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The D to K and D to π semileptonic decay form factors from Lattice QCD

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We present a new and very high statistics study of D and D_s semileptonic decay form factors on the lattice. We work with MILC $N_f = 2+1$ lattices and use the Highly Improved Staggered Action (HISQ) for both the charm and the light valence quarks. We use both scalar and vector currents to determine the form factors $f_0(q^2)$ and $f_+(q^2)$ for a range of D and D_s form factors including those for D to π and D to K semileptonic decays. By using a phased boundary condition we are able to tune accurately to $q^2 = 0$. We also compare the shape in q^2 to that from experiment. We show that the form factors are very insensitive to the spectator quark: D to K and D_s to η_s form factors are essentially the same, and the same is true for D to π and D_s to K. This has important implications when considering the corresponding B/B_s processes.

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1. Scalar and vector currents

Form factors f_0 and f_+ can be extracted from scalar and vector 3-point correlators — see diagram in Fig. 1. The scalar current is a local, conserved current

$$\langle K|S|D\rangle = f_0^{D\to K}(q^2)\frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}} \tag{1.1}$$

with $S = \bar{\Psi}\Psi$. Here q is the difference of the four momenta of the mesons, $q = p_D - p_K$, and the process $D \to Kl\nu$ is used as an example. One or both of the mesons are given a spatial momentum p using so called twisted boundary conditions (i.e. a phase at the boundary):

$$\Phi(x + \hat{e}_j L) = \mathrm{e}^{i2\pi\theta}\Phi(x) \tag{1.2}$$

where L is the size of the lattice. This gives the quark a momentum $2\pi\theta/L$. Note that θ can be tuned to get a desired value for q^2, e.g. $q^2 = 0$. We use different kinematical set-ups: kinematics A, where only one of the mesons has spatial momentum and the other one is at rest (in Fig. 1, s quark would have the momentum), and kinematics C, where both mesons have the same spatial momentum (in Fig. 1, the light quark would have the momentum). We have tested this method carefully, for example by checking that the speed of light is one (see Fig. 2), and that the amplitude of the meson correlator depends on the momentum like $1/\sqrt{E}$.

The vector current can be written as

$$\langle K|V^\mu|D\rangle = f_+^{D\to K}(q^2)\left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2}q^\mu\right] + f_0^{D\to K}(q^2)\frac{M_D^2 - M_K^2}{q^2}q^\mu. \tag{1.3}$$

We have chosen to use $V^\mu = \gamma^\mu$, a tasteless, spatial vector current. This has to be a 1-link current, if we have Goldstone mesons. We also tested a local, temporal vector current γ^t with a non-Goldstone D_s ($\gamma_5\gamma_t$) for $D_s \to \eta_s$ [1] (denoted as V_t in Fig. 6). We use MILC $N_f = 2+1$ lattices to do the calculations — see Table 1 for more details.

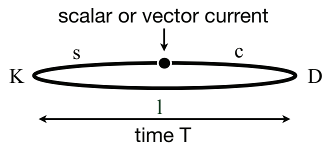

Figure 1: Diagram of the 3-point correlator setup.

1.1 Fitting

To extract the form factors we do a simultaneous least χ^2 fit to both 3-point correlators and the corresponding 2-point meson correlators. For a given semileptonic decay, say $D \to K$, we also fit all q^2 values simultaneously. For the 2-point correlators the fit function is the usual sum of exponentials (with the oscillating states, as we are dealing with staggered quarks): e.g. for the D meson

$$C_D(t) = \sum_j (b_j^D)^2\left(\mathrm{e}^{-E_j^D t} + \mathrm{e}^{-E_j^D(N_t - t)}\right) - \sum_k (d_k^D)^2(-1)^t\left(\mathrm{e}^{-E_k'^D t} + \mathrm{e}^{-E_k'^D(N_t - t)}\right). \tag{1.4}$$

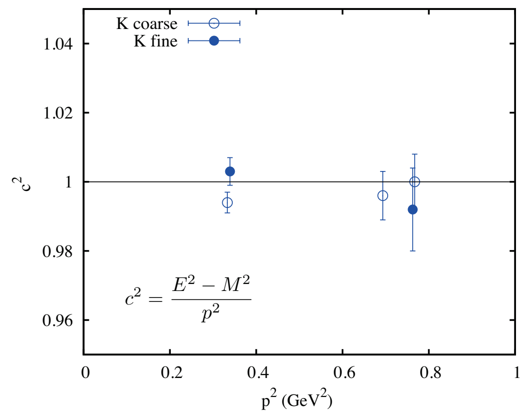

Figure 2: Test: Speed of light.

