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Confinement in multiparton sectors of SYM_2 with adjoint matter

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A hierarchical scheme to attack a coupled system of Bethe Salpeter equations for QCD bound states in the infinite momentum frame is discussed. Some results for Fock wave functions, in SYM_2 with various parton multiplicities, are presented. The strong localization of these states in the LC configuration space is found. This allowes to confirm directly the string interpretation od two dimensional gauge theories and to determine the string tension directly from the spectrum. A new, sensitive method to look for the screening was also proposed. Its advantages were tested on the well known example of the massless Schwinger Model.

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In parallelling the steady and impressive progress in lattice methods, which are revealing and confirming more and more subtle predictions of QCD as the theory of strong interactions, there is still a considerable interest in understanding the theory by other, nonperturbative techniques. The most prominent one is the Light Cone approach (LC) which over years has provided a lot of insight in the structure of QCD [1],[2]. Especially when coupled with large N and reduction techniques, it offers an interesting path of approaching QCD problems [3].

In this talk I will review some results of a recursive scheme of understanding (and possibly solving), LC based, "QCD equations" for the spectrum [4]. We begin with the two dimensional supersymmetric YM theory in the large N limit obtained by the dimensional reduction of the $\mathcal{N} = 1 \quad SYM_4$. Even though in two dimensions, the model has a rich structure of multi parton Fock sectors and has not been solved up to date.

The spectrum of the model follows from the eigen equation for the mass operator

$$M^2|\Phi\rangle \equiv 2P_-P^+|\Phi\rangle \equiv 2PH_{LC}|\Phi\rangle = \lambda|\Phi\rangle, \quad |\Phi\rangle = \sum_p \int_{\{x\}} f_p(x_1,\ldots,x_p)Tr[A^\dagger{}_{x_1}\ldots A^\dagger{}_{x_p}]|0\rangle,$$

which in terms of the individual Fock components has a generic form

$$M^2\Phi_n(x_1\ldots x_n) = A\otimes\Phi_n + B\otimes\Phi_{n-2} + C\otimes\Phi_{n+2}, \tag{1}$$

with A,B and C being simple (rational in fact) functions of momenta fractions x_r of partons involved in the elementary processes: elastic $(2\to 2)$, creation $(2\to 4)$, fusion $(4\to 2)$, which are induced by the LC Hamiltonian H_{LC}.

Depending on the dimension of the unreduced theory, its matter content and the color group representation, there are many detailed versions of these equations [5]. We quote only one explicit example (QCD_2 with scalar, adjoined matter [6]).

$$\begin{aligned} M^2\phi_n(x_1\ldots x_n) &= \frac{m^2}{x_1}\phi_n(x_1\ldots x_n) + \frac{\lambda}{\pi}\frac{1}{(x_1+x_2)^2}\int_0^{x_1+x_2} dy\phi_n(y,x_1+x_2-y,x_3\ldots x_n) \\ &+ \frac{\lambda}{\pi}\int_0^{x_1+x_2}\frac{dy}{(x_1-y)^2}\left\{\phi_n(x_1,x_2,x_3\ldots x_n)-\phi_n(y,x_1+x_2-y,x_3\ldots x_n)\right\} \\ &+ \frac{\lambda}{\pi}\int_0^{x_1}dy\int_0^{x_1-y}dz\phi_{n+2}(y,z,x_1-y-z,x_2\ldots x_n)\left[\frac{1}{(y+z)^2}-\frac{1}{(x_1-y)^2}\right] \\ &+ \frac{\lambda}{\pi}\phi_{n-2}(x_1+x_2+x_3,x_4\ldots x_n)\left[\frac{1}{(x_1+x_2)^2}-\frac{1}{(x_1-x_3)^2}\right] \\ &\pm\ cyclic\ \ permutations\ \ of\ \ (x_1\ldots x_n). \end{aligned} \tag{2}$$

The common features of these equations are: elastic, Coulomb contributions, first time found by ’t Hooft (in the model with fundamental fermions), hierarchical structure of mixings with Fock sectors of different multiplicity of partons, and logarithmic divergencies in theories with scalar matter. In four dimensions above convolutions involve also integrals over transverse momenta.

