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M_b and f_B from non-perturbatively renormalized HQET with $N_\mathrm{f} = 2$ light quarks

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We present an updated analysis of the non-perturbatively renormalized b-quark mass and B meson decay constant based on CLS lattices with two dynamical non-perturbatively improved Wilson quarks. This update incorporates additional light quark masses and lattice spacings in large physical volume to improve chiral extrapolations and to reach the continuum limit. We use Heavy Quark Effective Theory (HQET) including $1/m_\mathrm{b}$ terms with non-perturbative coefficients based on the matching of QCD and HQET developed by the ALPHA collaboration during the past years.

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1. Introduction

In the current area of flavor physics, LHC's 2011 data taking pushes the amount of available experimental data in B physics to a new level and expands the window of precision tests of the Standard Model. Due to confinement the processes considered at the experiments must involve hadronic initial states. The impact on the indirect search of New Physics is therefore limited by the uncertainties on long distance effects. A natural way to estimate these non-perturbative hadronic contributions within a few percent of theoretical error is given by Lattice QCD. To carefully treat B physics on the lattice one has to keep under control simultaneously the finite size effects and, particularly, the discretisation effects (the lattice spacing should be smaller than the Compton wavelength of the b-quark) induced by the simulation. In practice it is not possible to control both effects in one simulation.

The ALPHA Collaboration has followed a strategy discussed in detail in [1]: in HQET the hard degrees of freedom $\sim m_{\rm b}$ are removed and taken into account by an expansion in the inverse b-quark mass $m_{\rm b}^{-1}$. As discussed in those papers and also in earlier work, the benefit is the suppression of large discretisation effects which may arise in hadronic quantities when the theory is regularised on the lattice. The difficult aspect of that method is that a matching with QCD, which is the field theory believed to describe the strong interactions, is needed to fix the parameters of the effective theory. This step also takes care of removing all UV divergencies appearing in the effective theory. In lattice HQET those come as inverse powers of the lattice spacing and thus have to be removed non-perturbatively before the continuum limit can be taken.

The main advantages of this method compared to other existing ones are: i) the theoretical soundness of the approach, in particular the existence of a continuum limit which is in addition numerically reachable, ii) the completely non-perturbative treatment at any order in the $1/m_{\rm b}$ expansion, including the matching between QCD and HQET, iii) the self-consistency of the method, meaning that HQET is tested rather than assumed as opposed to what other approaches have to do for masses around the charm, iv) the numerical cost, which is comparable to that for other setups, as the extra computations needed for the small-volume matching between QCD and HQET are very cheap compared to large-volume simulations.

2. Parameters and observables of Heavy Quark Effective Theory

The HQET Lagrangian including terms of order $1/m_{\rm b}$ reads

$$\mathscr{L}_{\rm HQET}(x)=\mathscr{L}_{\rm stat}(x)-\omega_{\rm kin}\mathscr{O}_{\rm kin}(x)-\omega_{\rm spin}\mathscr{O}_{\rm spin}(x)\,, \tag{2.1}$$

$$\mathscr{L}_{\rm stat}(x)=\overline{\psi}_{\rm h}(x)\,D_0\,\psi_{\rm h}(x)\,,\quad \mathscr{O}_{\rm kin}(x)=\overline{\psi}_{\rm h}(x)\mathbf{D}^2\psi_{\rm h}(x)\,,\quad \mathscr{O}_{\rm spin}(x)=\overline{\psi}_{\rm h}(x)\sigma\!\cdot\!\mathbf{B}\psi_{\rm h}(x)\,, \tag{2.2}$$

with the lowest order (static) Lagrangian $\mathscr{L}_{\rm stat}$ and the first order $(1/m_{\rm b})$–corrections $\mathscr{O}_{\rm kin}$, $\mathscr{O}_{\rm spin}$, giving the kinetic and spin contribution respectively. These are sufficient to compute the b-quark mass to subleading order, but in order to compute the pseudo-scalar decay constant we also introduce the zero momentum projected time component of the heavy-light axial vector current,

