PROCEEDINGS OF SCIENCE

Quark masses from $N_\mathrm{f} = 2$ Clover fermions – an update

QCDSF Collaboration: G. Bali,[a] S. Collins,[a] B. Glässle,[a] M. Göckeler,[a] R. Horsley,[b] Y. Nakamura,[c] J. Najjar,[a] A. Nobile,[a] D. Pleiter*,[d,a] P.E.L. Rakow,[e] A. Schäfer,[a] G. Schierholz,[f] A. Sternbeck,[a] H. Stüben,[g] F. Winter[b] and J.M. Zanotti[b]

[a] *Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany*
[b] *School of Physics, University of Edinburgh, Edinburgh EH9 3JZ, UK*
[c] *RIKEN Advanced Institute for Computational Science, Kobe, Hyogo 650-0047, Japan*
[d] *JSC, Jülich Research Centre, 52425 Jülich, Germany*
[e] *Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK*
[f] *Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany*
[g] *Konrad-Zuse-Zentrum für Informationstechnik Berlin, 14195 Berlin, Germany*
Email: `d.pleiter@fz-juelich.de`

We use $N_\mathrm{f} = 2$ non-perturbatively improved clover fermions and the standard Wilson gluonic action to compute the masses of the light quarks. After results for much smaller quark masses have become available we provide here an update on earlier published results [1]. The renormalization constants have been determined non-perturbatively. Partially quenched chiral perturbation theory is used to extra-/interpolate to the physical quark masses.

The XXIX International Symposium on Lattice Field Theory - Lattice 2011
July 10-16, 2011
Squaw Valley, Lake Tahoe, California

*Speaker.

PoS(Lattice 2011)235

1. Introduction

Quarks are not asymptotic states of QCD and thus cannot be observed directly. Lattice calculations in principle allow us to calculate their masses from first principles. A precise determination of these fundamental parameters of QCD is however limited by the control on systematic errors, like discretization effects. To convert the results obtained on the lattice into continuum numbers it is necessary to convert the bare quark masses to renormalized masses in some standard renormalization scheme:

$$m_q^{\mathscr{S}}(\mu) = Z^{\mathscr{S}}(\mu)\, m_q^{\mathrm{bare}}, \tag{1.1}$$

where $\mathscr{S}$ and μ denote the renormalization scheme and scale, respectively.

The simulations have been done using $N_\mathrm{f} = 2$ flavours of degenerate, non-perturbatively $O(a)$ improved Wilson fermions plus the standard Wilson gluonic action. We have generated a large number of ensembles with different lattice spacings in the range $0.060\,\mathrm{fm} \lesssim a \lesssim 0.075\,\mathrm{fm}$. Our smallest quark mass corresponds to a pseudo-scalar meson mass $m_\mathrm{PS} \simeq 180\,\mathrm{MeV}$. For our smallest quark masses we use lattices of size $L \simeq 3\,\mathrm{fm}$ while for heavier quark masses our lattices are smaller. To scale our lattice results we employ the Sommer parameter r_0 extrapolated to the chiral limit [2].Since there is no precise experimental value for r_0 available we use the nucleon mass to determine the conversion factor needed to convert the lattice results to physical units. Within errors this is consistent with a value $r_0 = 0.5\,\mathrm{fm}$ [3], which we will use throughout this paper.

We compute the bare quark mass using the axial Ward identity (AWI). On the lattice the identity can be written as

$$\partial_\mu \mathscr{A}_\mu = 2\tilde{m}_q \mathscr{P} + \mathscr{O}(a^2), \tag{1.2}$$

where $\mathscr{A}$ and $\mathscr{P}$ are the (unrenormalized) axial current and pseudo-scalar density. The (bare) quark mass $\tilde{m}_q$ can be obtained by computing the ratio

$$a\tilde{m}_q = \frac{\langle \partial_4 \mathscr{A}_4(t) \mathscr{O}(0)\rangle}{2\langle \mathscr{P}(t)\mathscr{O}(0)\rangle}. \tag{1.3}$$