Needless to say that full solutions of these equations are not available. Instead one uses the numerical approach namely the Discretized Light Cone Quantization (DLCQ) [7].

In this situation we propose an approximation scheme which is based on the degree of the infrared divergence of various terms. Namely, at zeroth order we consider only the most divergent

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(i.e. linearly) Coulomb terms and attempt to solve such a problem for all parton multiplicities. Subsequently one might imagine including logarithmic ones (i.e. radiation) and eventually finite terms. Retaining only the most divergent terms in the full LC Hamiltonian gives:

$$H_{LC} = \frac{\lambda}{\pi}\sum_A \int_0^\infty dk \mathrm{Tr}\left[\int_0^k \frac{dq}{q^2} A_k^\dagger A_k\right] - \sum_{A,B} \frac{g^2}{2\pi}\int_0^\infty dp_1 dp_2 \quad \left[\int_0^{p_1} \frac{dq}{q^2}\mathrm{Tr}(A_{p_1}^\dagger B_{p_2}^\dagger B_{p_2+q} A_{p_1-q}) + \int_0^{p_2} \frac{dq}{q^2}\mathrm{Tr}(A_{p_1}^\dagger B_{p_2}^\dagger B_{p_2-q} A_{p_1+q})\right],$$

where $\lambda \equiv g^2 N_c$ and A denotes any one of the parton species existing in the model. Here $A = a, b, f, g$, i.e., after the dimensional reduction we are left with two bosons and two fermions.

In addition to considerable reduction of various terms, the above, most IR divergent, Hamiltonian does not mix different parton multiplicities, hence it can be diagonalized separately in every Fock sector.

Second, the eigen equations (2) generate, in the Coulomb approximation, the generalizations of 't Hooft equation for many variables.

We have diagonalized Coulomb Hamiltonian up to four partons, using the DLCQ approach. Matrix elements were calculated with the large N technique [1], [8]. For complete presentation and more details the reader is referred to [4]. Our results reveal a very simple and intuitively appealing structure.

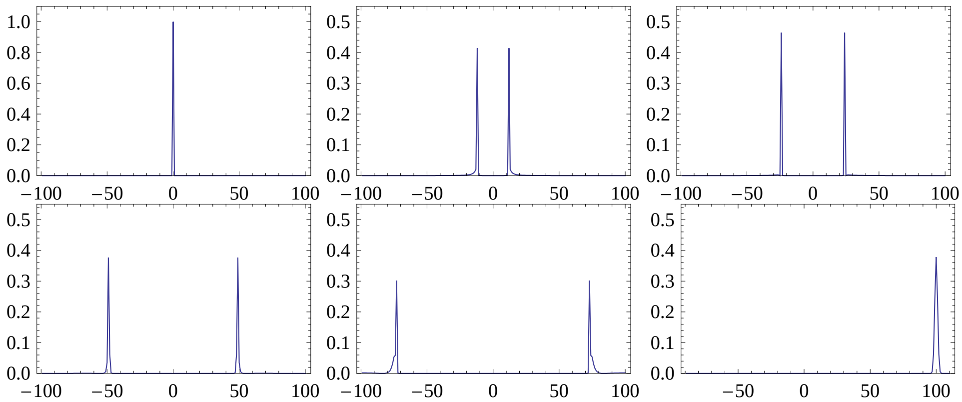

Figure 1: Probability distributions of the relative distance, $d_{12} = |x_2^- - x_1^-|P/2\pi$, in the two parton sector.

In Figure 1 the x^- profiles of a sample of eigenstates with two partons are shown. Evidently, solutions are well localized in the (LC) configuration space and the distance between two partons increases with the eigenenergy of the state.

