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Non-perturbative renormalization for general improved staggered bilinears

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We present results for non-perturbative renormalization (NPR) factors for staggered fermion bilinears of arbitrary spin and taste. We use "covariant" bilinears which transform irreducibly under the lattice translation and rotation group, and thus do not mix. We form ~ 30 ratios which have no anomalous dimensions, and compare the NPR results to those from 1-loop perturbation theory. We also compare the absolute renormalization factors (which, in general, do have anomalous dimensions) to 1-loop perturbation theory. We use asqtad and HYP-smeared staggered valence fermions on the coarse MILC asqtad lattices.

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1. Introduction

Precise knowledge of matching factors between lattice operators and their continuum counterparts is necessary for phenomenological applications of lattice QCD. In this work we present the status of our calculations of matching factors using non-perturbative renormalization (NPR) [1] as applied to valence asqtad and HYP-smeared improved staggered fermions. These actions have been used, respectively, in the calculation of m_s from light-hadron quantities [2, 3, 4] and of B_K [5]. In both of these calculations uncertainties in matching factors are the dominant source of error.

Although NPR is widely used for other types of fermions, it has been much less used for staggered fermions. We have chosen to gain experience by first calculating matching factors for fermion bilinears before proceeding to the four-fermion operators needed for B_K. Specifically, we calculate matching factors for all bilinears residing on a 2^4 hypercube, and make a detailed comparison with one-loop perturbation theory (PT).

The bilinears are also of interest in their own right as part of a calculation of m_s. In particular, with asqtad valence and sea quarks one finds $m_s(\overline{\text{MS}}, 2\text{ GeV}) = 76 \pm 8$ MeV using 1-loop matching [2], 87 ± 6 MeV with 2-loop matching [3],[1] while replacing perturbative matching factors with our preliminary NPR result gives $103 \pm 3(\text{stat})$ [6]. One purpose of the present study is to determine the systematic error in the latter result. This is important in order to understand whether there is a significant deviation from the two-loop result. We would also like to check consistency with the more precise determination of m_s using HISQ fermions via the ratio m_s/m_c, which yields $m_s(\overline{\text{MS}}, 2\text{ GeV}) = 92.4 \pm 1.5$ MeV [4].

2. Methodology

We consider flavor non-singlet valence bilinears with arbitrary Dirac matrix γ_S and taste matrix ξ_F. The matching or "Z-factors" appear in the relation

$$\overline{Q}(x)(\gamma_S \otimes \xi_F) Q'(x) \cong Z_{\gamma_S\otimes\xi_F} \sum_{A,B} \overline{\chi}_A(n)\, \overline{(\gamma_S \otimes \xi_F)}_{AB} U_{n+A,n+B}\, \chi'_B(n)\,, \tag{2.1}$$

where the symbol $\cong$ indicates that matrix elements of the continuum operators on the left, evaluated with $\overline{\text{MS}}$ regularization, equal those of the lattice operators on the right, evaluated with lattice regularization. The continuum fields Q have four (implicit) tastes. $\chi_B(n)$ is a staggered field at a lattice position $n+B$ determined by the hypercube label, n, and the offset within the hypercube, B. $U_{n+A,n+B}$ is the gauge matrix connecting $\overline{\chi}$ to χ, and is constructed by averaging over the product of links along all paths of minimal length.

We consider only bilinears with vanishing four-momentum, which fall into 35 irreps under the lattice symmetry group (as enumerated below). While this proliferation of operators is often the bane of staggered fermions, here it is a boon as there are 35 different Z-factors to compare to PT.

One innovation we introduce is to use "covariant" bilinears. Normally one sums over non-overlapping positions of the hypercubes. This has the disadvantage that the summed operators do not transform irreducibly under lattice translations, and results in mixing between a subset of

[1] Here we are quoting older results based on the two lattice spacings $a \approx 0.12$ fm and 0.09 fm, since these are the spacings we use in our calculation.

the operators. Instead, if one sums over all translations (including appropriate signs), one creates bilinears that live in irreps of the full staggered translation and rotation group and thus can be shown not to mix (at any order in PT). A bonus is that it is simpler to implement such operators numerically. More details will be given in Ref. [7].

The different irreps are listed in Table 1. They are organized according to the minimal number of gauge links connecting $\overline{\chi}$ and χ, and by their spin. Matching factors of operators related by multiplication by $(\gamma_5 \otimes \xi_5)$ are the same, leading to the relationships discussed in the caption. In total there are 19 independent Z-factors for the 35 operators.