$$A_0^{\rm HQET}(x_0)=Z_{\rm A}^{\rm HQET}\,a^3\textstyle\sum_{\mathbf{x}}\,[A_0^{\rm stat}(x)+c_{\rm A}^{(1)}A_0^{(1)}(x)]\,, \tag{2.3}$$

$$A_0^{\rm stat}(x)=\overline{\psi}_{\rm l}(x)\gamma_0\gamma_5\psi_{\rm h}(x)\,,\qquad A_0^{(1)}(x)=\overline{\psi}_{\rm l}(x)\frac{1}{2}\gamma_5\gamma_i(\nabla_i^{\rm S}-\overleftarrow{\nabla}_i^{\rm S})\psi_{\rm h}(x)\,, \tag{2.4}$$

CLS based

β	a (fm)	L/a	Lm_π	m_π (MeV)	no. of cnfg.s	separ. (MD u.)	label
5.2	0.075	32	4.7	386	800	8	A4
		32	4.0	331	200	4	A5
5.3	0.065	32	4.7	438	1000	16	E5
		48	4.8	312	500	8	F6
		48	4.2	267	600	8	F7
5.5	0.048	48	5.2	442	400	8	N5
		64	4.2	268	700	4	O7

Table 1: Presently used large volume ensembles of the Coordinated Lattice Simulations (CLS) consortium.

where ∇_i^S denotes the spatial components $(i=1,2,3)$ of the symmetric covariant derivative. The effective parameters of HQET that need to be known hence are

$$\omega = \left(m_\mathrm{bare}, \ln[Z_\mathrm{A}^\mathrm{HQET}], c_\mathrm{A}^{(1)}, \omega_\mathrm{kin}, \omega_\mathrm{spin}\right) . \tag{2.5}$$

The additional parameter m_bare in the static theory, which does not appear in (2.1)-(2.4), is the energy shift which absorbs the $1/a$ divergence of the static energy. If the matching is performed in HQET to order $1/m_\mathrm{b}$, it absorbs a $1/a^2$ term. The non-perturbative determination of these parameters has been presented at last years conference [2] and closely follows the general strategy originally discussed in [1]. We refer the reader to these papers for any unexplained notation and further explanations. What still needs to be emphasised at this point is:

- light quarks are simulated with two-flavor non-perturbatively improved Wilson fermions,
- for the static action we use the so-called HYP2 discretisation [3],[1]
- in the course of the non-perturbative (NP) matching to QCD in small volume of extent $L_1 \sim 0.5\,\mathrm{fm}$, various heavy quark masses have been simulated relativistically at renormalization group invariant (RGI) heavy quark mass M fixed to [4]

$$z \equiv L_1 M \in \{4,6,7,9,11,13,15,18,21\} , \tag{2.6}$$

ranging from slighty above the charm to beyond the bottom quark region.

To summarize, the parameters of HQET, $\omega(M,a)$, are known for the masses given in (2.6) and the lattice spacings a corresponding to $\beta \in \{5.2,5.3,5.5\}$ in large volume, c.f. Table 1.

Now we use the effective parameters to compute the B meson mass m_B, the hyperfine mass splitting $m_{\mathrm{B}^*}-m_\mathrm{B}$ and the pseudo-scalar heavy-light meson decay constant f_B ($f_{\mathrm{B_s}}$ is left for a future application). To first order in the $1/m_\mathrm{b}$ expansion our main observables are defined by

$$m_\mathrm{B} = m_\mathrm{bare} + E^\mathrm{stat} + \omega_\mathrm{kin}\, E^\mathrm{kin} + \omega_\mathrm{spin}\, E^\mathrm{spin} , \tag{2.7}$$

$$m_{\mathrm{B}^*} - m_\mathrm{B} = -\tfrac{4}{3}\omega_\mathrm{spin}\, E^\mathrm{spin} , \tag{2.8}$$

$$\begin{aligned} \ln(a^{3/2} f_\mathrm{B}\sqrt{m_\mathrm{B}/2}) &= \ln(Z_\mathrm{A}^\mathrm{HQET}) + \ln(a^{3/2}p^\mathrm{stat}) + b_\mathrm{A}^\mathrm{stat} a m_\mathrm{q} \\ &\quad + \omega_\mathrm{kin} p^\mathrm{kin} + \omega_\mathrm{spin} p^\mathrm{spin} + c_\mathrm{A}^{(1)} p^{\mathrm{A}^{(1)}} , \end{aligned} \tag{2.9}$$

[1] A final analysis with increased statistics will also take into account results for HYP1 which are fully compatible.