Similar analysis in the three parton sector results in Fig. 2, which shows the profiles in two independent relative distances, for a series of eigenstates and at fixed value of the DLCQ resolution

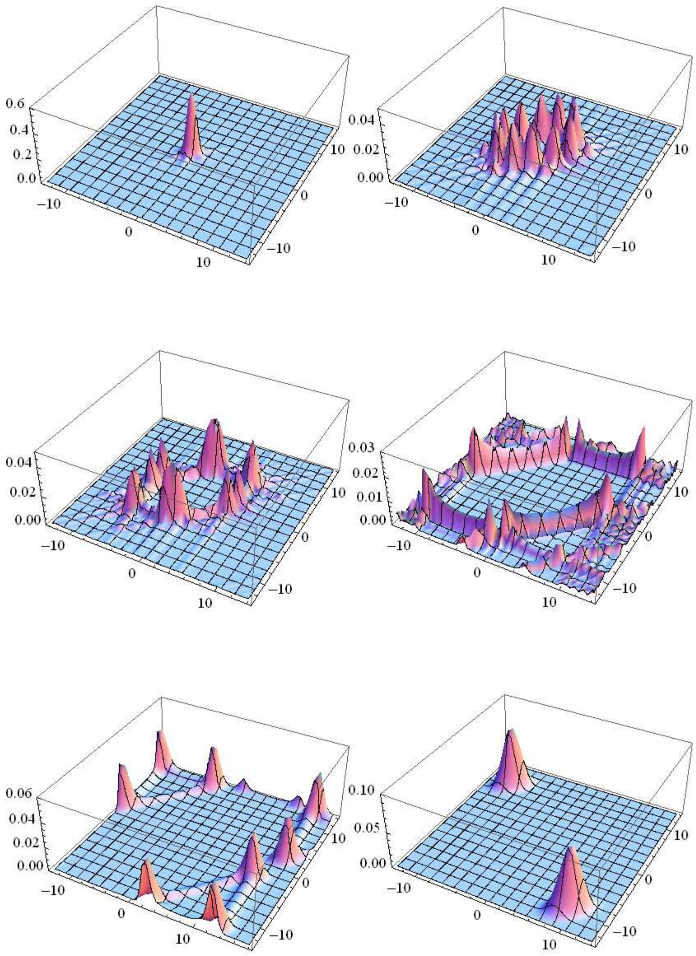

Figure 2: Distributions of (d_{13}, d_{23}) for three partons.

parameter K. Again configuration space wave functions describe three partons located at well defined positions with the inter-parton distances increasing with the eigenenergy of a state. The same picture emerges in the four parton sector.

This approximate localization in the configuration space can be readily understood if one realizes that the Hamiltonian of massless particles does not have the kinetic energy in the infinite momentum frame, hence we basically see the eigenstates of the potential. Since however, the momentum fractions are bound, the localization is not exact.

The large N Coulomb Hamiltonian of n partons has the Z_n symmetry. This symmetry is not manifest in Fig., since the set of the two independent relative coordinates (d_{13}, d_{23}) is not. Indeed when plotted on the massless Dalitz plot, where the constraint $d_{12} + d_{23} + d_{31} = 0$ is automatically satisfied, the profiles show the required Z_3 symmetry (c.f. Fig. 3).

Since the eigenstates found above are well localized, one can ask which is the relevant variable controlling their eigenenergies. It turns out that it is just the length of the effective string joining the partons: $x = |d_{12}| + |d_{21}|$ for two partons, and $x = |d_{12}| + |d_{23}| + |d_{31}|$, in the three parton case.