To eliminate $O(a)$ discretization errors the operators need to be improved, i.e. we have to use the improved axial current $\mathscr{A}_\mu = (1 + b_\mathrm{A}\, am_\mathrm{q}^{(\mathrm{S})})(A_\mu + c_\mathrm{A} a\partial_\mu P)$ and pseudo-scalar density $\mathscr{P} = (1 + b_\mathrm{P}\, am_\mathrm{q}^{(\mathrm{S})})P$. Here $m_\mathrm{q}^{(\mathrm{S})} = \frac{1}{2}\left(\frac{1}{\kappa^{(\mathrm{S})}} - \frac{1}{\kappa_c^{(\mathrm{S})}}\right)$ refers to the vector Ward identity (VWI) quark mass. Since we will consider partially quenched results where valence and sea quark masses differ we introduced the superscript (S) for the sea quark masses. The improvement coefficient c_A is known non-perturbatively [4], while the coefficients b_A and b_P are only known in one-loop perturbation theory [5]. Since to leading order both of them are equal we can assume $1 + (b_\mathrm{A} - b_\mathrm{P})am_\mathrm{q}^{(\mathrm{S})} \simeq 1$, i.e. we can ignore them in Eq. (1.3). We have some freedom to select the operator $\mathscr{O}$. We use the sink smeared pseudo-scalar density P^{smeared}. The quantities we compute on the lattice are

$$a\tilde{m}_q^{(0)}(t) \;=\; \frac{\langle \partial_4 A_4(t) P^{\mathrm{smeared}}(0)\rangle}{2\langle P(t) P^{\mathrm{smeared}}(0)\rangle}, \tag{1.4}$$

$$a\tilde{m}_q^{(1)}(t) \;=\; a\frac{\langle \partial_4^2 P(t) P^{\mathrm{smeared}}(0)\rangle}{2\langle P(t) P^{\mathrm{smeared}}(0)\rangle}. \tag{1.5}$$

For timeslices sufficiently far from the source, i.e. $0 \ll t \ll T$, we expect these ratios to be constant. The improved AWI quark mass is given by $a\tilde{m}_q = a\tilde{m}_q^{(0)} + c_\mathrm{A}\, a\tilde{m}_q^{(1)}$.

To renormalize the quark masses it is convenient to first compute the renormalization group invariant quark masses. The renormalization constant $Z_m^{\mathrm{RGI}}(a)$ has been computed in [6] using the Rome-Southampton method. In a second step the renormalization invariant quark mass is translated into a different scheme. Conventionally light quark masses are defined in the $\overline{\mathrm{MS}}$ scheme at a scale $\mu = 2\,\mathrm{GeV}$. We use the 4- and 3-loop results for the β- and γ-function using $r_0\Lambda^{\overline{\mathrm{MS}}} = 0.73$ [7]. At the relevant scale we observe a good convergence of the perturbative series.

2. Computational strategy

Since in our simulations we have $N_\mathrm{f} = 2$ flavours of dynamical quark masses we have to partially quench the strange quark. To extrapolate/interpolate the lattice results towards the point where the quark masses take their physical values we fit our data to expressions obtained from partially quenched chiral perturbation theory (χPT). For $N_\mathrm{f} \geq 1$ sea quarks in LO and NLO the quark mass dependence of the pseudo-scalar meson mass[1] can be written as [8]

$$\left(\frac{m_{\mathrm{PS}}^{(A,B)}}{4\pi f_0}\right)^2 = \chi^{(A,B)} \Bigg[1 + \chi^{(S)} N_\mathrm{f}(2\alpha_6 - \alpha_4) + \chi^{(A,B)}(2\alpha_8 - \alpha_5) + \frac{1}{N_\mathrm{f}} \frac{\chi^{(A)}(\chi^{(S)} - \chi^{(A)})\ln\chi^{(A)} - \chi^{(B)}(\chi^{(S)} - \chi^{(B)})\ln\chi^{(B)}}{\chi^{(B)} - \chi^{(A)}}\Bigg] \tag{2.1}$$