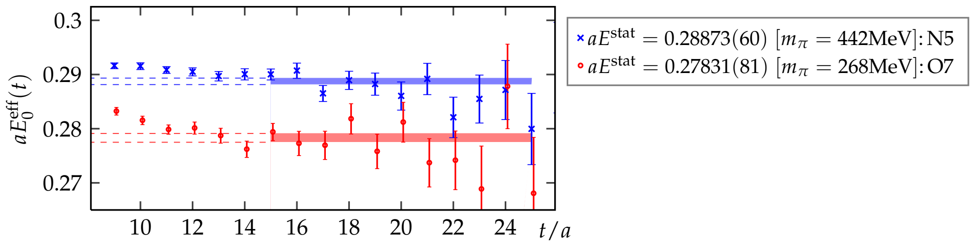

Figure 1: Results for static ground state energies (HYP2) after applying the GEVP method on 2 ensembles.

while their equivalents at static order are given by choosing $\omega = \left(m_{\rm bare}^{\rm stat}, \ln[Z_{\rm A}^{\rm stat}], ac_{\rm A}^{\rm stat}, 0, 0\right)$ compared to (2.5). The improvement coefficient $b_{\rm A}^{\rm stat}$ is determined to one loop in [5] and the HQET energies $E^{\rm stat}, E^{\rm kin}, E^{\rm spin}$ as well as the HQET hadronic matrix elements $p^{\rm stat}, p^{\rm kin}, p^{\rm spin}, p^{{\rm A}^{(1)}}$ depend on the set (m_π, a), i.e. the simulated pion masses m_π and lattice spacings a. They have been measured on a subset of ensembles produced within the CLS effort [6] with $N_{\rm f} = 2$ flavors of O(a)-improved Wilson-Clover fermions, obeying

$$m_\pi L \gtrsim 4.0\,, \qquad\qquad 270\,{\rm MeV} \lesssim m_\pi \lesssim 450\,{\rm MeV}\,. \tag{2.10}$$

For this reason we expect finite volume effects to be negligible. Details of these large volume ensembles including the actual size of the statistics used here are summarized in Table 1. We get HQET energies & hadronic matrix elements by solving the generalised eigenvalue problem, GEVP,

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)\,, \tag{2.11}$$

for an $N\times N$ correlator matrix C. The $n^{\rm th}$ state has eigenvalue λ_n with eigenvector v_n. As has been shown in [7], a systematic expansion of C in HQET is given by $C(t) = C^{\rm stat}(t) + \sum_{\rm X}\omega_{\rm X} C^{\rm X}(t) + {\rm O}(\omega^2)$ in terms of small expansion parameters $\omega_{\rm X} \propto 1/m_{\rm b}$ with ${\rm X} \in \{{\rm kin}, {\rm spin}, {\rm A}^{(1)}\}$, depending on the actual quantity. The eigenvalue λ_n determines the effective energy of the $n^{\rm th}$ state while the eigenvectors enter the expressions for the matrix elements [7]. As a variance reduction technique we employ stochastic all-to-all propagators. The heavy-light interpolating quark bilinears used,

$$\begin{aligned} O_k(x) &= \overline{\psi}_{\rm h}(x)\gamma_0\gamma_5\psi_{\rm l}^{(k)}(x)\,, & \psi_{\rm h}(x)&\text{: static quark field} \\ O_k^*(x) &= \overline{\psi}_{\rm l}^{(k)}(x)\gamma_0\gamma_5\psi_{\rm h}(x)\,, & \psi_{\rm l}^{(k)}(x) &= (1+a^2\Delta/10)^{R_k}\psi_{\rm l}(x) \end{aligned} \tag{2.12}$$

