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Semileptonic B to D decays at nonzero recoil with 2+1 flavors of improved staggered quarks

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(Fermilab Lattice and MILC Collaborations)

The Fermilab Lattice-MILC collaboration is completing a comprehensive program of heavy-light physics on the MILC (2+1)-flavor asqtad ensembles with lattice spacings as small as 0.045 fm and light-to-strange-quark mass ratios as low as 1/20. We use the Fermilab interpretation of the clover action for heavy valence quarks and the asqtad action for light valence quarks. The central goal of the program is to provide ever more exacting tests of the unitarity of the CKM matrix. We give a progress report on one part of the program, namely the analysis of the semileptonic decay B to D at both zero and nonzero recoil. Although final results are not presented, we discuss improvements in the analysis methods, the statistical errors, and the parameter coverage that we expect will lead to a significant reduction in the final error for $|V_{cb}|$ from this decay channel.

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1. Introduction

Precision tests of the standard model from flavor factory and intensity frontier experiments can reveal new physics even if the accelerator energy isn't sufficient to create the new particles associated with the new physics. Here preliminary results of a lattice-QCD calculation of the nonzero-recoil form factor for the semileptonic process $B \to D\ell\overline{\nu}$ are presented. The principal goal is to determine $|V_{cb}|$ to a high precision. The theoretical uncertainty on $|V_{cb}|$ limits the precision of the unitarity triangle constraint from neutral kaon mixing.

There are several methods for determining $|V_{cb}|$. (1) From the inclusive decay $b \to c\ell\overline{\nu}$, perturbation theory and the operator product expansion provide an estimate of the inclusive decay rate, which is then combined with the measured decay rate to obtain $|V_{cb}|$ [1]. (2) From the exclusive semileptonic decay $B \to D^*\ell\overline{\nu}$, lattice-QCD methods provide the hadronic contribution to the decay rate [2]. For the special case of zero recoil, we have presented an unquenched calculation for this process three years ago [3] with a preliminary update reported at the Lattice 2010 [4] and CKM 2010 conferences [5] (3) Likewise, from the exclusive semileptonic decay $B \to D\ell\overline{\nu}$, lattice-QCD methods provide the hadronic contribution [8]. Here we report the first unquenched lattice-QCD calculation for this process at nonzero recoil.

At present there is a 1.6 σ disagreement between the value of $|V_{cb}|$ determined from inclusive decays and our recent preliminary value based on the decay $B \to D^*\ell\overline{\nu}$ at zero recoil, as illustrated in Fig. 1. Thus, further cross checks of and improvements in the theoretical calculations are needed. The value of $|V_{cb}|$ can be determined from the ratio of the measured decay rate to the calculated hadronic form factor at any chosen recoil energy. Typically, the uncertainty in the experimental measurement increases as the recoil energy vanishes [12], whereas the uncertainty in the lattice-QCD calculation decreases [6, 7, 8]. Thus, to minimize the uncertainty in the ratio, it is best to combine results of the experimental measurement with those of the lattice-QCD calculation over the full range of available recoil energies. For this reason we undertake an analysis over a broad range.

2. Methodology for determining $|V_{cb}|$

The hadronic weak matrix element for this process is commonly parameterized as

$$\langle D(p')|\mathcal{V}_\mu|B(p)\rangle = \sqrt{m_B m_D}[h_+^{B\to D}(w)(v+v')_\mu + h_-^{B\to D}(w)(v-v')_\mu] \,. \tag{2.1}$$

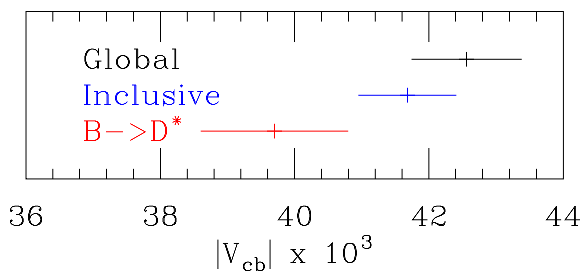

Figure 1: The central values and errors resulting from three determinations of $|V_{cb}|$: (1) A "global fit" to the unitarity triangle including all inputs except $|V_{cb}|$ [9] [10], (2) inclusive measurements summarized by the HFAG [11], and (3) our $B \to D^*$ lattice-QCD calculation [4, 5].

