattices (large N_τ in finite-temperature setup) and/or use actions with the smallest discretization effects. Analysis of the discretization effects for asqtad and HISQ/tree used in this study is presented in Ref. [2].

2. Chiral transition

For vanishing light quark masses there is a chiral phase transition which is expected to be of second order and in the $O(4)$ universality class [7]. However, universal scaling allows to define pseudo-critical temperatures for the chiral transition even for non-zero light quark masses, provided they are small enough. For staggered fermions that preserve only a part of the chiral symmetry there is a complication: in the chiral limit at finite lattice spacing the relevant universality class is $O(2)$ rather than $O(4)$. Fortunately, in the numerical analysis the differences between $O(2)$ and $O(4)$ universality classes are small so when referring to scaling we will use the term $O(N)$ scaling. Previous studies with the p4 action provided evidence for $O(N)$ scaling [8, 9]. Similar analysis for the asqtad and HISQ/tree action establishing if the $O(N)$ scaling is applicable is performed in [5] and explained below.

The order parameter for the chiral transition is the chiral condensate

$$M_b \equiv \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} \ . \tag{2.1}$$

Its temperature and quark mass dependence near the critical temperature can be parametrized by a universal scaling function f_G and a regular function $f_{M,reg}$ that describes corrections to scaling:

$$M_b(T, m_l, m_s) = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + f_{M,reg}(T,H), \ \ t = \frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0}\right), \ \ h = \frac{1}{h_0}H, \ \ H = \frac{m_l}{m_s} \tag{2.2}$$

and T_c^0 is the critical temperature in the chiral limit. The pseudo-critical temperature can be defined as the peak position of the chiral susceptibility

$$\chi_{m,l} = \frac{\partial}{\partial m_l}\langle \bar{\psi}\psi \rangle_l \tag{2.3}$$

[1] The lightest mass $m_l = m_s/40$ is used for the analysis of the chiral condensate and susceptibility [6], while all other quantities are calculated at $m_l = m_s/20$ for HISQ/tree.

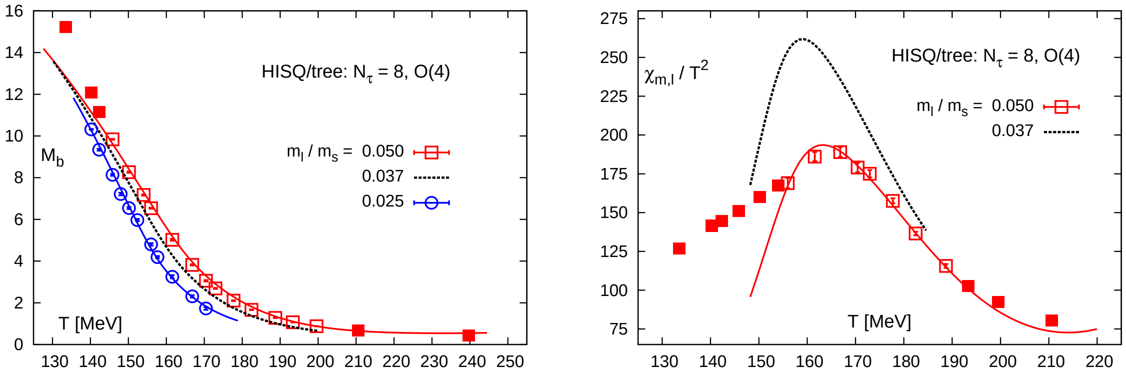

Figure 1: An example of a simultaneous fit to the chiral condensate (left) and susceptibility (right) for HISQ/tree on $N_\tau = 8$ lattices. Open symbols indicate the range included in the fit. Dotted black line is an extrapolation to the physical light quark mass.

whose scaling behavior is also described by f_G and $f_{M,reg}$ as

$$\frac{\chi_{m,l}}{T^2} = \frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1}f_\chi(z) + \frac{\partial f_{M,reg}(T,H)}{\partial H}\right), \quad f_\chi(z) = \frac{1}{\delta}[f_G(z) - \frac{z}{\beta}f'_G(z)], \quad z = \frac{t}{h^{1/\beta\delta}}. \tag{2.4}$$

