PROCEEDINGS OF SCIENCE

Renormalization constants for Iwasaki action

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By numerically integrating the differential equations of Stochastic Perturbation Theory, Numerical Stochastic Perturbation Theory can perform high order perturbative calculations in lattice gauge theory. We report on the computation of renormalization constants for Iwasaki gauge action and Wilson fermions. We generated configurations at different lattice volumes V=12^4, 16^4, 20^4, 24^4, and 32^4. To remove the effect of finite time step in the integration of stochastic differential equations, for each volume we generate configuration at different time step τ=0.010, 0.02, and 0.030. Renormalization constants are defined in the RI'-MOM scheme. We extrapolate them to the continuum limit and also correct for finite volume effects. Here we present one loop results, checking that they are consistant with analytical results.

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1. Introduction

This is the era of high precision Lattice calculations, aiming at taking all the errors under control. The computation of renormalization constants are one important source of errors. The debate on how to keep them under control often appears to boil down to the issue of Perturbative vs Non-Perturbative computations. We think the real issue is to correctly enumerate all the sources of systematic errors, which for sure include the following:

1. In the perturbative case, one must tackle truncation errors.
2. Since one often defines renormalization constants in massless schemes, in the Non-perturbative case there are often chiral extrapolations in place.
3. In both the Perturbative and Non-Perturbative case, one aims at extrapolating results to the continuum limit.
4. As a separate issue, finite size effects are often in place as well: this holds for example in the RI'-MOM scheme, which is defined in infinite volume and necessarely simulated on finite volumes.

We compute renormalization constants using our method: Numerical Stochastic Perturbative Theory (NSPT). This enables us to reach three loop results. Here our goal is to compute renormalization constants for Iwasaki gauge action and Wilson fermions. A final goal is to bridge between Perturbative and Non-Perturbative results.

2. Methods

We briefly enumerate a few points about our method, enlighting the control on errors.

2.1 Numerical Stochastic Perturbation Theory

In order to get high order perturbative expansions, we use Numerical Stochastic Perturbation Theory. For a comprehensive introduction, the reader is referred to the literature[1]. Here we simply sketch the very basics facts.

In the Stochastic Quantization framework

$$\frac{\partial}{\partial t}\phi_\eta(x,t) = -\frac{\delta S[\phi]}{\delta \phi_\eta(x,t)} + \eta(x,t).$$

$$\lim_{t\to\infty}\langle \phi(x_1,t)\ldots\phi(x_n,t)\rangle_\eta = \langle \phi(x_1)\ldots\phi(x_n)\rangle.$$

we expand the solution to Langevin equation

$$\phi_\eta(x,t) = \phi_\eta^{(0)}(x,t) + \sum_{n>0} g^n \phi_\eta^{(n)}(x,t)$$

and compute observables order by order

$$O\left[\sum_n g^n \phi_\eta^{(n)}(x,t)\right] = \sum_n g^n O^{(n)}(x,t).$$

The method (which can be taylored to lattice gauge theories) is thus a numerical one, altohough perturbative. It has already proved to be very effective in enabling high loops computations, thus circumventing to a large extent truncation errors.

2.2 RI'-MOM scheme

We compute quark bilinears bracketted in fixed momentum states in Landau gauge and amputate them to get Γ functions

$$\int dx \langle p|\, \overline{\psi}(x)\Gamma\psi(x)\, |p\rangle = G_\Gamma(p) \qquad G_\Gamma(p) \;\rightarrow\; \Gamma_\Gamma(p).$$

We then project on tree-level structures

$$O_\Gamma(p) = Tr\left(\hat{P}_{O_\Gamma}\Gamma_\Gamma(p)\right).$$

We define the field renormalization

$$Z_q(\mu,g) = -i\frac{1}{12}\frac{Tr(\not{p}S^{-1}(p))}{p^2}$$

and finally define renormalization constants

$$Z_{O_\Gamma}(\mu,g)Z_q^{-1}(\mu,g)O_\Gamma(p)|_{p^2=\mu^2} = 1.$$

The resulting renormalization scheme is RI'-MOM, for which anomalous dimensions are know to three loops[2]. This is a very important advantage for us: by taking the relevant anomalous dimensions from continuum computations, we do not need to fit logarithms in our procedure (see later, subsection 2.5). This also means that three loops is the highest we can go: not having at our disposal anomalous dimensions at higher loops, computing them would require an irrealistic numerical precision.

