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SU(3) centre vortices underpin both confinement and dynamical chiral symmetry breaking

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The mass function of the nonperturbative quark propagator in $SU(3)$ gauge theory shows only a weak dependence on the vortex content of the gauge configurations. Of particular note is the survival of dynamical mass generation on vortex-free configurations having a vanishing string tension. This admits the possibility that mass generation associated with dynamical chiral symmetry breaking persists without confinement. In this presentation, we examine the low-lying ground state hadron spectrum of the π, ρ, N and Δ and discover that while dynamical mass generation persists in the vortex-free theory, it is not connected to dynamical chiral symmetry breaking. In this way, centre vortices in $SU(3)$ gauge theory are intimately linked to both confinement and dynamical chiral symmetry breaking. We conclude that centre vortices are the essential underlying feature of the QCD vacuum.

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Numerical simulations of QCD on a space-time lattice reveal that the essential, fundamentally-important, nonperturbative features of the QCD vacuum fields are:

1. The dynamical generation of mass through chiral symmetry breaking (χSB), and

2. The confinement of quarks.

However, there exists no derivation of quark confinement starting from first principles, nor is there a totally convincing explanation of the effect.

The questions that dominate the field centre around gaining an understanding on how these fundamentally important features of QCD come about. The question is: *What is the essence of QCD vacuum structure?* That is, what is it about the field fluctuations of the QCD vacuum that causes quarks to be confined? What aspects of the QCD vacuum are responsible for dynamical mass generation? Do the underlying mechanisms share a common origin?

The prevailing view is that quark confinement and dynamical χSB is the work of some special class of gauge field configurations which dominate the QCD vacuum on large distance scales. Candidates have included instantons, abelian monopoles, and centre vortices. In recent years, algorithms have been invented which can locate these types of objects in thermalized lattices, generated by the lattice Monte Carlo technique. This is an important development enabling *ab initio* investigations of the underlying mechanism of quark confinement and dynamical chiral symmetry breaking.

Centre vortices are exposed by gauge-fixing. A gauge transformation is applied which brings each lattice link as close as possible to a centre element of the gauge group. This is the set of N $SU(N)$ group elements $\{Z_m\}$, with

$$Z_m = \exp\left(i\frac{2\pi}{N}m\right)\mathbf{I}, \quad (m = 0,1,2,\ldots,N-1)\,. \tag{1}$$

Vortices are identified as the defects in the centre-projected gauge field. Again, the idea of centre dominance is that the centre degree of freedom encodes all the long-distance nonperturbative physics.

In $SU(2)$ gauge theory, a clear link between centre vortices, confinement and mass generation via dynamical chiral symmetry breaking is manifest [1]. Centre vortices are the single underlying mechanism giving rise to both chiral symmetry breaking and quark confinement in $SU(2)$ gauge theory.

Whether this is the case for $SU(3)$ Yang-Mills theory relevant to QCD is not as clear. As outlined in Refs. [2–4] the relation between centre vortices and dynamical chiral symmetry breaking is much more complicated in $SU(3)$ gauge theory. Ref. [4] explores the role of centre vortices identified by gauge fixing Monte Carlo generated configurations to maximal centre gauge [5], clearly illustrating how dynamical mass generation survives the removal of these vortices. This admits the possibility that the underlying mechanisms generating confinement and dynamical chiral symmetry breaking are decoupled.

We proceed to investigate the low-lying hadron mass spectrum in this unique centre-vortex free environment lacking confinement and retaining dynamical mass generation. Our aim is to search for evidence of dynamical chiral symmetry breaking and thus provide further insight into the role of centre vortices in QCD.

Here we focus on vortices identified by gauge fixing the original Monte-Carlo generated configurations directly to Maximal Centre Gauge [6 – 8] without any preconditioning [9], as done in Ref. [4] illustrating the survival of dynamical mass generation on such vortex-free configurations.

First the links $U_\mu(x)$ are gauge transformed to be brought close to the centre elements of $SU(3)$,

$$Z = \exp\left(2\pi i \frac{m}{3}\right) \mathbf{I}, \text{ with m } = -1, 0, 1. \tag{2}$$

On the lattice this is implemented by searching for the gauge transformation Ω which maximizes

$$\sum_{x,\mu} \left| \mathrm{tr}\, U_\mu^\Omega(x) \right|^2 \xrightarrow{\Omega} \max . \tag{3}$$

One can then project the gluon field to a centre-vortex only configuration where each link is a number, one of the roots of unity, times the identity matrix.

$$U_\mu(x) \to Z_\mu(x) \text{ where } Z_\mu(x) = \exp\left(2\pi i \frac{m_\mu(x)}{3}\right) \mathbf{I}, \tag{4}$$

where $m_\mu(x) = -1, 0, 1$. Vortices are removed by removing the centre phase. This is done by making the transformation

$$U_\mu(x) \to U'_\mu(x) = Z^*_\mu(x)\, U_\mu(x) . \tag{5}$$

An examination of the mass function of the nonperturbative quark propagator in $SU(3)$ gauge theory reveals only small differences in dynamical mass generation between the original and vortex-removed configurations. Figure 1 shows a direct comparison of the quark propagator mass function on the original and vortex-free configurations as reported in Ref. [4]. Data has been cylinder cut to facilitate a detailed comparison.