The fit functions for the 3-point correlators have similar form: e.g. for $D \to K$ we have

$$C_{D\to K}(t,T) = \sum_j \sum_k A_{jk}^{D\to K} e^{-E_j^K t} e^{-E_k^D (T-t)} - \sum_j \sum_k B_{jk}^{D\to K} e^{-E_j^K t} e^{-E_k^{\prime D}(T-t)} (-1)^{T-t} \quad (1.5)$$

$$- \sum_j \sum_k C_{jk}^{D\to K} e^{-E_j^{\prime K} t} e^{-E_k^D (T-t)} (-1)^t + \sum_j \sum_k D_{jk}^{D\to K} e^{-E_j^{\prime K} t} e^{-E_k^{\prime D}(T-t)} (-1)^T, \quad (1.6)$$

where $A_{00} = b_0^K b_0^D \langle K|S|D\rangle/(2\sqrt{M_D E_K})$ gives the desired form factor f_0 at a given q^2. We use three or four time separations T for the mesons (see Fig. 1), as the 3-point correlators are oscillating.

2. Renormalization of the currents

The scalar current is absolutely normalized (note the bare quark masses in Eq. (1.1) — the renormalization factors cancel). However, the vector current does need to be renormalized. We extract the renormalization factor Z from the symmetric vector current by demanding $f_+^{H\to H}(0) = 1$ for $H = D$, D_s, η_s, and η_c. The extracted Z factors agree over a range of momenta and different mesons for both charm-charm and charm-strange currents — see Fig. 3. This is essential, as we want to use Z to renormalize a charm-strange and a charm-light current.

The local, temporal current V_t is renormalized using f_0 at $q^2_{\rm max}$ extracted from the scalar current. Note that at $q^2_{\rm max}$ the temporal vector current gives the form factor f_0 directly, as the coeffi-

ensemble	size, $L^3 \times N_t$	physical size	# configs.	# time sources	m_l
coarse	$20^3 \times 64$	$\approx (2.4\ \rm fm)^3$	2259	8	$\approx m_s/3.5$
fine	$28^3 \times 96$	$\approx (2.4\ \rm fm)^3$	1911	4	$\approx m_s/4.2$

Table 1: Details of the MILC 2+1 flavor lattice configurations used in this study: lattice size, number of configurations, number of time sources per configuration, and light valence quark mass m_l (compared to strange quark mass m_s). The mass values for HISQ valence light quarks are tuned to match the same goldstone pion mass as those for the asqtad sea light quarks. The HISQ valence s quark masses are tuned to the physical value — see [2] for more details.

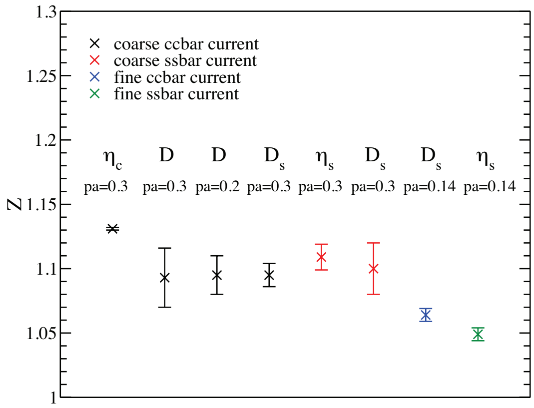

Figure 3: Renormalization factor Z for the 1-link vector current.

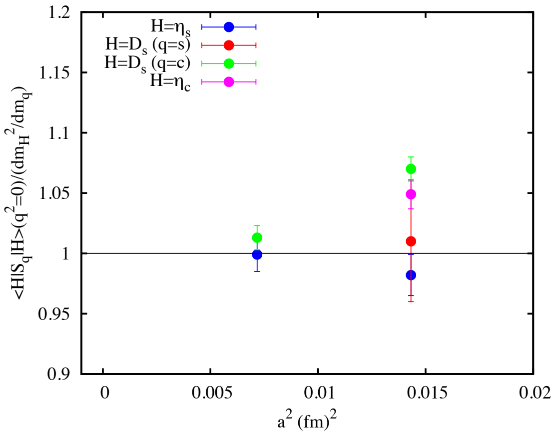

Figure 4: Test of the symmetric scalar current.

cient multiplying f_+ vanishes (see Eq. 1.3). We can thus calculate $f_0(q^2_{\rm max})$, and set $f_{0,V_t}(q^2_{\rm max}) = f_{0,S}(q^2_{\rm max})$.

