

## Studies of $B$ and $B_s$ Meson Leptonic Decays with NRQCD Bottom and HISQ Light/Strange Quarks

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We present a progress report on new calculations of  $B$  and  $B_s$  meson decay constants employing NRQCD heavy and HISQ light valence quarks and using MILC  $N_f = 2 + 1$  AsqTad lattices. Bare quark masses have been retuned in accord with HPQCD's new  $r_1$  scale. We find significant reductions in discretization effects compared to previous calculations with AsqTad light valence quarks. Matching of the NRQCD/HISQ heavy-light axial vector current is carried out at one-loop order including all relevant dimension 4 current corrections.

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Set	$r_1/a$	$m_l/m_s$ (sea)	$am_{l,s}$ (valence)	$aM_b$ (valence)	$N_{conf}$	$N_{tsrc}$	$L^3 \times N_t$
C1	2.647	0.005/0.050	0.0070 0.0489	2.650	1200 1200	2 2	$24^3 \times 64$
C2	2.618	0.010/0.050	0.0123 0.0492	2.688	1200 1200	2 2	$20^3 \times 64$
C3	2.644	0.020/0.050	0.0246 0.0491	2.650	600 600	2 2	$20^3 \times 64$
F1	3.699	0.0062/0.031	0.00674 0.0337	1.832	1200 1200	4 4	$28^3 \times 96$
F2	3.712	0.0124/0.031	0.0135 0.0336	1.826	600 600	4 4	$28^3 \times 96$
F0	3.695	0.0031/0.031	0.00339 0.0339	1.832	work in progress	4 4	$40^3 \times 96$

**Table 1:** Simulations details on three “coarse” and three “fine” MILC ensembles.

## 1. Introduction

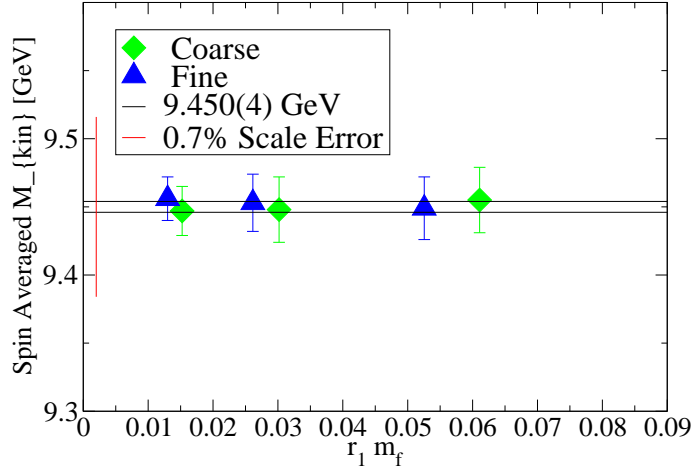
Leptonic decays of charged  $B$ 's such as  $B^+ \rightarrow \tau^+ \nu_\tau$ , are important processes for Cabbibo-Kobayashi-Maskawa (CKM) and Unitarity Triangle (UT) physics. There is currently some tension between  $\epsilon_K$ ,  $\sin(2\beta)$ ,  $|V_{ub}|$  and  $\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau)$  and the  $B$  meson decay constant  $f_B$  plays an important role in such analyses. For instance global fit results for  $f_B$  from precision electroweak data are being compared with “direct Standard Model predictions” of the decay constant from lattice QCD [1, 2, 3]. Reducing errors in the lattice determinations of  $f_B$  is, hence, a worthwhile high priority goal.

For the  $B_s$  meson there are no tree-level leptonic decays in the Standard Model (SM). Nevertheless the decay constant  $f_{B_s}$  is a useful parameter in many decay and mixing rates both within and beyond the SM. Furthermore, lattice QCD determinations of  $f_{B_s}$  can be achieved with smaller errors than for  $f_B$  since no light valence quarks are involved here. This combined with the fact that the ratio  $\frac{f_{B_s}}{f_B}$  is also known more accurately (due to cancellation of statistical and many systematic errors) than either of the decay constants on their own, opens the possibility of getting at precision  $f_B$  via precision values for  $f_{B_s}$  and  $\frac{f_{B_s}}{f_B}$ .