The singular function f_G is well studied in spin models and has been parametrized for $O(2)$ and $O(4)$ groups. For the regular part we consider leading-order (linear) dependence in H and quadratic in T:

$$f_{M,reg}(T,H) = \left(a_0 + a_1\frac{T-T_c^0}{T_c^0} + a_2\left(\frac{T-T_c^0}{T_c^0}\right)^2\right)H. \tag{2.5}$$

Then we are left with 6 parameters to be determined from fitting the data, T_c^0, t_0, h_0, a_0, a_1 and a_2.

We perform simultaneous fits to M_b and $\chi_{m,l}$ for the asqtad action on $N_\tau = 8$, 12 and the HISQ/tree action on $N_\tau = 6$, 8 and 12. An example of such a fit for HISQ/tree, $N_\tau = 8$ is shown in Fig. 1.

Then, performing a combined $1/N_\tau^2$ extrapolation of T_c values obtained with the asqtad and HISQ/tree action as shown in Fig. 2 we obtain

$$T_c = (154 \pm 8 \pm 1)\ \text{MeV}, \tag{2.6}$$

where the first error is from the fit and the second is the overall error on the lattice scale determination. The fits for asqtad and HISQ/tree are constrained to have a common intercept. (See Ref. [5] for more details on the fitting procedure and analysis of systematic errors.) To present a combined error we add the two errors giving our final value $T_c = 154 \pm 9$ MeV.

3. Deconfinement aspects of the transition

The deconfinement phenomenon in pure gauge theory is governed by breaking of the $Z(N_c)$ symmetry. The order parameter is the renormalized Polyakov loop, obtained from the bare Polyakov loop as

$$L_{ren}(T) = z(\beta)^{N_\tau} L_{bare}(\beta) = z(\beta)^{N_\tau}\left\langle \frac{1}{N_c}\mathrm{Tr}\prod_{x_0=0}^{N_\tau-1} U_0(x_0,\vec{x})\right\rangle, \tag{3.1}$$

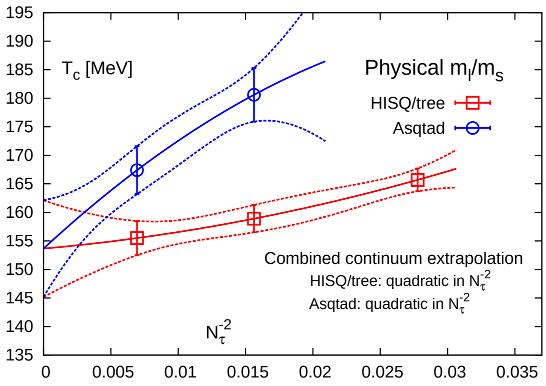

Figure 2: Example of a combined fit to the asqtad and HISQ/tree data. The fit is constrained to have a common intercept at $1/N_\tau^2 \to 0$.

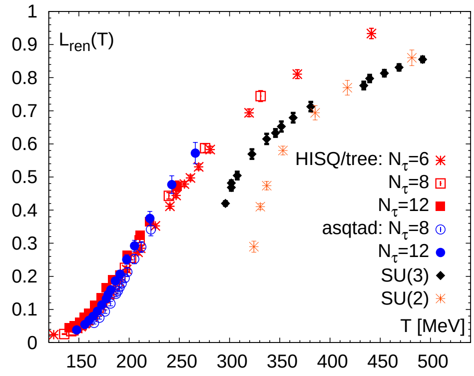

Figure 3: The renormalized Polyakov loop in pure gauge $SU(2)$, $SU(3)$ and full QCD with asqtad and HISQ/tree.

where $z(\beta) = \exp(-c(\beta)/2)$. $c(\beta)$ is the additive normalization of the static potential chosen such that it coincides with the string potential at distance $r = 1.5r_0$ with r_0 being the Sommer scale.