As yet another test we calculated the symmetric scalar current, $\langle H|S|H\rangle$, for $H = D_s$, η_s, and η_c. At $q^2 = 0$, and when lattice spacing $a \to 0$, this is expected to give

$$\langle H|S_q|H\rangle(q^2 = 0) = \frac{dm_H^2}{dm_q}. \tag{2.1}$$

This is indeed the case, as can be seen in Fig. 4.

3. Preliminary results: form factors f_0 and f_+

Our preliminary results for the $D \to \pi$ and $D \to K$ semileptonic decay form factors f_0 and f_+ are presented in Figs. 5, 6, and compared to experimental results in Section 3.2. We consider different mesons: For example, we calculate the charm-strange current 3-point correlator with different spectator quarks, light, strange and charm. This allows us to compare the form factors for

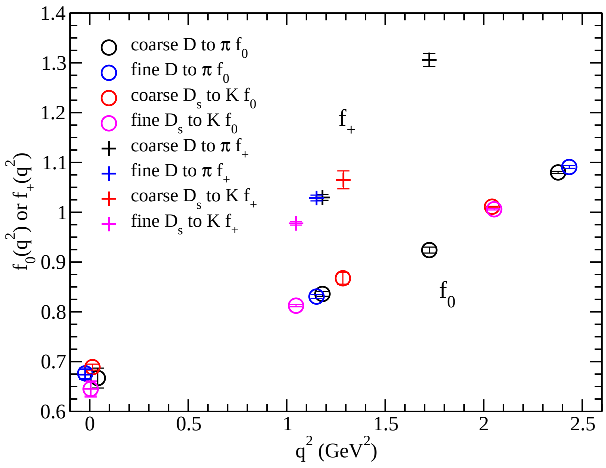

Figure 5: Preliminary results: form factors, charm to light decay.

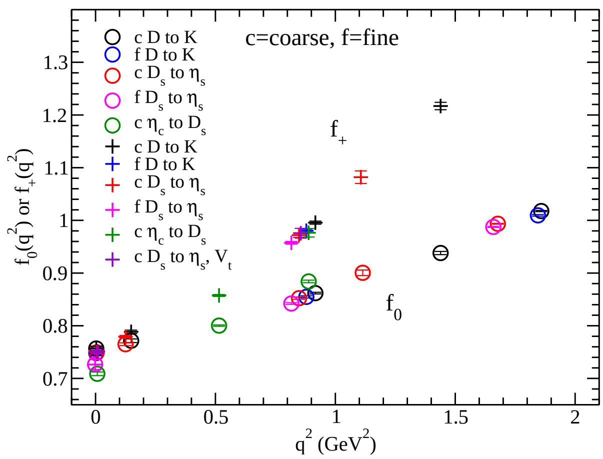

Figure 6: Preliminary results: form factors, charm to strange decay.

$D \to K$, $D_s \to \eta_s$ and $\eta_c \to D_s$. The semileptonic decay form factors are very insensitive to the spectator quark: $D \to K$ and $D_s \to \eta_s$ form factors are almost identical. However, if the spectator quark is as heavy as the charm quark, as in $\eta_c \to D_s$, the form factors do have a noticeably different shape. The insensitiveness of the form factors to the spectator quark is also seen in the light-charm current: $D \to \pi$ and $D_s \to K$ form factors have the same shape. One would expect to see similar behaviour in the corresponding B/B_s form factors, i.e. basically no dependence on the spectator quark.

3.1 z-expansion and continuum extrapolation

It is convenient to transform the form factors to z-space to do the continuum extrapolation. This is done as follows: First we remove the poles from the form factors,

$$\tilde{f}_0^{D\to K}(q^2) = \left(1 - \frac{q^2}{M^2_{D^*_{s0}}}\right) f_0^{D\to K}(q^2), \quad \tilde{f}_+^{D\to K}(q^2) = \left(1 - \frac{q^2}{M^2_{D^*_s}}\right) f_+^{D\to K}(q^2). \tag{3.1}$$