The HPQCD collaboration has initiated new calculations of the  $B$  and  $B_s$  meson decay constants based on NRQCD bottom and HISQ light and strange valence quarks and employing MILC  $N_f = 2 + 1$  configurations. This improves on previous determinations of  $f_B$ ,  $f_{B_s}$  and  $\frac{f_{B_s}}{f_B}$  by HPQCD that used AsqTad light and strange valence quarks [4]. Simulation details are summarized in Table 1.

## 2. Tuning of Quark Masses

We use the static potential quantity  $r_1 = 0.3133(23)\text{fm}$  [5] to set absolute scales and  $r_1/a$  from



**Figure 1:** Tuning of the  $b$ -quark mass using the spin averaged  $\Upsilon$  mass.

MILC [6] for relative scales between different MILC ensembles. To fix the bare  $b$ -quark mass in lattice units  $aM_b$  we use the spin averaged  $\Upsilon$  mass. One calculates,

$$\overline{M}_{b\bar{b}} \equiv \frac{1}{4} [3M_{kin}(^3S_1) + M_{kin}(^1S_0)], \quad (2.1)$$

with

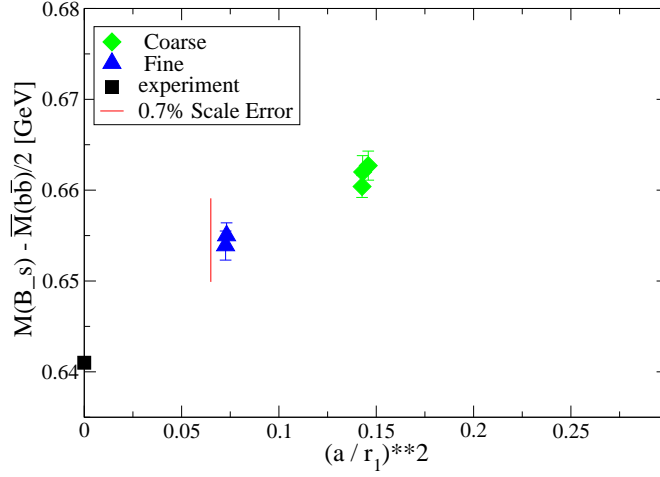
$$M_{kin} = \frac{p^2 - \Delta E_p^2}{2\Delta E_p}, \quad \Delta E_p = E(p) - E(0), \quad (2.2)$$

and compares with the experimental value (adjusted for the absence of electromagnetic, annihilation and sea charm quark effects in our simulations) of 9.450(4)GeV [7]. Results from this tuning are shown in Fig.1. Errors in the data points include statistical and  $r_1/a$  errors. One sees that these are much smaller than the 0.7% error in the absolute physical value of  $r_1$ . To achieve small statistical errors in  $M_{kin}$  it was crucial to employ random wall sources for the NRQCD  $b$ -quark propagators. With point sources errors would have been about 4~5 times larger. The  $s$ -quark mass was tuned to the (fictitious)  $\eta_s$  mass of 0.6858(40)GeV [5]. Having fixed the bottom and strange quark masses on each ensemble one can check how well  $M_{B_s} - \overline{M}_{b\bar{b}}/2$  is reproduced in the continuum limit. The leading dependence on the heavy quark mass cancels in this difference, so one is testing how well the lattice actions are simulating QCD boundstate dynamics. Results for this mass difference are shown in Fig.2. Within the  $r_1$  scale error and additional  $\sim 10$ MeV uncertainty from relativistic corrections to  $\overline{M}_{b\bar{b}}$  one sees agreement with experiment in the continuum limit.

### 3. The Currents and Matching

In the  $B_q$  meson rest frame ( $q = \text{light or strange}$ ) the decay constant is defined in terms of the temporal component of the  $bq$  heavy-light axial vector current  $A_0$  as,

$$\langle 0 | A_0 | B_q \rangle_{QCD} = M_{B_q} f_{B_q}. \quad (3.1)$$



**Figure 2:** The mass difference  $M_{B_s} - \overline{M}_{b\bar{b}}/2$  versus the square of the lattice spacing for the first five ensembles of Table 1. One sees negligible sea quark mass dependence but a noticeable lattice spacing dependence.