# links	S	V	T
4	$(\mathbf{1} \otimes \xi_5)$	$(\gamma_\mu \otimes \xi_\mu \xi_5)$	$(\gamma_\mu \gamma_\nu \otimes \xi_\mu \xi_\nu \xi_5)$
3	$(\mathbf{1} \otimes \xi_\mu \xi_5)$	$(\gamma_\mu \otimes \xi_5)$ $(\gamma_\mu \otimes \xi_\nu \xi_\rho)$	$[(\gamma_\mu \gamma_\nu \otimes \xi_\mu \xi_5)$ $(\gamma_\mu \gamma_\nu \otimes \xi_\rho)]$
2	$(\mathbf{1} \otimes \xi_\mu \xi_\nu)$	$(\gamma_\mu \otimes \xi_\nu)$ $(\gamma_\mu \otimes \xi_\nu \xi_5)$	$[(\gamma_\mu \gamma_\nu \otimes \mathbf{1})$ $(\gamma_\mu \gamma_\nu \otimes \xi_5)]$ $(\gamma_\mu \gamma_\nu \otimes \xi_\nu \xi_\rho)$
1	$(\mathbf{1} \otimes \xi_\mu)$	$(\gamma_\mu \otimes \mathbf{1})$ $(\gamma_\mu \otimes \xi_\mu \xi_\nu)$	$[(\gamma_\mu \gamma_\nu \otimes \xi_\nu)$ $(\gamma_\mu \gamma_\nu \otimes \xi_\rho \xi_5)]$
0	$(\mathbf{1} \otimes \mathbf{1})$	$(\gamma_\mu \otimes \xi_\mu)$	$(\gamma_\mu \gamma_\nu \otimes \xi_\mu \xi_\nu)$

Table 1: Covariant bilinears forming irreps of the lattice symmetry group. Indices μ, ν and ρ are summed from $1-4$, except that all are different. Pseudoscalar and axial bilinears are not listed: they can be obtained from scalar and vector, respectively, by multiplication by $\gamma_5 \otimes \xi_5$. Bilinears related in this way have the same matching factors. This operation also implies the identity of the Z-factors for the tensor bilinears within square brackets. Bilinears marked in blue are used as the denominators of the ratios discussed in the text.

The only addition to the standard NPR methodology required for staggered fermions is that one needs 16 lattice momenta for each physical momenta, so that propagators and vertices are all 16×16 matrices. See Ref. [8] for more details. We use momentum sources which allows us to work with a small number of configurations (8-16). We calculate Z-factors in the RI$'$ scheme, and extrapolate linearly to $am = 0$. As is well known, this method fails for psuedoscalar bilinears due to the pion pole. A more sophisticated analysis is needed, and we do not present results for these bilinears here. We use ~ 10 different physical momenta for each choice of quark masses. Our code is an adaptation of Chroma.

All our calculations use the MILC lattices generated with the asqtad staggered fermion action and Symanzik glue. Results presented here are from the "coarse" ensembles with $a \approx 0.12$ fm, size $20^3 \times 64$, light sea-quark masses $am_\ell = 0.01, 0.02, 0.03$ and strange sea-quark mass $am_s = 0.05$. We are presently running on the $a \approx 0.09$ fm "fine" ensembles and will present combined results in Ref. [7]. For valence quarks we use both the asqtad action, with unquenched masses $am_{\rm val} = am_\ell$, and the HYP-smeared action, also with $am_{\rm val} = am_\ell$. In the latter case the *physical* valence and sea quark masses differ, since they are renormalized differently, but this difference vanishes in the chiral limit.

One must also choose the links to be used in the bilinears. We have tried various choices, but focus here on those giving the best agreement with PT. For asqtad valence quarks we use thick "Fat 7+Lepage" links, while for HYP valence quarks we use HYP-smeared links.

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3. Perturbative Predictions

We compare our NPR matching factors with those from PT. The perturbative results take the form:

$$Z_{\mathscr{O}}^{\mathrm{RI}',\mathrm{LAT}}(\mu,a) = e^{-\int_{a(\mu_0)}^{a(\mu)} da\,\gamma_{\mathscr{O}}/\beta(a)}\, Z_{\mathscr{O}}^{\mathrm{RI}',\overline{MS}} \left(1 + a(\mu_0)[-2\gamma_{\mathscr{O}}^{(0)} \log(\mu_0 a) + C_{\mathscr{O}}^{\overline{\mathrm{MS}}} - C_{\mathscr{O}}^{\mathrm{lat}}]\right). \quad (3.1)$$

Here μ is the scale of the NPR renormalization, $a = \alpha/4\pi$, $\gamma_{\mathscr{O}}$ is the RI$'$ anomalous dimension (known to 4 loops for S, P, V and A bilinears, and to 3 loops for tensors), $\beta(a)$ is the corresponding beta-function (known to four loops), and the $C_{\mathscr{O}}$ are the finite parts of the one-loop matrix elements. In words, the above equation says "match (at 1-loop) from lattice to $\overline{\mathrm{MS}}$ at scale μ_0 (which we take to be our highest momentum with $\mu_0 \approx 3$ GeV), then match (at 3-loop order) from $\overline{\mathrm{MS}}$ to RI$'$ in the continuum at scale μ_0, and finally run in the continuum down to the scale μ."