This shape indicates the retention of dynamical mass generation, despite the absence of confinement. Dynamical mass generation has only a weak dependence on the vortex content of the theory. This leads to the key question under investigation. Is the persistence of dynamical mass generation a manifestation of dynamical chiral symmetry breaking in the absence of confinement?

We note that at large momenta, the mass function of the propagator of the vortex-removed configurations experiences a vertical shift upwards of approximately 60 MeV. This may be attributed to a roughening of the configurations at short distances associated with the removal of centre vortices via Eq. 5.

A statistical ensemble of 200 $SU(3)$ gauge-field configurations is generated using the Lüscher-Weisz [10] mean-field improved action on a $20^3 \times 40$ lattice with a lattice spacing of 0.125 fm. We use the FLIC fermion action [11] providing nonperturbative $\mathcal{O}(a)$ improvement [12] with improved chiral properties allowing efficient access to the light quark-mass regime [13].

Initially we consider four different values for the Wilson hopping parameter, κ, selected to provide a wide view of the mass dependence of the spectrum. The associated quark mass can be estimated by linearly extrapolating the squared pion mass to zero as a function $1/\kappa$, to identify the critical hopping parameter $\kappa_{\rm cr}$ where the pion mass vanishes. Then

$$m_q = \frac{1}{2a}\left(\frac{1}{\kappa} - \frac{1}{\kappa_{\rm cr}}\right) . \tag{6}$$

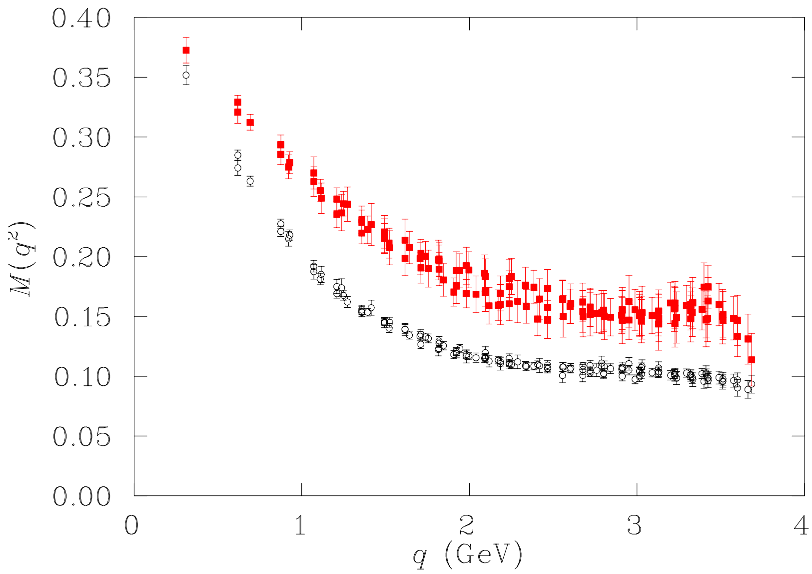

Figure 1: Mass function (in GeV) of the Landau gauge quark propagator from Ref. [4]. Open circles denote the propagator obtained from the original gauge field configurations whereas the (red) filled squares denote the mass function following the removal of center vortices.

An examination of the pion masses of the vortex removed configurations as a function of the inverse hopping parameter in Fig. 2 reveals that it is possible to perform simulations at hopping parameters smaller than the $\kappa_{\rm cr}$ obtained from the original configurations. This is in accord with Fig. 1 from Ref. [4], where the mass function for the vortex removed configurations is shifted higher by about 60 MeV indicating smaller bare quark masses are required to obtain the same renormalised quark mass. We consider two lower quark masses for the vortex-free configurations which are unphysical for the normal configurations.

This necessarily leads to a different $\kappa_{\rm cr}$ for the vortex-free configurations when using Wilson-style fermions. Taking the lightest three masses and assuming $m_\pi^2 \propto 1/\kappa$ in the vortex-free theory provides the linear extrapolation and vortex-free $\kappa_{\rm cr}$ illustrated in Fig. 2. Note that the heavier quark masses in the vortex-free theory show a clear deviation from linear behaviour.