In QCD $Z(N_c)$ symmetry is explicitly broken by dynamical quarks, therefore there is no obvious reason for the Polyakov loop to be sensitive to the singular behavior close to the chiral limit. Indeed, the temperature dependence of the Polyakov loop in pure gauge theory and in QCD is quite different, as one can see from Fig. 3. Also note, that in this purely gluonic observable there is very little sensitivity (through the sea quark loops) to the cut-off effects coming from the fermionic sector. While losing the status of the order parameter in QCD, the Polyakov loop is still a good probe of screening of static color charges in quark-gluon plasma [10, 11].

Other probes of deconfinement are fluctuations and correlations of various charges that can signal liberation of degrees of freedom with quantum numbers of quarks and gluons in the high-temperature phase. Here we consider quadratic fluctuations and correlations of conserved charges:

$$\frac{\chi_i(T)}{T^2} = \frac{1}{T^3V}\frac{\partial^2 \ln Z(T,\mu_i)}{\partial(\mu_i/T)^2}\bigg|_{\mu_i=0}, \quad \frac{\chi_{11}^{ij}(T)}{T^2} = \frac{1}{T^3V}\frac{\partial^2 \ln Z(T,\mu_i,\mu_j)}{\partial(\mu_i/T)\partial(\mu_j/T)}\bigg|_{\mu_i=\mu_j=0}. \tag{3.2}$$

Fluctuations are also sensitive to the singular part of the free energy density. The light and strange quark number susceptibilities are often considered in connection with deconfinement. These quantities show a rapid rise in the transition region and for the strange quark number susceptibility the behavior is consistent with the Hadron Resonance Gas (HRG) model, as shown in Fig. 4. For the light quark number susceptibility, a quantity dominated by pions and therefore very sensitive to the taste symmetry, cut-off effects are still significant, and comparing to HRG is more subtle and requires further study, Fig. 5. In the left panels of Fig. 4 and 5 temperature is set by Sommer scale (r_1 in this case).

In part cut-off effects in the observables directly related to hadrons can be accounted for if the temperature scale is set with a hadronic observable. Following the insight by the Budapest-Wuppertal collaboration [12, 13] we use the kaon decay constant f_K for this purpose. For the strange and light quark number susceptibility the results with f_K scale are shown in the right panels of Fig. 4 and 5.

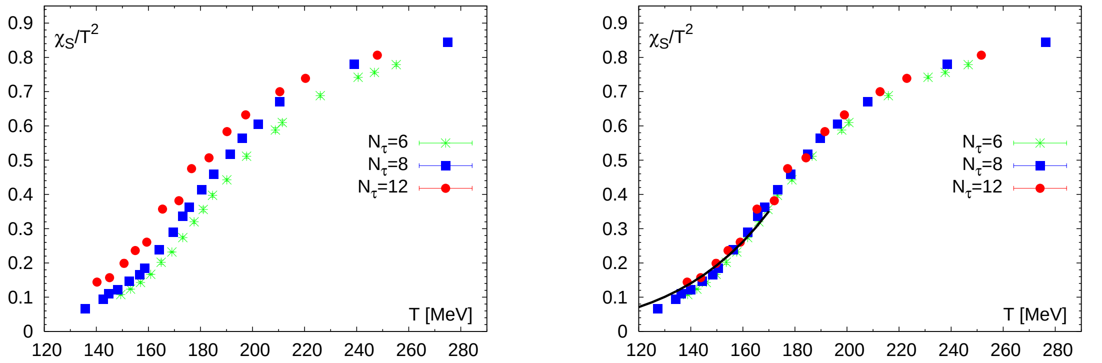

Figure 4: The strange quark number susceptibility for the HISQ/tree action at $m_l = m_s/20$ with r_1 (left) and f_K (right) temperature scale. The solid curve in the right panel is from the HRG model.