represent different levels of Gaussian smearing [8] for the light quark field $\psi_{\rm l}^{(k)}$, $k=1,\ldots,N$, with APE smeared links [9, 10] in the lattice Laplacian Δ. Numerical experiments have shown that choosing $N=3$ with $R_k \times (a/0.3{\rm fm})^2 \in \{1,4,10\}$ fixed, gives good results. For further details about the general procedure see [11]. Special care has been taken to control the contribution of excited states. In our analysis we have chosen a time range to extract the plateaux such that the corrections to $E_1^{\rm stat}$ are small compared to its statistical error; we found this to be the case for $t > t_0 > 0.3\,{\rm fm}$. Figure 1 shows two ground state energies obtained in this way. An autocorrelation analysis has shown that existing data can be considered decorrelated to a sufficient degree.

We arrange all data sets for a combined jackknife analysis with 100 estimators and compute (2.7)-(2.9) which now depend on (z, m_π, a). For each quantity we perform a joint, continuum $(a\to 0)$ and chiral $(m_\pi \to m_\pi^{\rm exp} \equiv m_{\pi^0} = 135\,{\rm MeV}$ [12]) extrapolation — χ+CL in short form.

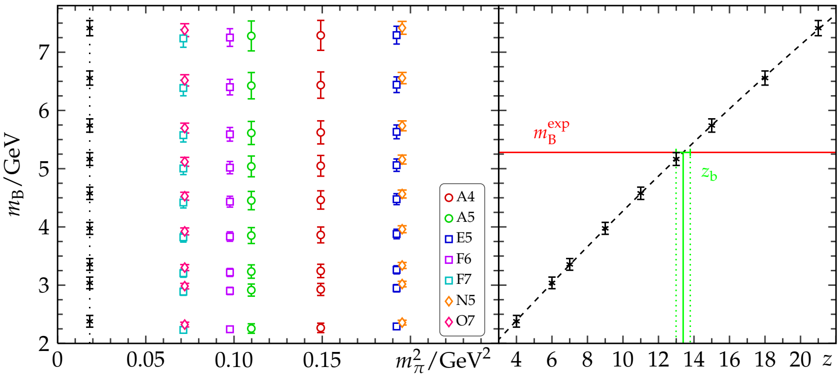

Figure 2: *Left:* HQET to $\mathrm{O}(1/m_{\rm b})$ data and results of χ+CL extrap. of B meson mass to the physical point for all values of z. *Right:* Dependence $m_{\rm B}^{\rm HQET}(z)$ in the continuum and its matching to the physical B meson.

3. Results

b-quark mass: We determine $m_{\rm B}$ by combining the HQET parameters and energies using the static or the $\mathrm{O}(1/m_{\rm b})$ expressions in eq. (2.7). In both cases we then use the global fit ansatz

$$m_{\rm B}\,(z,m_\pi,a) = B(z) + C\cdot m_\pi^2 + C'\cdot m_\pi^3 + D\cdot a^2\,, \tag{3.1}$$

where we either set $C'=0$ or $C'=-3\hat{g}^2/(16\pi f_\pi^2)$ computed with $f_\pi \equiv f_{\pi^0} = 130.4\,\mathrm{MeV}$ [12] and $\hat{g}=0.51(2)$ [13]. The data points and resulting values of $m_{\rm B}$ at the physical point (from $C'=0$) are shown on the left of Figure 2, while its z-dependence is shown on the right. Knowing the latter allows to match our computations of the B meson mass to its physical value,

$$m_{\rm B}(z,m_\pi^{\rm exp},0)\big|_{z=z_{\rm b}} \equiv m_{\rm B}^{\rm exp}\,, \qquad\qquad \text{taking} \quad m_{\rm B}^{\rm exp} = 5279.50\,\mathrm{MeV}\ [12]\,. \tag{3.2}$$