Of course an alternative scenario is also possible. One could argue that dynamical chiral symmetry breaking is spoiled in the vortex-free theory with m_π^2 no longer proportional to $1/\kappa$ or m_q. When a quark of mass zero is placed in the vortex-removed configurations, the pion still has mass. A comparison of the π and ρ meson masses will reveal the correct scenario.

In figure 3 the pion mass from the original and vortex-free configurations are plotted as a function of bare quark mass, m_q, determined with reference to the critical κ value from the original configurations. While it seems the vortex-free mass will approach zero as the quark mass decreases, the relationship between the quark mass and m_π is evidently different between the original and vortex-free configurations.

The Gell-Mann-Oakes-Renner relationship, $m_\pi^2 \propto m_q$ can be seen in the results from the original configurations as the points have the shape of a typical square-root function. While the pion masses obtained in the vortex-free configurations at the two lightest quark masses considered are

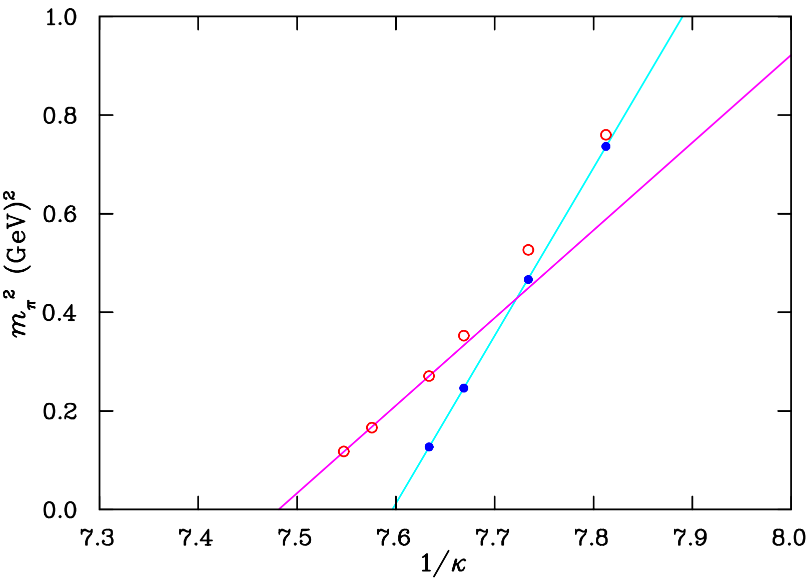

Figure 2: Pion mass squared in terms of the inverse hopping parameter, κ^{-1}. The lines illustrate fits to the original configurations and the vortex-free configurations, the latter addressing only the lightest three quark masses considered where there is some promise that $m_\pi^2 \propto \kappa^{-1}$.

of a similar magnitude to that of the lightest pion mass from the original configurations, there is no evidence of the curvature associated with $m_\pi^2 \propto m_q$.

In the vortex-free configurations the data appears linear with $m_\pi \propto m_q$ over a wide range of m_q, indicating a significant difference between the two types of configurations and a loss of the Goldstone nature of the pion in the vortex-free theory.

This loss of a pseudo-Goldstone boson in the vortex-free theory becomes very clear once one compares the masses of the π and ρ mesons in the vortex-free theory. Figure 4 plots masses for the π and ρ mesons obtained from the original configurations (full symbols) and the vortex-free configurations (open symbols). This figures clearly illustrates how the π and ρ mesons become nearly degenerate on the vortex-free configurations. Thus the vortex-free pion is not associated with dynamical chiral symmetry breaking. It is not a pseudo-Goldstone boson.

The degeneracy of the π and ρ meson is somewhat surprising. For example, in a simple quark model the ρ-meson mass sits much higher than that of the pion due to a large hyperfine interaction between the quark and anti-quark. The degeneracy of the masses on the vortex removed configurations implies that any hyperfine interactions have also been removed with the removal of the centre vortices.

An analysis of the nucleon and Δ masses reveals similar degeneracies of the baryon masses following centre vortex removal The apparent degeneracy of the masses from the vortex-free configurations indicates perhaps that the hadron mass being measured is merely the sum of the dressed constituent-quark-like masses of the quarks composing the hadron. Taking into account the number of constituent quarks composing each hadron, one finds that all hadrons have the same mass per quark. The vortex-free theory is simply a theory of weakly interacting constituent quarks.