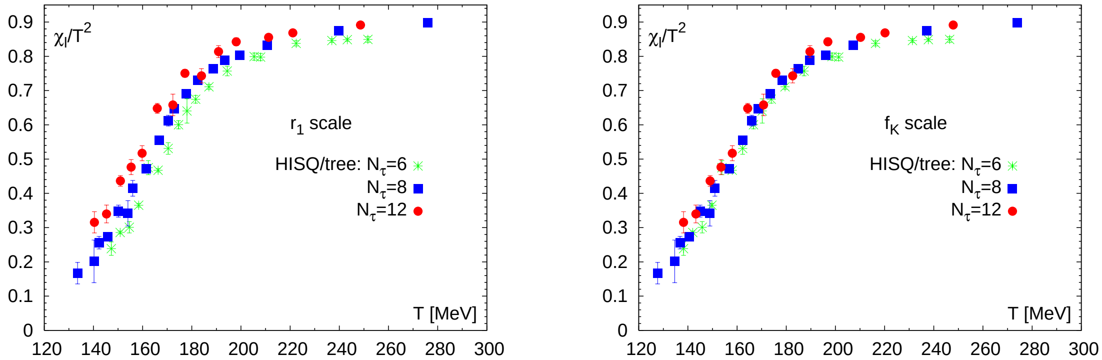

Figure 5: The light quark number susceptibility for the HISQ/tree action at $m_l = m_s/20$ with r_1 (left) and f_K (right) temperature scale.

As one can see, for the strange quark number susceptibility the f_K scale eliminates virtually all cut-off dependence and the data from different lattices are hardly distinguishable. For the light quark number susceptibility some residual cut-off dependence remains. This may be expected for, at least, two reasons: a) χ_l is very sensitive to taste symmetry breaking (even at our finest, $N_\tau = 12$ lattice the root-mean-squared pion mass is still about 200 MeV for the HISQ/tree action [2]), b) there is no a priori reason for the lattice spacing dependence in χ_l, χ_s to be the same as in f_K.

Next, we consider correlations of the electric charge and strangeness. With the r_1 scale we observe substantial cut-off dependence, which is largely reduced with f_K scale, compare left and right panel in Fig. 6. At high temperatures these correlations are close to the value expected in the non-interacting ideal gas (Stefan-Boltzmann limit), while at low temperatures they are well described by HRG.

Correlations of different quark numbers are also a convenient way to study the deconfinement aspects of the transition. At high temperatures such correlations are expected to be very small, while at low temperatures hadronic degrees of freedom give naturally rise to such correlations. In Fig. 7 we present our results for the light-strange and light-light quark number correlations. We divide them, correspondingly, by the strange and light quark number susceptibility in attempt to

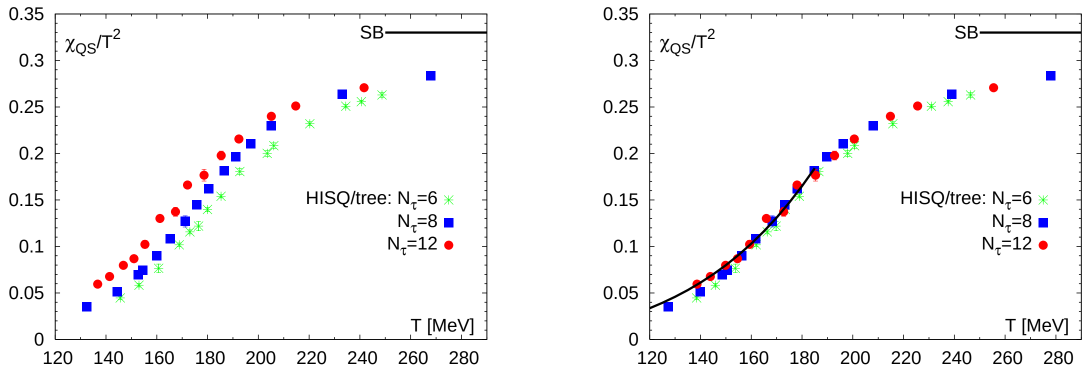

Figure 6: Electric charge–strangeness correlations with r_1 (left) and f_K (right) temperature scale for HISQ/tree. The solid curve in the right panel is from the HRG model.