As a result we obtain the dimensionless RGI b-quark mass $z_{\rm b} = L_1 M_{\rm b}$. Taking the recent estimate of $L_1 = 0.405(18)\,\mathrm{fm}$ [14] and applying the 4-/3-loop running of the coupling/mass in the $\overline{\rm MS}$ scheme, we obtain the b-quark mass to

$$\overline{m}_{\rm b}(\overline{m}_{\rm b})\big|^{\rm stat}_{N_{\rm f}=2} = 4.21(13)_{\rm stat}(3)_a(6)_z\,\mathrm{GeV}\,, \qquad \left[z_{\rm b}^{\rm stat} = 13.46(21)_{\rm stat}(14)_a(18)_z\right] \tag{3.3}$$

$$\overline{m}_{\rm b}(\overline{m}_{\rm b})\big|^{\rm HQET}_{N_{\rm f}=2} = 4.23(13)_{\rm stat}(3)_a(6)_z\,\mathrm{GeV}\,. \qquad \left[z_{\rm b}^{\rm HQET} = 13.40(22)_{\rm stat}(16)_a(18)_z\right] \tag{3.4}$$

We explicitly separate the statistical error and the error coming from the scale setting. The last uncertainty is mainly due to the quark mass renormalisation constant $Z_{\rm M}$, entering z in QCD [4], i.e. the accuracy to which the values of z could be fixed. Since the difference of the static and HQET result of $\overline{m}_{\rm b}$ is already small, we can conclude that the truncation error of the latter $\sim \mathrm{O}(\Lambda^3/m_{\rm b}^2)$ is negligible compared to the statistical error. Comparing our new result for example with $\overline{m}_{\rm b}(\overline{m}_{\rm b}) = 4.163(16)\,\mathrm{GeV}$ [15] or $4.19^{+0.18}_{-0.06}\,\mathrm{GeV}$ [12] shows good agreement given our current uncertainty. Besides an increase in our statistics and the inclusion of two new ensembles, we are currently improving our estimate of L_1 which is the dominating source of error in $m_{\rm b}$ and are reducing the error in the lattice spacings.

With the physical value of $z_{\rm b}$ our theory is now uniquely defined and we can make further predictions to compare with experiment. There are two ways to proceed from here: a) one can

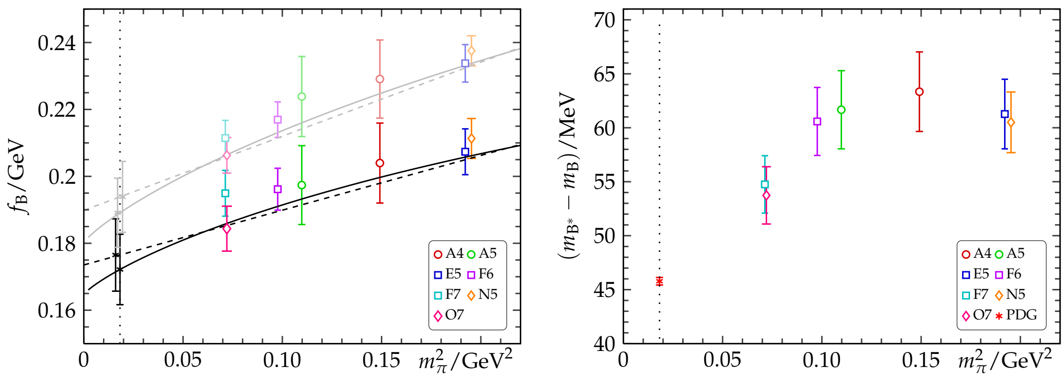

Figure 3: χ+CL extrapolations to the physical point. *Left:* Decay constant f_B. *Right:* Spin splitting.

combine the HQET parameters with matrix elements and/or energies to build new quantities, perform a χ+CL extrapolation and a subsequent interpolation to z_b *or* b) one once interpolates the HQET parameters to get $\varpi(a) \equiv \omega(z_b, a)$, use them in new observables and perform just the χ+CL extrapolation. Doing both serves as a consistency check.