where $\chi^{(A,B)} = B_0^{\mathscr{S}}(m^{(A)} + m^{(B)})^{\mathscr{S}}/\Lambda_\chi^2$ $(A,B \in V_1, V_2, S)$ is related to the sea quark mass $m^{(S)}$ or the (possibly non-degenerate) valence quark masses $m^{(V_1)}$ and $m^{(V_2)}$. $B_0^{\mathscr{S}}$ is related to the chiral condensate via $B_0^{\mathscr{S}} = -\langle\overline{q}q\rangle/f_0^2$. The low-energy constants (LECs) α_i are evaluated at the scale $\Lambda_\chi = 4\pi f_0$, where f_0 is the pion decay constant in the chiral limit. (We use the convention where $f_\pi = 92.4\,\mathrm{MeV}$.) We now perform a generic rescaling of the variables $\chi^{(A,B)} = c_\chi y^{(A,B)}$ and $m_{\mathrm{PS}}^{(A,B)}/\Lambda_\chi = c_m M_{\mathrm{PS}}^{(A,B)}$. Eq. (2.1) can then be written in the following form:

$$\frac{y^{(A,B)}}{(M_{\mathrm{PS}}^{(A,B)})^2} = c_a + \left(\frac{c_b - c_d(1+\ln c_a)}{c_a}\right) y^{(S)} + \left(\frac{c_c + c_d(1+2\ln c_a)}{c_a}\right) y^{(A,B)} - \left(\frac{c_d}{c_a}\right) \frac{y^{(A)}(y^{(S)} - y^{(A)})\ln y^{(A)} - y^{(B)}(y^{(S)} - y^{(B)})\ln y^{(B)}}{y^{(B)} - y^{(A)}}, \tag{2.2}$$

which in the case of degenerate valence quark masses reduces to

$$\frac{y^{(V)}}{(M_{\mathrm{PS}}^{(V)})^2} = c_a + c_b (M_{\mathrm{PS}}^{(S)})^2 + c_c (M_{\mathrm{PS}}^{(V)})^2 + c_d \left((M_{\mathrm{PS}}^{(S)})^2 - 2(M_{\mathrm{PS}}^{(V)})^2\right) \ln(M_{\mathrm{PS}}^{(V)})^2 . \tag{2.3}$$

Note that the coefficients c_i are scheme and scale dependent. We make this explicit after setting

$$y^{(V)} = r_0 \tilde{m}_q^{\mathrm{RGI}}, \quad M_{\mathrm{PS}}^{(S)} = r_0 m_{\mathrm{PS}}^{(S)}, \quad M_{\mathrm{PS}}^{(V)} = r_0 m_{\mathrm{PS}}^{(V)} \tag{2.4}$$

[1]We do not consider higher order results (see, e.g., [9]) here as our data is not sufficiently precise to fix the large number of low-energy constants.

in our final fit formula:

$$\frac{r_0\tilde{m}_q^{\rm RGI}}{(r_0 m_{\rm PS}^{(V)})^2} = c_a^{\rm RGI} + c_b^{\rm RGI}(r_0 m_{\rm PS}^{(S)})^2 + c_c^{\rm RGI}(r_0 m_{\rm PS}^{(V)})^2 + c_d^{\rm RGI}\left((r_0 m_{\rm PS}^{(S)})^2 - 2(r_0 m_{\rm PS}^{(V)})^2\right)\ln(r_0 m_{\rm PS}(V))^2 . \tag{2.5}$$

Once the coefficients have been determined one can obtain the strange quark mass from

$$\begin{aligned} r_0\tilde{m}_s^{\rm RGI} =\; & c_a^{\rm RGI}\left[(r_0 m_{K^+})^2 + (r_0 m_{K^0})^2 - (r_0 m_{\pi^+})^2\right] \\ & + (c_b^{\rm RGI} - c_d^{\rm RGI})\left[(r_0 m_{K^+})^2 + (r_0 m_{K^0})^2\right](r_0 m_{\pi^+})^2 \\ & + \frac{1}{2}(c_c^{\rm RGI} + c_d^{\rm RGI})\left[(r_0 m_{K^+})^2 + (r_0 m_{K^0})^2\right]^2 - (c_b^{\rm RGI} + c_c^{\rm RGI})(r_0 m_{\pi^+})^4 \\ & - c_d^{\rm RGI}\left[(r_0 m_{K^+})^2 + (r_0 m_{K^0})^2\right]\left[(r_0 m_{K^+})^2 + (r_0 m_{K^0})^2 - (r_0 m_{\pi^+})^2\right] \\ & \qquad \times \ln\left((r_0 m_{K^+})^2 + (r_0 m_{K^0})^2 - (r_0 m_{\pi^+})^2\right) \\ & + c_d^{\rm RGI}(r_0 m_{\pi^+})^4 \ln(r_0 m_{\pi^+})^2 . \end{aligned} \tag{2.6}$$