2.3 No chiral extrapolation: we stay at zero mass

We do not need any chiral extrapolation. In the (Wilson) quark self-energy there is an addittive counter term (critical mass)

$$\begin{aligned} a\Gamma_2(\hat{p},\hat{m}_{cr},\beta^{-1}) &= aS(\hat{p},\hat{m}_{cr},\beta^{-1})^{-1} \\ &= i\hat{\not{p}} + \hat{m}_W(\hat{p}) - \hat{\Sigma}(\hat{p},\hat{m}_{cr},\beta^{-1}) \end{aligned}$$

$$\hat{\Sigma}(\hat{p},\hat{m}_{cr},\beta^{-1}) = \hat{\Sigma}_c(\hat{p},\hat{m}_{cr},\beta^{-1}) + \hat{\Sigma}_V(\hat{p},\hat{m}_{cr},\beta^{-1}) + \hat{\Sigma}_o(\hat{p},\hat{m}_{cr},\beta^{-1})$$

$$\hat{\Sigma}(0,\hat{m}_{cr},\beta^{-1}) = \hat{\Sigma}_c(0,\hat{m}_{cr},\beta^{-1}) = \hat{m}_{cr}$$

which we plug in order by order (in our notation $\hat{p} = pa$), since it is known from analytical computations to two loops[3]. At three loops we get a novel result from our computations.

2.4 Hyper cubic Taylor expansion and Continuum limit

Continuum limit comes from fitting expansions[4], e.g. for the quark self-energy ($\hat{p} = pa$)

$$\hat{\Sigma}(\hat{p}, \hat{m}_{cr}, \beta^{-1}) = \hat{\Sigma}_c(\hat{p}, \hat{m}_{cr}, \beta^{-1}) + \hat{\Sigma}_V(\hat{p}, \hat{m}_{cr}, \beta^{-1}) + \hat{\Sigma}_o(\hat{p}, \hat{m}_{cr}, \beta^{-1})$$

Let's H4-Taylor expand it

$$\hat{\Sigma}_V = i \sum_\mu \gamma_\mu \hat{p}_\mu \left(\hat{\Sigma}_V^{(0)} + \hat{p}_\mu^2 \hat{\Sigma}_V^{(1)} + \hat{p}_\mu^4 \hat{\Sigma}_V^{(2)} + \dots \right)$$

$\Sigma^{(n)}$ are also H4-Taylor expanded

$$\hat{\Sigma}_V^{(n)} = \alpha_1^{(n)} 1 + \alpha_2^{(n)} \sum_\nu \hat{p}_\nu^2 + \alpha_3^{(n)} \sum_\nu \hat{p}_\nu^4 + \alpha_4^{(n)} \sum_{\nu \neq \rho} \hat{p}_\nu^2 \hat{p}_\rho^2 + \mathscr{O}(a^6)$$

The only term surviving the $a \to 0$ limit is $\alpha_1^{(0)}$. Notice that this expansion is free of logarithmic contributions, since one loop anomalous dimension for the quark field vanishes in Landau gauge. In a general one loop case a logarithm would be present, whose coefficient is known from continuum computation and thus does not need to be fitted.

2.5 Evaluating finite size effects: get to $L \to \infty$

Taming finite size effects is a key issue: RI'-MOM is actually defined in infinite volume. On dimensional grounds we expect (take once again $\Sigma^{(n)}$) pL effects

$$\begin{aligned} \hat{\Sigma}_V^{(n)}(\hat{p}, pL) &= \hat{\Sigma}_V^{(n)}(\hat{p}, \infty) + \left(\hat{\Sigma}_V^{(n)}(\hat{p}, pL) - \hat{\Sigma}_V^{(n)}(\hat{p}, \infty) \right) \\ &= \hat{\Sigma}_V^{(n)}(\hat{p}, \infty) + \Delta \hat{\Sigma}_V^{(n)}(\hat{p}, pL) \end{aligned}$$

so that a better expansion to fit is

$$\begin{aligned} \hat{\Sigma}_V^{(n)}(\hat{p}, pL) = \; & \alpha_1^{(n)} 1 + \alpha_2^{(n)} \sum_\nu \hat{p}_\nu^2 + \alpha_3^{(n)} \sum_\nu \hat{p}_\nu^4 + \\ & + \alpha_4^{(n)} \left(\sum_\nu \hat{p}_\nu^2 \right)^2 + \Delta \hat{\Sigma}_V^{(n)}(\hat{p}, pL) + \dots \end{aligned}$$

In first approximation

$$\Delta \hat{\Sigma}_V^{(n)}(\hat{p}, pL) \sim \Delta \hat{\Sigma}_V^{(n)}(pL)$$

But $p_\mu L = \frac{2\pi n_\mu}{L} L = 2\pi n_\mu$, i.e. we expect the same correction on different lattice sizes for the same $\{n_1, n_2, n_3, n_4\}$. This enable us to fit only a few extra parameters[5].