Here, w is velocity transfer, $w = v \cdot v'$. The differential rate for the semileptonic decay $B \to D\ell\overline{\nu}$ is

$$\frac{d\Gamma(B \to Dl\overline{\nu})}{dw} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}(m_B+m_D)^2 m_D^3[w^2-1]^{3/2}|\mathscr{G}_{B\to D}(w)|^2 \,, \tag{2.2}$$

where $\mathscr{G}_{B\to D}(w)$ is defined as

$$\mathscr{G}_{B\to D}(w) = h_+^{B\to D}(w) - \frac{m_B - m_D}{m_B + m_D} h_-^{B\to D}(w) \,. \tag{2.3}$$

Thus, once we know $h_+^{B\to D}$ and $h_-^{B\to D}$, we can determine the form factor $\mathscr{G}_{B\to D}(w)$. Then, combining it with the result of the decay rate from experiment, we will obtain $|V_{cb}|$. In order to construct $h_+^{B\to D}$ and $h_-^{B\to D}$, we need the following quantities:

$$h_+(w) = R_+(\mathbf{p})[1 - x_f(\mathbf{p})R_-(\mathbf{p})], \tag{2.4}$$
$$h_-(w) = R_+(\mathbf{p})[1 - R_-(\mathbf{p})/x_f(\mathbf{p})], \tag{2.5}$$
$$R_+(\mathbf{p}) = \langle D(\mathbf{p})|\mathscr{V}^4|B(0)\rangle, \tag{2.6}$$
$$R_-(\mathbf{p}) = \frac{\langle D(\mathbf{p})|\mathscr{V}^1|B(0)\rangle}{\langle D(\mathbf{p})|\mathscr{V}^4|B(0)\rangle}, \tag{2.7}$$
$$x_f(\mathbf{p}) = \frac{\langle D(\mathbf{p})|\mathscr{V}^1|D(0)\rangle}{\langle D(\mathbf{p})|\mathscr{V}^4|D(0)\rangle} \,, \tag{2.8}$$

where $\mathscr{V}^\mu$ is the continuum hadronic weak vector current.

In our calculation we use the local (nonconserved) lattice vector current $V_\mu(x) = \bar{\Psi}_b(x) i\gamma_\mu \Psi_c(x)$ with $\mathscr{O}(a)$ improved heavy quark fields $\Psi_h(x)$ following [13], and we renormalize it following a partly nonperturbative method, namely

$$Z_{Vbc} = \rho_V \sqrt{Z_{Vbb} Z_{Vcc}} \,, \tag{2.9}$$

where the flavor-diagonal renormalization coefficients are computed nonperturbatively on the lattice via the conditions

$$Z_{Vbb}\langle B|V^4|B\rangle = 1, \tag{2.10}$$
$$Z_{Vcc}\langle D|V^4|D\rangle = 1, \tag{2.11}$$

and ρ_V is computed perturbatively [7]. Because of cancellations among similar loop diagrams, we expect that ρ_V is nearly equal to 1 [7]. Preliminary results presented here omit the ρ_V factor.

At zero recoil ($w = 1$) we use the double ratio method [2]:

$$|h_+(1)|^2 = \rho_V^2 \frac{\langle D|V^4|B\rangle\langle B|V^4|D\rangle}{\langle D|V^4|D\rangle\langle B|V^4|B\rangle} \,. \tag{2.12}$$

This ratio suppresses a large part of the statistical fluctuations, and it builds in the current renormalization $\sqrt{Z_{V^{bb}} Z_{V^{cc}}}$.

a(fm)	size	m_l/m_h
≈0.15	$16^3 \times 48$	0.2
≈0.12	$20^3 \times 64$	0.14
≈0.12	$20^3 \times 64$	0.2
≈0.12	$20^3 \times 64$	0.4
≈0.12	$24^3 \times 64$	0.1
≈0.09	$28^3 \times 96$	0.2
≈0.09	$32^3 \times 96$	0.15

a(fm)	size	m_l/m_h
≈0.09	$40^3 \times 96$	0.1
≈0.09	$64^3 \times 96$	0.05
≈0.06	$48^3 \times 144$	0.2
≈0.06	$48^3 \times 144$	0.4
≈0.06	$56^3 \times 144$	0.14
≈0.06	$64^3 \times 144$	0.1
≈0.045	$64^3 \times 192$	0.2

Table 1: Summary of all ensembles to be included in the full analysis. The ensembles in bold have been analyzed for this report. Light(strange) sea quark masses are denoted by $m_l(m_h)$.

3. Results

We report on results from an analysis of a large set of gauge field ensembles generated in the presence of $2+1$ flavors of improved staggered (asqtad) quarks [14, 15]. Further ensembles will be included in the future. Some key ensemble parameters are listed in Table 3.