Similarly, for the light quark masses one gets the following expression:

$$r_0\tilde{m}_{ud}^{\rm RGI} = c_a^{\rm RGI}(r_0 m_{\pi^+})^2 + (c_b^{\rm RGI} + c_c^{\rm RGI})(r_0 m_{\pi^+})^4 - c_d^{\rm RGI}(r_0 m_{\pi^+})^4 \ln(r_0 m_{\pi^+})^2 . \tag{2.7}$$

To compute the strange quark mass it is, however, better to use a modified fit function of the form

$$\frac{r_0\tilde{m}_q^{\rm RGI}}{(r_0 m_{\rm PS})^2} = c_{a'}^{\rm RGI} + c_b^{\rm RGI}[(r_0 m_{\rm PS}^{(S)})^2 - d_b] + c_c^{\rm RGI}[(r_0 m_{\rm PS}^{(V)})^2 - d_c] + c_d^{\rm RGI}\left[\left((r_0 m_{\rm PS}^{(S)})^2 - 2(r_0 m_{\rm PS}^{(V)})^2\right)\ln(r_0 m_{\rm PS}(V))^2 - d_d\right] . \tag{2.8}$$

This expression is obtained by eliminating $c_a^{\rm RGI}$ from Eq. (2.5) in terms of

$$c_{a'}^{\rm RGI} \equiv \frac{r_0 m_s^{\rm RGI}}{(r_0 m_{K^+})^2 + (r_0 m_{K^0})^2 - (r_0 m_{\pi^+})^2} , \tag{2.9}$$

i.e. $c_{a'}^{\rm RGI}$ is directly related to the strange quark mass.

The results obtained so far are valid for ‘pure’ QCD. To match these with experimental numbers electromagnetic effects have to be taken into account. This can be done using Dashen’s theorem [10]:

$$\begin{aligned} m_{\pi^+}^2 &= m_{\pi^0}^2 = (m_{\pi^0}^{\rm EXP})^2 \\ m_{K^+}^2 &= (m_{K^+}^{\rm EXP})^2 - (m_{\pi^+}^{\rm EXP})^2 + (m_{\pi^0}^{\rm EXP})^2 \\ m_{K^0}^2 &= (m_{K^0}^{\rm EXP})^2 \end{aligned} \tag{2.10}$$

3. Results

We fitted the results for the ratio $r_0\tilde{m}_q^{\rm RGI}/(r_0 m_{\rm PS}^{(V)})^2$ to Eq. (2.5) restricting our fit range to $r_0 m_{\rm PS} \leq 1.8$. This range has been chosen such that it includes a fictitious pseudo-scalar meson with

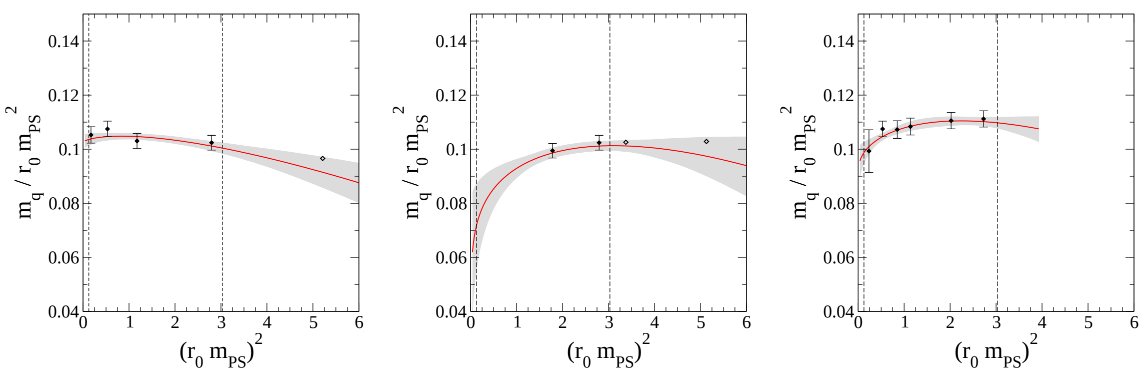

Figure 1: Fits to Eq. (2.5) at $\beta = 5.29$. The left plot shows the results for $\kappa^{(\mathrm{V})} = \kappa^{(\mathrm{S})}$ while the other plots show partially quenched results at $\kappa^{(\mathrm{S})} = 0.13590$ (middle) and 0.13632 (right). Open symbols have not been included in the fit. The vertical dashed lines indicate the mass of a pion and of a fictitious pseudo-scalar meson with 2 strange quarks, respectively.