3. Results

We are running our simulations at both $n_f = 0$ and $n_f = 4$ (the latter being relevant for phenomelogy; it could be cross-checked with Non-Perturbative results from the ETM Collaboration).

We generate configurations at different lattice volumes V=12^4, 16^4, 20^4, 24^4, and 32^4. Since we adhere to the simplest (Euler) scheme in the integration of our (stochastic) differential equations, in order to remove the effect of finite time step we generate configurations at different time step τ=0.010, 0.02, and 0.030 (for each volume).

We are still in the process of generating configurations and measuring. The preliminary results which we presented at the Lattice 2011 Conference and that we report on here come from only a few measurements as the table below shows:

	$\tau = 0.010$	$\tau = 0.020$	$\tau = 0.030$
V=12^4	80	80	80
V=16^4	57	57	57
V=20^4	30	30	29
V=24^4	20	20	20
V=32^4	3	2	2

To verify the correctness of our codes we first measured one loop results, to be checked against the known analytical results[6].

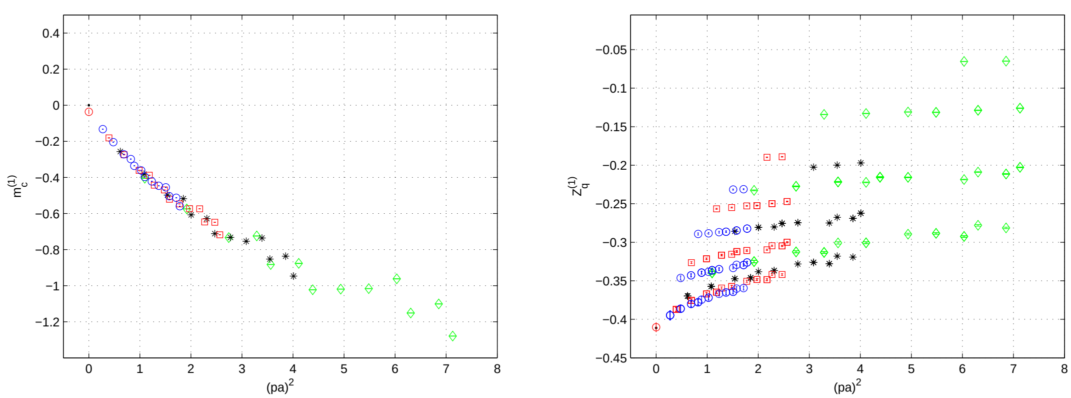

Figure 1: First loop critical mass and Z_q. The lattice volumes are $V = 12^4$ (green diamonds), 16^4 (black stars), 20^4 (red squares), and 24^4 (blue circles). Analytical result is black star, while the red circle is our result, as obtained by fitting data to the function.

4. Conclusions and Future

This is work in progress. We are still in the process of enlarging our statistics, focusing most on the $n_f = 4$ case. We also point out that there is one result missing to perform three loops computations. In order to benefit from the continuum results, we need to compute the two loops matching of the lattice Iwasaki coupling to the continuum coupling (a similar computation has been performed for the Symanzick action[7]).