On each ensemble we compute the three-point correlation functions relevant to the weak matrix elements and the two-point functions relevant to the propagation of the B and D mesons. For the heavy-light mesons we use heavy clover quarks in the Fermilab interpretation and light, improved staggered (asqtad) quarks [14, 15]. We use two types of interpolating operators for the B and D mesons, namely local and smeared using the 1S Richardson wave function. We set the valence-quark masses equal to the sea-quark masses. The bare lattice charm and bottom quark masses are fixed by matching the kinetic masses of the D_s and B_s mesons, respectively, to their experimental values. Some small adjustments will be needed to refine this tuning but are not yet included in this preliminary analysis.

In terms of interpolating operators $\mathscr{O}_B$ and $\mathscr{O}_D$ for the B and D mesons and the vector current, the two-point and three-point functions are given by

$$\begin{aligned} C^{2pt}(t;\mathbf{p}) &= \sum_{\mathbf{x}} \exp(i\mathbf{p}\cdot\mathbf{x})\langle \mathscr{O}(\mathbf{x},t)\mathscr{O}^\dagger(\mathbf{0})\rangle \\ &= \sum_n s_n^t Z_n(\mathbf{p})[\exp(-E_n(\mathbf{p})t) + \exp(-E_n(\mathbf{p})(N_t - t))] \qquad (3.1) \\ C^{3pt,B\to D}_{V_\mu}(t,T;\mathbf{p}) &= \sum_{\mathbf{x},\mathbf{y}} \exp(i\mathbf{p}\cdot\mathbf{y})\langle \mathscr{O}_D(0)V_\mu(\mathbf{y},t)\mathscr{O}_B^\dagger(\mathbf{x},T)\rangle\,, \qquad (3.2) \end{aligned}$$

where $s_n = \pm 1$ accounts for contributions that oscillate in t, and N_t is the lattice extent in the t dimension. Note that in the three-point function above, we have put the D meson at the origin and the B meson at $(\mathbf{x},T)$.

The matrix elements we want from Eq. (2.6) to Eq. (2.8) are obtained by factorization and reduction of the three-point functions. The reduction is based on the overlap coefficients $Z_n(\mathbf{p})$, energies and masses obtained in fits to the two-point functions. For example, the $R_+(\mathbf{p})$ matrix element is obtained from

$$R_+(t,T;\mathbf{p}) = \frac{C^{3pt,B\to D}_{V_4}(t,T;\mathbf{p})}{\sqrt{C^{3pt,D\to D}_{V_4}(t,T;\mathbf{0})C^{3pt,B\to B}_{V_4}(t,T;\mathbf{0})}}\sqrt{\frac{Z_D(\mathbf{0})E_D}{Z_D(\mathbf{p})m_D}}e^{E_Dt-\frac{1}{2}m_DT}e^{-m_B(t-\frac{1}{2}T)} \qquad (3.3)$$

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in the limit $t \to \infty$ and $T - t \to \infty$. In that limit it is a constant, $R_+(\mathbf{p})$, independent of t. However, at finite t and T, complications from oscillating terms and excited states must be taken into account.

Because they contain a light staggered quark, the meson interpolating operators excite even- as well as odd-parity channels. The unwanted even-parity states manifest themselves as terms that oscillate in t, corresponding to terms in the two-point and three-point functions with $s_n = -1$. To suppress the effect of the oscillating terms in the three-point functions, we average in both t and T (after removing the dominant exponential dependence on t as in Eq. (3.3)) as follows:

$$\overline{R} \equiv \frac{1}{2}R(0,t,T) + \frac{1}{4}R(0,t,T+1) + \frac{1}{4}R(0,t+1,T+1)\ . \tag{3.4}$$

Here, R refers to any of the quantities derived from three-point functions, such as R_+, R_-, and x_f.

Having suppressed the contributions of the oscillating states to the three-point correlators, we must still account for contributions from excited states. For example, for $D \to D$, the leading contributions are

$$\begin{aligned} C_{V4}^{3ptD\to D}(p,t) &= \sqrt{Z_D(p)}\frac{e^{-E_Dt}}{\sqrt{2E_D}}\langle D|V_4|D\rangle\frac{e^{-m_D(T-t)}}{\sqrt{2m_D}}\sqrt{Z_D(0)} + \sqrt{Z_{D'}(p)}\frac{e^{-E_{D'}t}}{\sqrt{2E_{D'}}} \\ &\times\langle D'|V_4|D\rangle\frac{e^{-m_D(T-t)}}{\sqrt{2m_D}}\sqrt{Z_D(0)} + \sqrt{Z_D(p)}\frac{e^{-E_Dt}}{\sqrt{2E_D}}\langle D|V_4|D'\rangle\frac{e^{-m_{D'}(T-t)}}{\sqrt{2m_{D'}}}\sqrt{Z_{D'}(0)}\ , \end{aligned} \tag{3.5}$$