The one-loop lattice-$\overline{\mathrm{MS}}$ matching with asqtad and HYP-smeared bilinears and the Symanzik gluon action has been worked out in Ref. [9] for standard hypercube bilinears. We have extended this to covariant bilinears, finding that (a) the mixing between bilinears vanishes as it should and (b) the diagonal mixing coefficients are the same as for the standard bilinears. We have also done the calculation both without and with mean-field improvement (MFI).

The PT predictions simplify considerably if we take ratios of Z-factors having the same spin but different tastes, for then the continuum matching and running cancels and the result depends only on the lattice part of the matching calculation

$$\frac{Z_{\gamma_S\otimes\xi_{F1}}(p)}{Z_{\gamma_S\otimes\xi_{F2}}(p)} = 1 + a(p)\left[C^{\mathrm{lat}}_{\gamma_S\otimes\xi_{F2}} - C^{\mathrm{lat}}_{\gamma_S\otimes\xi_{F1}}\right] + \mathscr{O}(a^2)\,. \quad (3.2)$$

We take the denominators in our ratios to be the $Z-$factors of the taste singlet operators (those shown in blue in Table 1).

4. Results

In Fig. 1 we compare the ratios for V, A, T and S bilinears composed of HYP-smeared quarks to PT. Results are for momentum "$(2,2,2,7)$" in units of $(2\pi/L_s, 2\pi/L_s, 2\pi/L_s, 2\pi/L_t)$, so that $|p| \approx 2$ GeV. We expect this momentum to be in the window, $\Lambda_{\mathrm{QCD}} \ll |p| \ll 1/a$, where both non-perturbative corrections and lattice artifacts are small. The color coding indicates the number of links in the operators in the numerators of the ratios; the denominators have 1-link operators for V and A, 2-link operators for T, and 0-link operators for S.

The level of agreement with 1-loop PT is quite different for V, A and T bilinears than for scalars. For the former, PT is accurate to $\sim 1\%$, capturing not only the ordering with link number but also the "fine structure" within a given link number. Note that statistical errors are very small, despite the use of relatively few configurations. Mean-field improvement leads to slightly better agreement with PT, although the shifts are small. An example of fine structure is that the two 2-link tensor ratios are predicted to be almost identical and are found to be very close. Non-perturbative effects (which lead to the differences between pairs of matched V and A bilinears) are at the subpercent level. A striking example of this is that the two 1-link T ratios are predicted to be the same and are indistinguishable on the plot. The same holds for the two 3-link T ratios.