2 strange quarks, which would have a mass $r_0 m_{\mathrm{PS}} = 1.74$. In Fig. 1 we compare some of the data to the fit at $\beta = 5.29$. Similar results have been obtained for $\beta = 5.25$ and 5.40. We observe for heavier quark masses that the data changes almost linearly as a function of the squared pseudo-scalar meson mass. Only for the smaller sea quark mass do we find a bending down indicating effects from the chiral logarithms.

In Fig. 2 we show the results for c_a^{RGI} and m_s^{RGI} obtained by fits to Eq. (2.5) and (2.8), respectively, as a function of the squared lattice spacing. Since we find the data to be consistent within statistical errors and do not observe a systematic dependence on the lattice spacing we fitted the results to a constant.

The coefficients c_i $(i = a, b, c, d)$ are directly related to the following combinations of LECs:

$$2\alpha_6 - \alpha_4 = \frac{1}{N_{\mathrm{f}}^2}\left[1 + \ln N_{\mathrm{f}} - \frac{c_b}{c_d} + \ln\frac{c_d}{c_a}\right], \tag{3.1}$$
$$2\alpha_8 - \alpha_5 = -\frac{1}{N_{\mathrm{f}}}\left[1 + 2\ln N_{\mathrm{f}} + \frac{c_c}{c_d} + \ln\frac{c_d}{c_a}\right].$$

In the following table we compare our results with those from [11]:[2]

	Bijnens	This work
$2\alpha_6 - \alpha_4$	0.0(6)	0.2(4)
$2\alpha_8 - \alpha_5$	0.29(48)	-1(2)

Results from other lattice QCD calculations reviewed in [12] also favour a small positive value for $2\alpha_6 - \alpha_4$. Less clear is the situation for $2\alpha_8 - \alpha_5$ where both positive and negative results have been found.

The coefficient c_a is related to the chiral condensate $\langle \bar{q}q \rangle$:

$$\frac{1}{2r_0 c_a^{\mathrm{RGI}}} = -\frac{1}{f_0^2}\langle \bar{q}q \rangle^{\mathrm{RGI}}. \tag{3.2}$$

[2]The results have been averaged by the Flavianet Lattice Averaging Group [12].

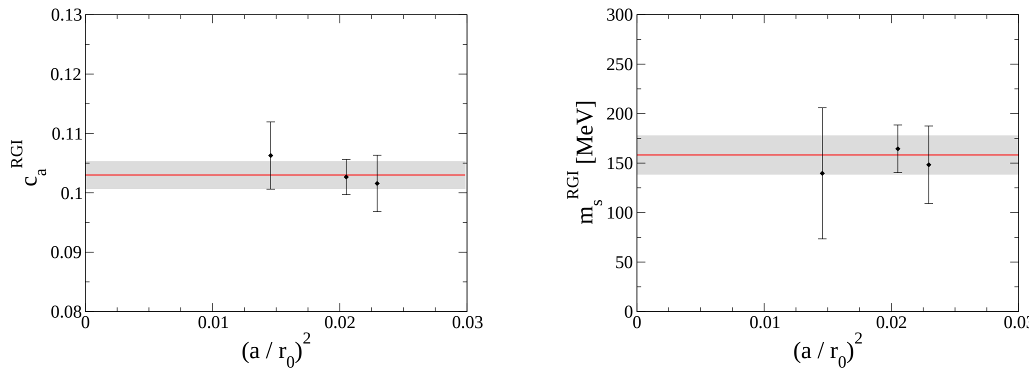

Figure 2: Continuum extrapolation of $c_a^{\rm RGI}$ (left) and $m_s^{\rm RGI}$ (right).