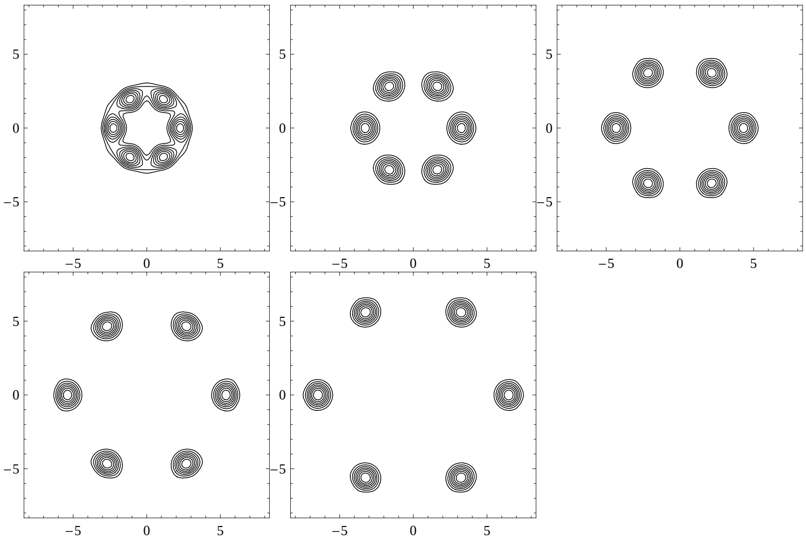

Figure 3: Contour plots of three parton eigenstates displayed on the massless Dalitz plot.

In Fig.4 dependence of the eigenenergies on that variable is shown for few values of K. In both

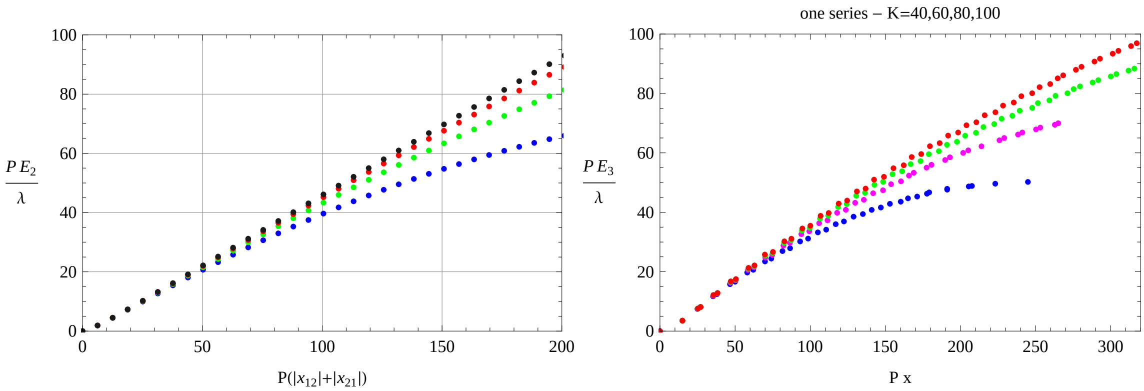

Figure 4: Eigenenergies vs. the combined length of strings, x, for two (left) and three (right) partons.

sectors the dependence approaches a linear form, at large K, and the slope is consistent with the theoretical prediction $\sigma = \lambda/2$ [9]. Quantitative determination of the string tension requires careful extrapolation to $K \to \infty$, especially for higher Fock sectors.

It is perhaps amusing that to recover the string tension, one does not have to measure Wilson loops. The accurate σ value follows directly from the spectrum of bound states. This is again due to the very good localization of massless partons on the light cone in two dimensions. Interestingly, this property persists also in higher parton sectors. It is expected that in the full theory nonabelian charges are screened, as in the massless Schwinger Model [10], [11]. Consequently the linear behavior just discussed gets modified at some energy scale. Technically this change should be driven by the creation and annihilation terms in the full Hamiltonian, which have been neglected in the Coulomb approximation. We plan to study this problem soon.

At the moment I would like to discuss, however, the new and very sensitive tool to demonstrate the screening in the LC formalism. The trick is also helpful in general when dealing with many body problems.

The multidimensional representations, like Fig., become cumbersome for higher multiplicities, see also Fig 7 in Ref.[4] for the $p = 4$ case. Standard way to copy with this problem is to use inclusive densities and correlations. In a given p parton sector, an inclusive (actually the semi-inclusive) single parton density is defined as

$$D_r(\Delta) = \int d^{p-1}\vec{\Delta}_p \sum_{i=1}^{p-1} \delta(\Delta - \Delta_{in}) |f_r(\vec{\Delta}_p)|^2, \tag{3}$$

and gives the number of partons at a distance Δ from, e.g., the last one. It can be easily calculated from our exclusive wave functions or, yet simpler, directly from the Fourier components. The latter representation reads, e.g. in the four parton case,

$$D_r(\Delta) = \int_{p_2,p_3,P-p_2-p_3>0}^{P} dp_2 dp_3 |f_r(\Delta, p_2, p_3)|^2 + cycl. \tag{4}$$

with $f_r(\Delta, p_2, p_3)$ standing for the partial Fourier transform - only in the first variable.