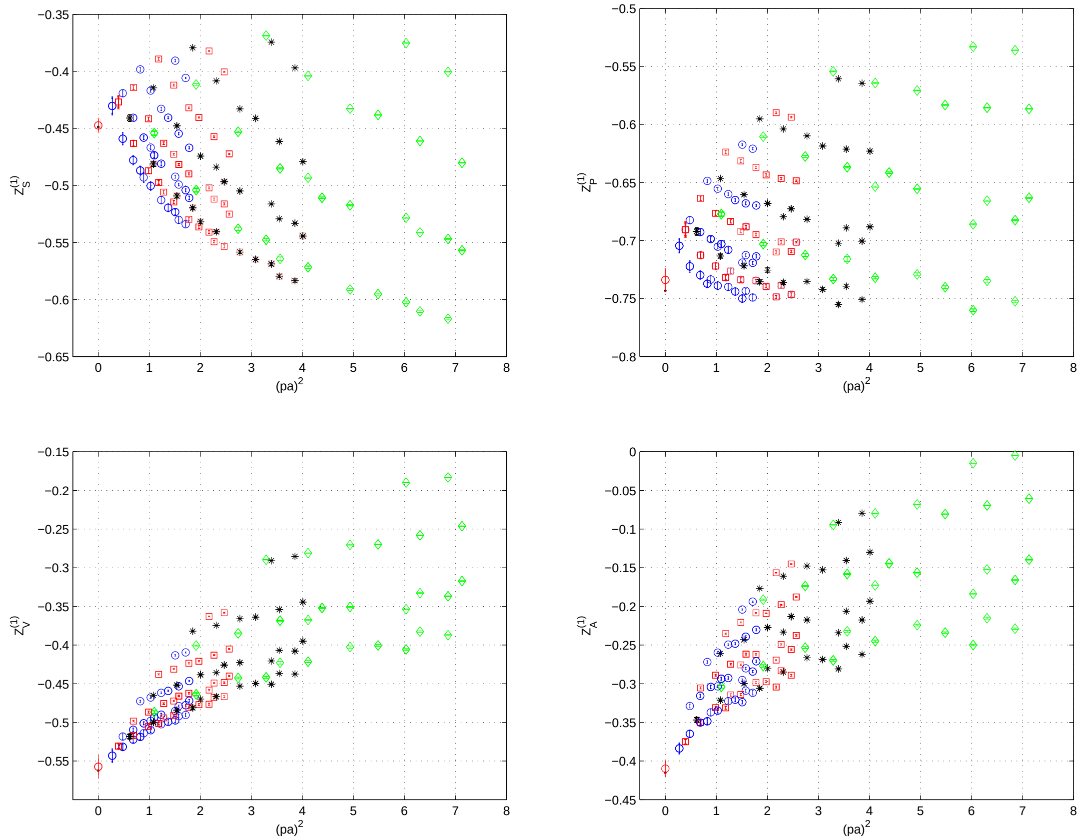

Figure 2: First loop results for Z_s, Z_p and Z_q, in good agreement with analytical results (same notations as Figure 1).

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References

[1] F. Di Renzo and L. Scorzato, *Numerical Stochastic Perturbation Theory for full QCD*, JHEP 0410 (2004) 073 [arXiv:hep-lat/0410010].

[2] J. A. Gracey, *Three loop anomalous dimension of nonsinglet quark currents in the RI-prime scheme*, Nucl. Phys. B **662**, 247 (2003) [arXiv:hep-ph/0304113].

[3] A. Skouroupathis, M. Constantinou, H. Panagopoulos, *Two-loop additive mass renormalization with clover fermions and Symanzik improved gluons*, Phys. Rev. **D77** (2008) 014513 [arXiv:0801.3146 [hep-lat]].

[4] F. Di Renzo, V. Miccio, L. Scorzato and C. Torrero, *High-loop perturbative renormalization constants for Lattice QCD. I. Finite constants for Wilson quark currents.*, Eur. Phys. J. **C51** (2007) 645-657 [arXiv:hep-lat/0611013].

[5] F. Di Renzo, E. -M. Ilgenfritz, H. Perlt, A. Schiller and C. Torrero, *Two-point functions of quenched lattice QCD in Numerical Stochastic Perturbation Theory. (I) The ghost propagator in Landau gauge*, Nucl. Phys. **B831** (2010) 262-284 [arXiv:0912.4152 [hep-lat]].
F. Di Renzo, E. -M. Ilgenfritz, H. Perlt, A. Schiller and C. Torrero, *Two-point functions of quenched lattice QCD in Numerical Stochastic Perturbation Theory. (II) The Gluon propagator in Landau gauge*, Nucl. Phys. **B842**, 122-139 (2011) [arXiv:1008.2617 [hep-lat]].

[6] S. Aoki, K. i. Nagai, Y. Taniguchi and A. Ukawa, *Perturbative renormalization factors of bilinear quark operators for improved gluon and quark actions in lattice QCD*, Phys. Rev. D **58**, 074505 (1998) [arXiv:hep-lat/9802034].

[7] M. Brambilla and F. Di Renzo, *Matching the lattice coupling to the continuum for the tree level Symanzik improved gauge action*, Pos (Lattice2010) 222.