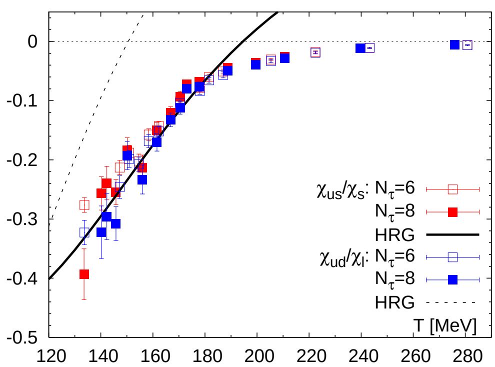

Figure 7: $u-$ and $s-$quark and $u-$ and $d-$quark correlations rescaled by the corresponding fluctuations with r_1 scale for HISQ/tree.

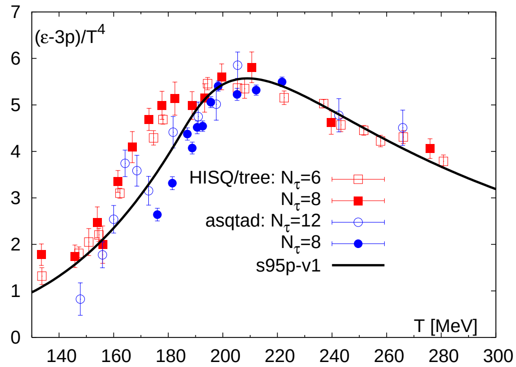

Figure 8: The trace anomaly for the asqtad and HISQ/tree action, $m_l = m_s/20$. "s95p-v1" parametrization is explained in the text.

get rid of the trivial mass effects. For the strange-light quark number correlations we see a good agreement with the HRG model, while the discrepancies with HRG predictions for χ_{ud}/χ_l are likely due to cut-off effects.

Comparing Figs. 4–7 we can conclude that quantities, where pions do not contribute, can be reproduced quite well on the lattice, while those, for which pions give dominant contribution, still demonstrate noticeable cut-off effects, that are not completely removed by switching to a hadronic (f_K) scale (at least, on the lattices typically in use, $N_\tau = 6-12$).

4. Update on the trace anomaly

The deconfinement transition is also often defined as the rapid rise in the energy density associated with liberation of new degrees of freedom. The energy density and other thermodynamic quantities are usually obtained by integrating the trace anomaly $(\varepsilon - 3p)/T^4$. This quantity is under extensive investigation on the lattice. We present an update of the results reported in [1] in Fig. 8. The solid line is a parametrization of $(\varepsilon - 3p)/T^4$ derived in Ref. [14] that combines HRG result at low T ($T < 160$ MeV) with the lattice data [15] at high T ($T > 220$ MeV). From the figure we conclude that the lattice results overshoot the HRG prediction for $T > 160$MeV since any cut-off effects could only decrease $(\varepsilon - 3p)/T^4$ at low T.

5. Conclusions

In this contribution we studied the chiral transition in 2+1 flavor QCD close to the physical point. Using the $O(N)$ scaling we determined the chiral transition temperature in the continuum limit at the physical light quark mass to be $154(9)$ MeV. We also studied the deconfinement phenomenon in terms of the renormalized Polyakov loop as well as in terms of fluctuations and correlations of conserved charges. We concluded that it is difficult to study the deconfinement aspects of the transition in terms of the Polyakov loop, while correlations and fluctuations of conserved charges are much better suited for this purpose. If f_K is used to set the scale we see a good agreement with the HRG model predictions for fluctuations and correlations that do not involve pions in their HRG expansion. For the quantities like χ_l and χ_{ud}, where the pion contribution is significant, cut-off effects are still too large for a meaningful comparison with the HRG model.

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