Let us see the power of the inclusive densities in revealing the screening in the Light Cone formulation of massless Schwinger Model. As is well known the solution of the model consists of a set of free bosons whose LC wave functions are constructed from the single composite bosonic creation operator. Its DLCQ version reads [12] $(K_2 = K/2)$

$$a_n^\dagger = \frac{1}{\sqrt{n}} \left[\sum_{r=0}^{\infty} b_{n+r}^\dagger b_r - \sum_{r=1}^{\infty} d_{n+r}^\dagger d_r + \sum_{r=0}^{n-1} b_r^\dagger d_{n-r}^\dagger \right], \tag{5}$$

This allows to construct e.g. the four parton Fock wave function of an eigenstate with four fermions (m - a half of a relative momentum of two bosons - labeles excited states).

$$f_K^{(m)}(k_1,k_2,k_3,k_4) = f_K^{(m)}(k_1,k_2,K_2+m-k_1,K_2-m-k_2) = \frac{1}{\sqrt{K_2^2 - m^2}}, \tag{6}$$

with $1 \le k_1 \le K_2 + m - 1, \quad 1 \le k_2 \le K_2 - m - 1$. From these one readily calculates the inclusive two-parton density introduced above. They are shown in Fig.5 for first few excited states. These figures directly confirm the well know picture: the four fermion state is composed of two tightly bound pairs of fermions. One of them is seen as a sharp peek while two fermions in the other one give rise to the flat distribution due to the free relative motion of bosons. Careful reader may check that the normalizations of both contributions are indeed correct.

Summarizing: the Coulomb approximation provides a lot of physical insight into the structure of the bound state equations. As such it offers a good starting point for more complete solution for the spectrum and hadron structure in general.

All results presented in this talk were obtained together with Gabriele Veneziano and Daniele Dorigoni. I thank them for the successful and continuing collaboration. I also would like to thank Herbert Neuberger for instructive discussions.

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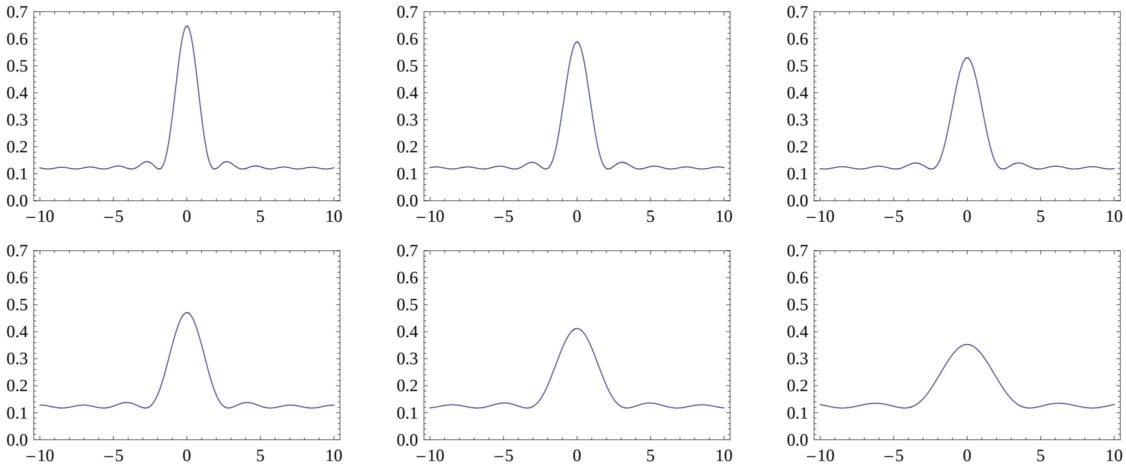

Figure 5: Inclusive densities, defined in the text, for the first few eigenstaes of the massless Schwinger Model

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