Simulations are carried out with effective lattice theory currents,

$$J_0^{(0)}(x) = \bar{q}(x) \Gamma_0 Q(x), \quad (3.2)$$

$$J_0^{(1)}(x) = \frac{-1}{2M_b} \bar{q}(x) \Gamma_0 \gamma \cdot \nabla Q(x), \quad (3.3)$$

$$J_0^{(2)}(x) = \frac{-1}{2M_b} \bar{q}(x) \gamma \cdot \overleftarrow{\nabla} \gamma_0 \Gamma_0 Q(x), \quad (3.4)$$

and matching through order  $\alpha_s$ ,  $\frac{\Lambda_{QCD}}{M}$ ,  $\frac{\alpha_s}{aM}$ ,  $a\alpha_s$ ,  $\alpha_s \frac{\Lambda_{QCD}}{M}$  gives,

$$\langle A_0 \rangle_{QCD} = (1 + \alpha_s \rho_0) \langle J_0^{(0)} \rangle + (1 + \alpha_s \rho_1) \langle J_0^{(1),sub} \rangle + \alpha_s \rho_2 \langle J_0^{(2),sub} \rangle, \quad (3.5)$$

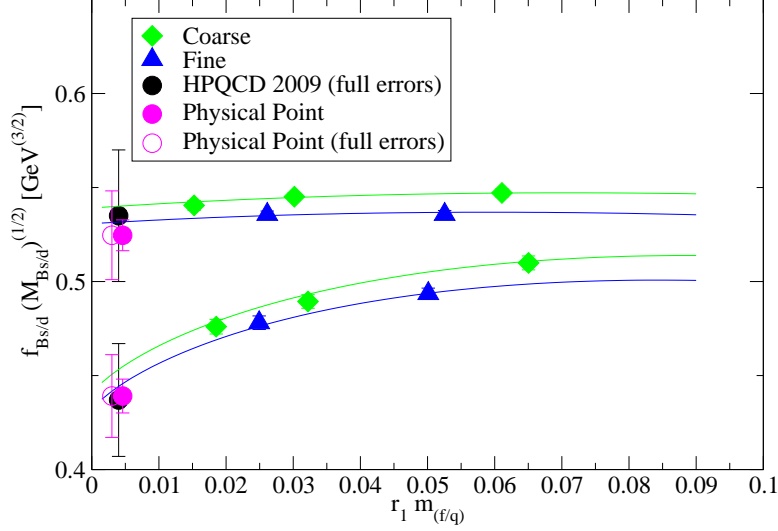
$$J^{(i),sub} = J^{(i)} - \alpha_s \zeta_{10} J^{(0)}. \quad (3.6)$$

$\rho_0$ ,  $\rho_1$ ,  $\rho_2$  and  $\zeta_{10}$  are the one-loop matching coefficients which have recently been calculated for NRQCD/HISQ currents.

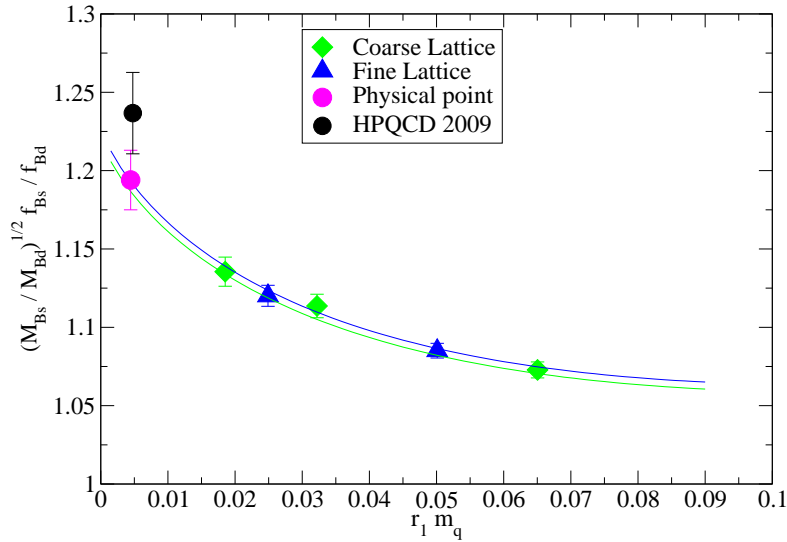
#### 4. Preliminary Results and Error Estimates