We have observed that the hadron masses of the vortex-free theory are simply a reflection of the number of quarks required to compose their quantum numbers. There is little evidence of quark

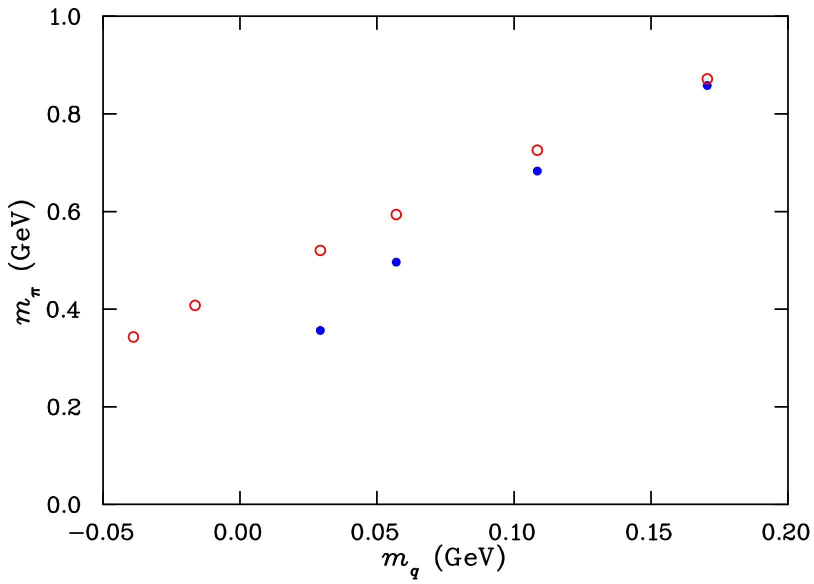

Figure 3: Pion mass in GeV as a function of the bare quark mass. Full symbols illustrate results from the original configurations while the open symbols illustrate results from the vortex-free configurations.

interactions in the mass spectrum and this is in accord with the general features of the Euclidean time evolution of the hadron effective masses in the vortex-free theory where the spectrum suggests a theory of free constituent quarks.

However a comparison of the input quark mass and the hadron masses reveals that dynamical mass generation is at work, in accord with Ref. [4]. The mass generation is reminiscent of the early constituent-quark model where current quarks are thought to be dressed by QCD-vacuum interactions giving rise to a constituent quark mass.

Of greatest importance is the complete absence of any remnant of dynamical chiral symmetry breaking. We find a pion degenerate with the ρ meson and a mass dependence of $m_\pi \propto m_q$ inconsistent with the properties of the pseudo-Goldstone boson of chiral symmetry.

Thus, centre-vortex removal spoils both confinement and chiral symmetry. Centre-vortices are the most fundamental degrees of freedom in QCD, essential to confinement and dynamical chiral symmetry breaking. Just as in SU(2), there is an intimate relationship between centre vortices, confinement and dynamical chiral symmetry breaking. Both confinement and dynamical chiral symmetry breaking are lost under centre vortex removal. Therefore, centre vortices *are* the essence of the QCD vacuum.

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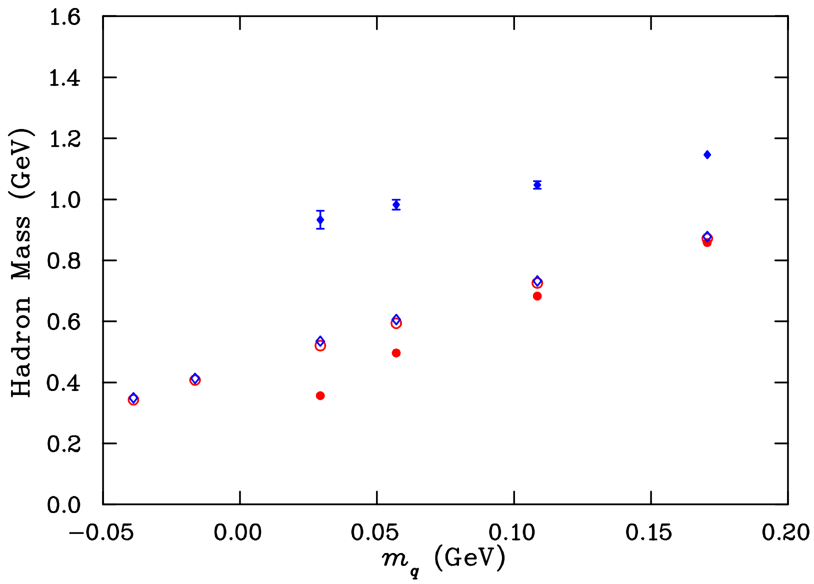

Figure 4: Masses for the π and ρ mesons obtained from the original configurations (full symbols) and the vortex-free configurations (open symbols).

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