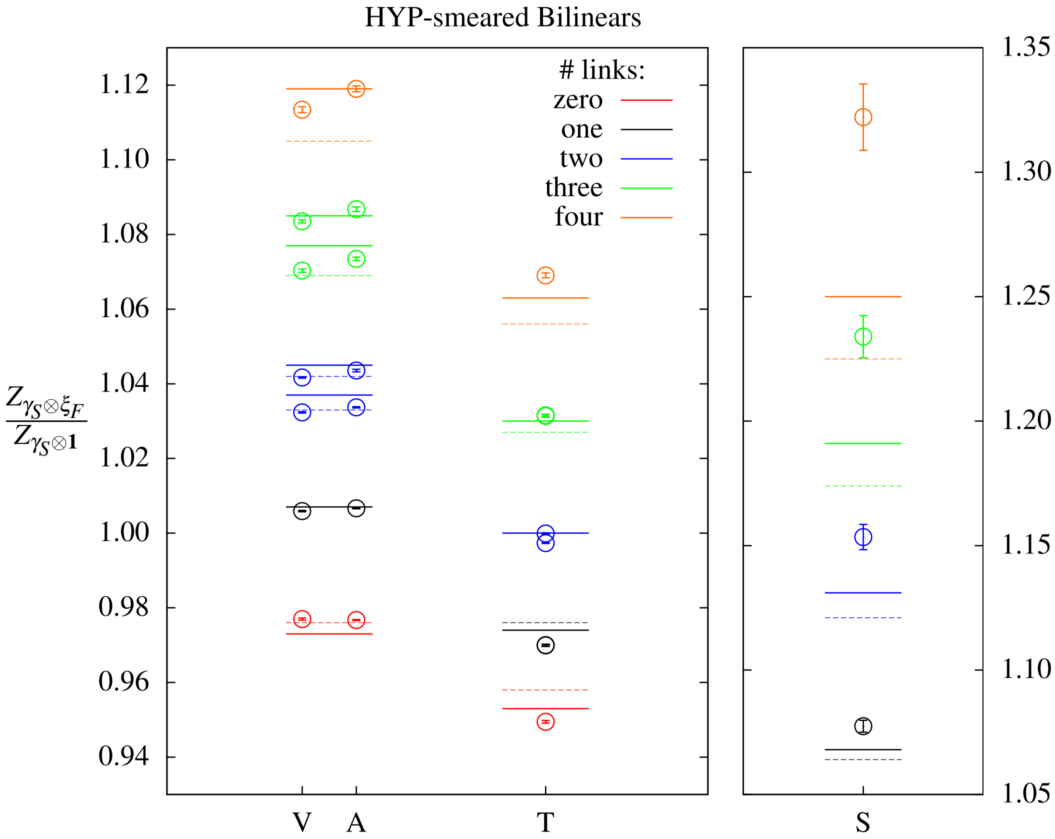

Figure 1: Comparison of Z-factor ratios for vector, axial, tensor and scalar bilinears to PT for HYP fermions. Horizontal lines show PT predictions, with solid/dotted lines showing results with/without MFI. Results are in the chiral limit for the momentum described in the text.

The agreement with PT is less good for the scalar bilinears. While the correct ordering is obtained, the 1-loop predictions differ by as much as $\sim 8\%$. This is, however, the difference that one would naively expect, because the missing two-loop term is of size $\alpha(p)^2 \approx 0.08$. We also note that we are less confident in the linear chiral extrapolations for the S bilinears than for the V, A and T bilinears.

Moving now to the asqtad bilinears, we find that mean-field improvement is necessary to get reasonably accurate predictions. This is not a surprise since, compared to HYP-smeared links, the asqtad fat links have (normalized) traces that are significantly further from unity. We show the results in Fig. 2. For V, A and T bilinears, the ordering with link number is predicted to be opposite to that for HYP bilinears, and this is borne out in the NPR results for tensors, and partially for the V and A bilinears. Overall, MFI PT works at the $\sim 2\%$ level here.

By contrast, PT does very poorly for the asqtad scalar bilinears. This is the reason why our NPR result for m_s quoted above lies significantly above those obtained using PT. We are presently investigating the systematics associated with the chiral and continuum extrapolations in order to firm up our result.