Hyperfine/spin splitting: The leading contribution to this quantity, eq. (2.8), is of $\mathrm{O}(1/m_b)$ and vanishes in the static limit. Its intrinsic truncation error thus is $\mathrm{O}(\Lambda^2/m_b)$. To construct this observable we use $\varpi_{\mathrm{spin}}(a)$ which has leading lattice artifacts of $\mathrm{O}(a)$. Since this quantity is the one most sensitive to systematic errors, we will also profit most from a combined analysis with the additional HYP1 data and the improvements mentioned above. Our results at finite a and simulated pion masses for the HYP2 action together with the PDG value of $45.78(35)\,\mathrm{MeV}$ at the physical point are shown in the right panel of Fig. 3.

***B* meson decay constant:** Finally we take eq. (2.9) and rewrite it to extract f_B using $\varpi(a)$, m_B^{exp} and a. Heavy Meson Chiral Perturbation Theory (HMχPT) predicts a dependence

$$f_B\,(m_\pi, a) = B\left[1 - \tfrac{3}{4}\tfrac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2)\right] + C\cdot m_\pi^2 + D\cdot a^2\,, \tag{3.5}$$

of f_B on m_π, taking $\hat{g}, f_\pi$ as in (3.1). The resulting extrapolation is shown left in Fig. 3 (solid curve). If the chiral logarithm is neglected one gets a leading order (LO) fit ansatz with a linear dependence on m_π^2 (dashed line). The additional, half-transparent data corresponds to results obtained in the pure static theory. In numbers this reads

$$\text{HM}\chi\text{PT:}\quad f_B|_{N_f=2}^{\mathrm{stat}} = 189(6)_{\mathrm{stat}}(5)_a\,\mathrm{MeV}\,,\qquad \text{LO:}\quad f_B|_{N_f=2}^{\mathrm{stat}} = 194(6)_{\mathrm{stat}}(5)_a\,\mathrm{MeV}\,, \tag{3.6}$$

$$\text{HM}\chi\text{PT:}\quad f_B|_{N_f=2}^{\mathrm{HQET}} = 172(6)_{\mathrm{stat}}(5)_a\,\mathrm{MeV}\,,\qquad \text{LO:}\quad f_B|_{N_f=2}^{\mathrm{HQET}} = 176(6)_{\mathrm{stat}}(5)_a\,\mathrm{MeV}\,. \tag{3.7}$$

Including the chiral logarithm of HMχPT or not, changes the value at the physical point by a small amount only. At the moment we take the average of the two extrapolations in HQET to $\mathrm{O}(1/m_b)$,

$$f_B|_{N_f=2}^{\mathrm{HQET}} = 174(11)(2)\,\mathrm{MeV}\,, \tag{3.8}$$

as the central value and include half of the difference as part of the systematic error. Note that our estimate of f_B is lower than other estimates presented at this conference, see [16] for a general review.

4. Conclusions

We have reported on the status of the ALPHA Collaboration's heavy quark project to extract relevant B physics quantities from $N_{\mathrm{f}} = 2$ lattice simulations in the framework of HQET expanded to $\mathrm{O}(1/m_{\mathrm{b}})$. The measurement of HQET energies and matrix elements has been done using the GEVP approach on ensembles produced by CLS. For the first time we quote results for $\overline{m}_{\mathrm{b}}$ and f_{B} in the continuum obtained in large volume with three lattice spacings and seven pion masses. These values are still preliminary since we are confident to decrease uncertainties further by using additional data that has not been taken into account yet.

Once we have further improved the accuracy of our results, we will also determine $f_{\mathrm{B_s}}$, hadronic parameters of the $B-\overline{B}$ mixing or $B \to \pi$ semileptonic form factors as well as more details of the spectrum of hadrons with a b-flavor. As a long term goal we would also like to include the dynamical strange and charm sea quark contributions such that the only remaining systematic effect comes from the truncation of HQET.

Acknowledgments

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