where the primes indicate the overlaps and energies of the excited states of the same parity. We have neglected the doubly excited contribution, since the singly excited contribution is found already to be small. Thus, at finite t and T the ratio $R_+(t,T;\mathbf{p})$ in Eq. (3.3) is not constant, but has a small contamination from excited states. We take this into account by fitting the resulting ratio to

$$R_+(\mathbf{p},t,T) = R_+(\mathbf{p})\exp(\delta m t) + A\exp(-\Delta E_D t) + B\exp(-\Delta m_B(T-t)) \tag{3.6}$$

as illustrated in Fig. 2. The fit parameter δm should be zero, but it is introduced to allow for a small error in determining E_D and m_B. The other terms involve $\Delta E_D = E_{D'} - E_D$ and $\Delta m_B = m_{B'} - m_B$. These parameters are obtained from the fits of two-point functions. The results of two-point function fits then become prior central values and widths for fitting $R_+(t,T;\mathbf{p})$. We use a similar method to obtain $R_-(\mathbf{p})$ and $x_f(\mathbf{p})$.

4. Chiral and continuum extrapolation

To complete the analysis we will extrapolate in the light quark masses to their physical values and take the continuum limit, using staggered chiral perturbation theory [16, 17] . At this preliminary stage, we don't show physical values and chiral fit in this report. The numerical data points for h_+ and h_- are shown in Fig. 3. We will need these data points to finish the extrapolation of physical value of h_+ and h_- and the chiral fit.

5. Future plans

To complete the project, we need to (1) finish analyzing the available ensembles, (2) make adjustments for the tuning of the heavy quark masses, (3) compute the current renormalization (ρ_V factor), and (4) combine the experimental results for the differential decay rate with our results for the form factor to obtain $|V_{cb}|$, using methods employed for $|V_{ub}|$ to extend w over the full kinematic range [18].

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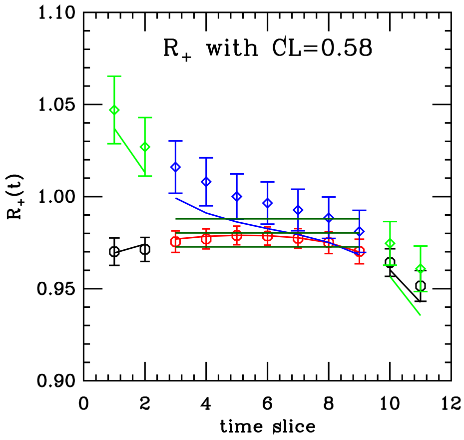

Figure 2: A sample fit to $R_+(t,T;\mathbf{p})$. Example three-point correlation fit for the 0.12 fm 0.14 m_l/m_h ensemble. The circles correspond to the local source, and diamonds, the smeared source. The fit result is obtained by a simultaneous fit to both correlators using Eq. (3.3). The horizontal lines represent the central value and error of the result for $R_+(\mathbf{p})$.

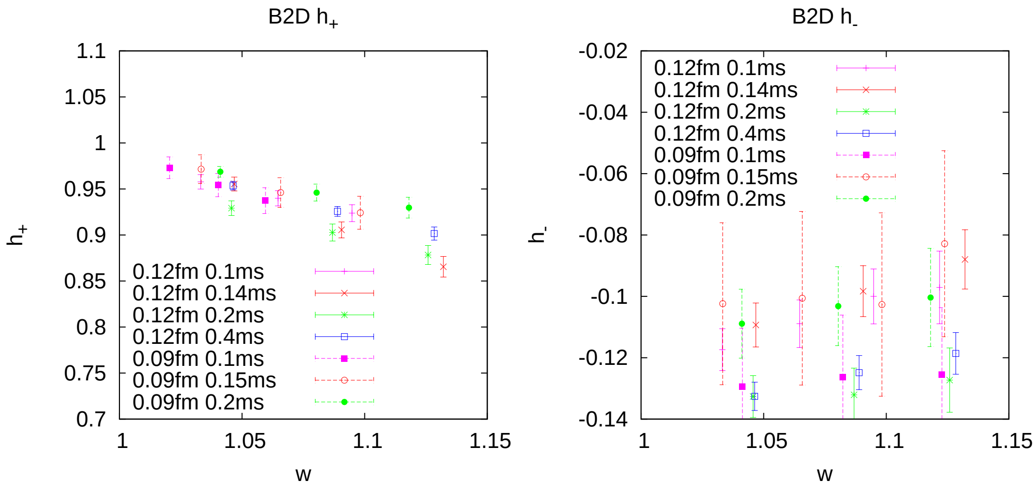

Figure 3: Form factors h_+ (left) and h_- (right) vs. w, omitting the matching factor ρ_V, on the bold ensembles in Table 3. The color code is given in the legends.

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