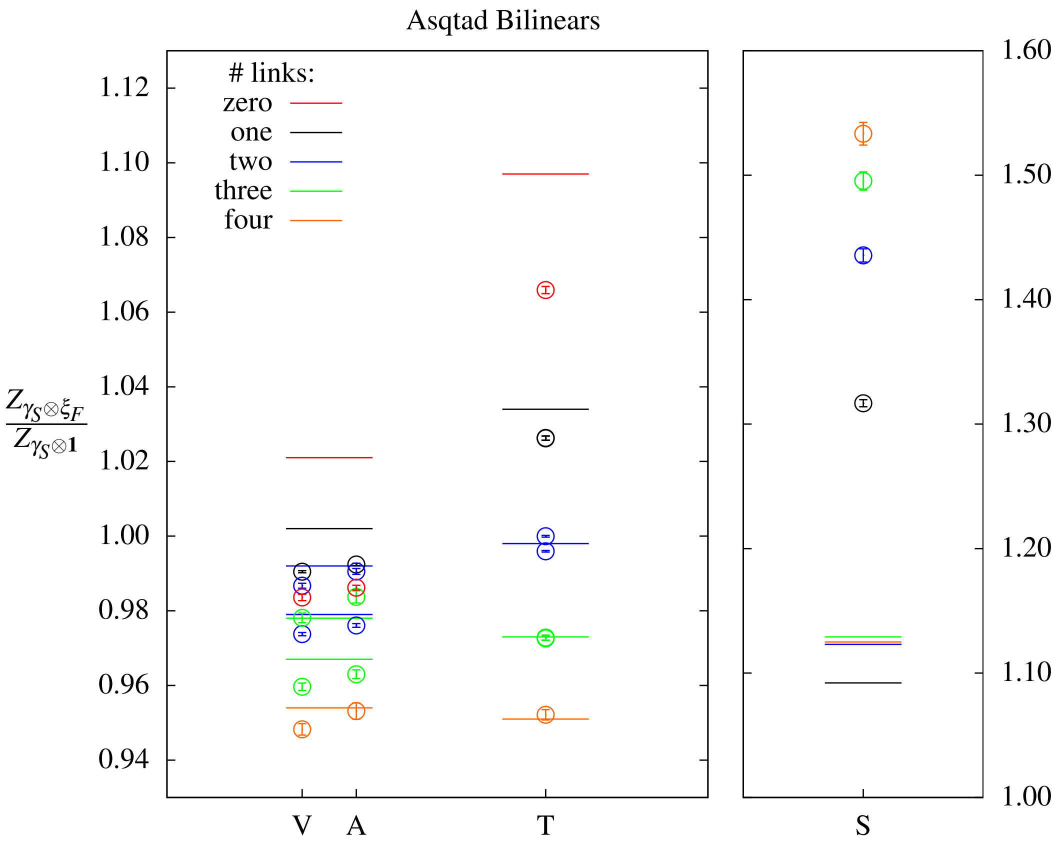

Figure 2: Comparison of the vector, axial, tensor, and scalar ratios for asqtad fermions to MFI PT. Results are in the chiral limit for the momentum described in the text.

Finally we show in Fig. 3 results for the three denominators, i.e. $Z_{\gamma_S\otimes\mathbf{1}}$ for spins S, V and T. These are plotted vs. $(ap)^2$ and compared to the PT result of Eq. (3.1). Note that all three operators have non-vanishing anomalous dimensions in the RI$'$ scheme, although that for the vector is very small. We expect that non-perturbative effects should be small for $p > 2$ GeV, which translates to $(ap)^2 > 1.4$. Indeed, we see that in this regime the NPR results track the PT predictions fairly well, up to overall rescalings of $\sim 5\%$.

5. Conclusions and Outlook

We have calculated all Z-factors of general “covariant" bilinears nonpertubatively and in one-loop PT for both HYP-smeared and asqtad fermions. For HYP-smeared fermions, one-loop PT predictions for the ratios give a good representation of the NPR results, with accuracy varying from $\sim 1\%$ for V, A, and T bilinears to $\sim \alpha^2$ for scalars. For the absolute Z-factors the one-loop predictions are also accurate to $\sim \alpha^2$ or better, and the predicted running with p is qualitatively described. The latter result is encouraging for the calculation of B_K with staggered fermions. This is because the tensor and scalar bilinears have anomalous dimensions which are similar in magnitude to that of the operator needed for B_K. If one-loop PT can represent these bilinears to an

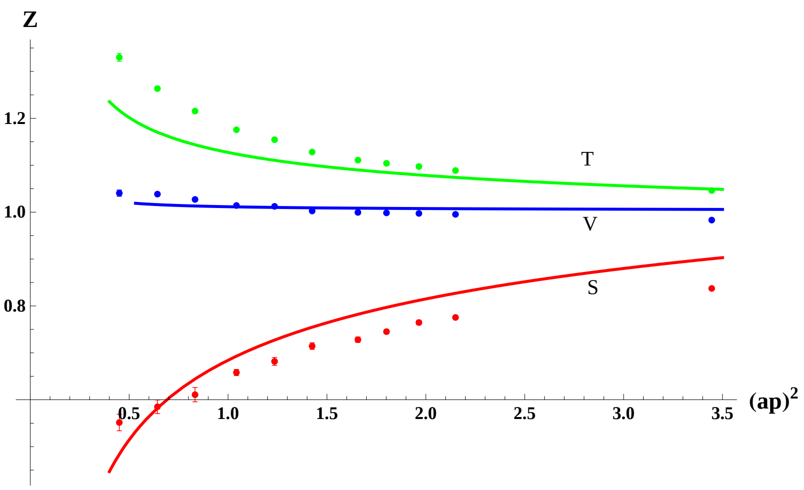

Figure 3: Comparison of the scalar, vector, and tensor HYP denominators for $m = 0$ to perturbation theory.

accuracy of at least $\sim \alpha^2$ then it is reasonable to expect the same to be true for the B_K operators. Thus the error estimate ($\delta B_K \approx \alpha(1/a)^2$) used in the B_K calculation is seen to be reasonable [5].

We find PT to be less accurate for asqtad bilinears, even after mean-field improvement, and to be very poor for the scalar bilinear. The accuracy gets considerably worse if one uses thin links in the operators. This raises a concern that the result for m_s obtained using PT, albeit at 2-loop order, is suspect. We hope to complete our calculation of m_s using NPR on two lattice spacings soon [7].

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