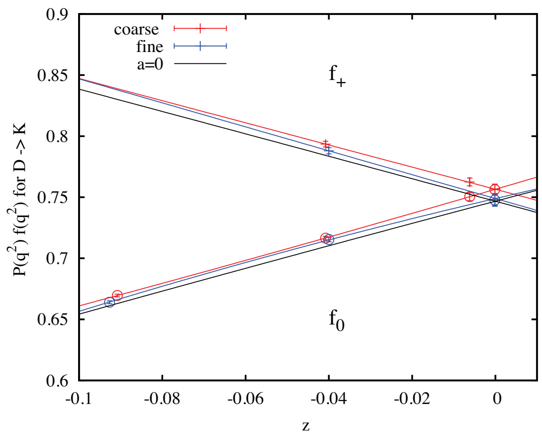

Figure 7: Form factors in z-space.

Here D to K is used as an example, so the poles are $M^2_{D^*_{s0}}$ and $M^2_{D^*_s}$. We then change from q to z,

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}, \quad t_+ = (m_D + m_K)^2 \tag{3.2}$$

— note that we have taken $t_0 = 0$ in the standard transformation formula — and fit the lattice data as power series in z,

$$\tilde{f}_0^{D\to K}(z) = \sum_{n\geq 0} b_n(a) z^n, \quad \tilde{f}_+^{D\to K}(z) = \sum_{n\geq 0} c_n(a) z^n. \tag{3.3}$$

We include terms up to z^4. Note that $b_0 = c_0$, because $f_0(q^2 = 0) = f_+(q^2 = 0)$. The lattice spacing dependence is very small, and the extrapolation to $a = 0$ is shown in Fig. 7.

3.2 Comparison with experimental results

In Figs. 8 and 9 we compare our new results to earlier results as well as experimental results. Earlier results for $D \to K$ form factor at $q^2 = 0$ from HPQCD Collaboration are from [3], and the experimental results by CLEO Collaboration are from [4]. Both the value at $q^2 = 0$ and the shape of the $D \to K$ form factors f_0 and f_+ agree very well with experimental results, where $|V_{cs}|$ is calculated from the CKM matrix assuming unitarity.

4. Conclusions

We have presented here the first results from a very high precision study of D meson semileptonic decay vector form factors. We have looked at semileptonic decays of different mesons, and have shown that the form factors are very insensitive to the spectator quarks — we expect the same to be true for B/B_s processes. Indeed our results indicate that B and B_s semileptonic form factors at a given q^2 should differ by less than 5%, whereas their decay constants differ by approximately 20%. QCD sum rules [5] predict SU(3) breaking effects at around 10% for both. We need to repeat our calculation with a lighter sea light quark mass to do a chiral extrapolation, and the simulations

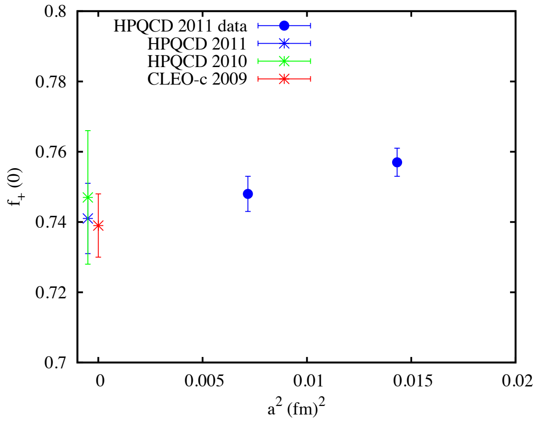

Figure 8: Form factor at $q^2 = 0$, extrapolated to $a = 0$.

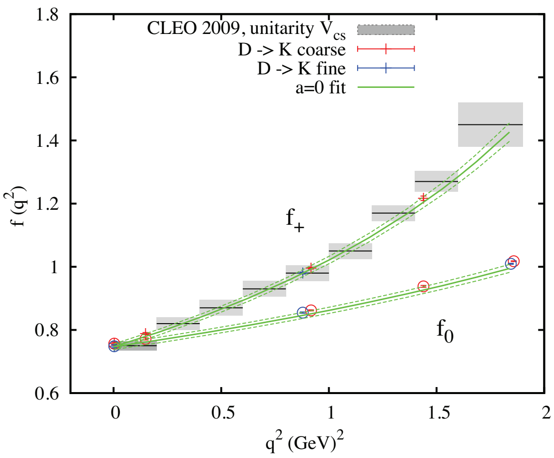

Figure 9: $D \to K$ form factors and experimental results from CLEO.

are already underway.

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