Using $f_0 = f_\pi$ we find $\langle \bar{q}q \rangle^{\overline{\rm MS}}(2{\rm GeV}) = -(284(2){\rm MeV})^3$. The error is purely statistical. This result is slightly large compared to results from other groups (see [12] for an overview).

Using the leading order in Eq. (2.7) we directly obtain the light quark mass from the continuum result for $c_a^{\rm RGI}$: $m_{\rm u/d}^{\overline{\rm MS}}(\mu = 2{\rm GeV}) = 3.5(1)\,{\rm MeV}$. The contributions from the higher orders are of relative size $\left|(r_0 m_{\pi^+})^2 (c_b^{\rm RGI} + c_c^{\rm RGI})/c_a^{\rm RGI}\right| \approx 0.0005$ and $\left|(r_0 m_{\pi^+})^2 \ln(r_0 m_{\pi^+})^2 c_d^{\rm RGI}/c_a^{\rm RGI}\right| \approx 0.007$ and therefore negligible. The situation is different for the strange quark mass. Using the leading order in Eq. (2.6) would lead to a very small strange quark mass $m_{\rm s}^{\overline{\rm MS}}(\mu = 2{\rm GeV}) = 88(2)\,{\rm MeV}$. Taking NLO into account we find an about 30% larger result: $m_{\rm s}^{\overline{\rm MS}}(\mu = 2{\rm GeV}) = 115(14)\,{\rm MeV}$.

There are further sources of systematic errors to be considered:

- If we allow the upper limit of our fit range to vary from $r_0 m_{\rm PS} = 1.8$ (which we use for our central values) to 2.5 we observe a change of our coefficients $c_i \lesssim 8\%$ resulting in a reduction of the strange quark mass.

- We estimate a 2% uncertainty for our method to set the scale. This systematic error affects both the determination of the renormalization group invariant quark masses (by about 2%) as well as the renormalization factor for converting the results to $\overline{\rm MS}$ at a scale $\mu = 2\,{\rm GeV}$. The latter is a 0.5% effect.

- Finally, we have to consider that electro-magnetic effects have not been included in our simulations. Attempts to calculate these effects on the lattice indicate that these are small (see, e.g., [13]). The use of Dashen's theorem, i.e. Eq. (2.10), has a negligible effect on the strange quark mass. In case of the up/down quark masses it is, however, expected to dominate the overall error budget.

4. Summary and conclusions

We have provided an update on ongoing work to determine the light quark masses on gauge configurations with $N_{\rm f} = 2$ flavours of Clover fermions. With results at very small quark masses becoming available we observe significantly clearer signatures of the effects from chiral logarithms. Since we have results for different lattice spacings we are in the position to check for discretization effects, which turn out to be negligible compared to statistical errors. The latter are still relatively large, but we are in the process of increasing our statistics (in particular for $\kappa^{(S)} \neq \kappa^{(V)}$). Our preliminary results are

$$m_{u/d}^{\overline{MS}}(\mu = 2\text{GeV}) = 3.5(1)(1)(1)\,\text{MeV}, \quad (4.1)$$

$$m_{s}^{\overline{MS}}(\mu = 2\text{GeV}) = 115(14)(9)(1)\,\text{MeV}. \quad (4.2)$$

The first error is the statistical error while the other errors take into account the fit range dependence and the uncertainties related to setting the scale.

Acknowledgements

The numerical calculations have been performed on the APEmille and apeNEXT systems at NIC/DESY (Zeuthen), the BlueGene/P at NIC/JSC (Jülich), the BlueGene/L at EPCC (Edinburgh), the Dell PC-cluster at DESY (Zeuthen), the QPACE systems of the SFB TR-55, the SGI Altix and ICE systems at LRZ (Munich) and HLRN (Berlin/Hannover). This work was supported in part by the DFG (SFB TR-55) and by the European Union (grants 238353, ITN STRONGnet and 227431, HadronPhysics2, and 256594). SC acknowledges support from the Claussen-Simon-Foundation (Stifter- verband für die Deutsche Wissenschaft). AS is supported by the EU International Reintegration Grant (IRG) 256594. JZ was supported by an Advanced Science & Technology Facilities Council Fellowship under Grant number ST/F009658/1.

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