Figs.3 & 4 show our preliminary chiral/continuum extrapolations of  $f_{B_s} \sqrt{M_{B_s}}$ ,  $f_B \sqrt{M_B}$  and the ratio  $f_{B_s} \sqrt{M_{B_s}} / f_B \sqrt{M_B}$  based on the first five ensembles of Table 1. Work on the sixth ensemble F0, a more chiral fine ensemble, is in progress. Extrapolations are carried out using continuum partially quenched ChPT for heavy-light decay constants augmented by lattice spacing dependent terms. Our preliminary numbers at the physical point are,

$$f_B = 0.191(9)\text{GeV}, \quad f_{B_s} = 0.226(10)\text{GeV}, \quad f_{B_s}/f_B = 1.184(19). \quad (4.1)$$



**Figure 3:** Continuum/chiral extrapolations of  $f_{B_s} \sqrt{M_{B_s}}$  (upper curves) and  $f_B \sqrt{M_B}$  (lower curves). The black circles are results at the physical point from reference [4] using AsqTad light and strange valence quarks.



**Figure 4:** Continuum/chiral extrapolation of the ratio  $f_{B_s} \sqrt{M_{B_s}}/f_B \sqrt{M_B}$ . The black circle is the result at the physical point from reference [4] using AsqTad light and strange valence quarks.

Source	$f_{B_s}$ (%)	$f_B$ (%)	$f_{B_s}/f_B$ (%)
Statistical	0.7	1.1	0.9
Scale $r_1^{3/2}$	1.1	1.1	—
continuum extrap.	0.9	0.9	0.8
chiral extrap.	0.3	1.0	1.0
$g_{B^*B\pi}$	0.1	0.1	0.1
mass tuning	0.2	0.1	0.2
relativistic correct.	1.0	1.0	0.0
operator matching	4.0	4.0	0.1
Total	4.4	4.6	1.6

**Table 2:** Preliminary error budget

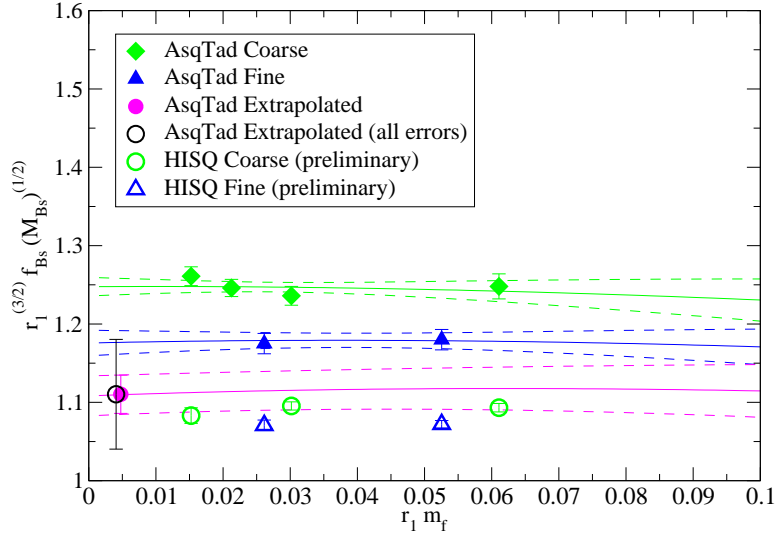
Table 2 shows a preliminary error budget.

The new NRQCD/HISQ numbers are consistent with HPQCD's NRQCD/AsqTad results of  $f_B = 0.190(13)\text{GeV}$ ,  $f_{B_s} = 0.231(15)\text{GeV}$ ,  $f_{B_s}/f_B = 1.226(26)$  [4]. The reduction in errors in the new calculations comes mainly from improvement in discretization errors, smaller  $r_1^{3/2}$  scale uncertainties and better fitting and extrapolation strategies. The striking decrease in lattice spacing dependence as one goes from AsqTad to HISQ strange quarks is demonstrated in Fig.5, where we compare  $f_{B_s}\sqrt{M_{B_s}}$  results on the same ensembles for the two different valence quark actions. Although this improvement in discretization errors is very welcome, in both the present and previous calculations the total error is dominated by the higher order operator matching uncertainty. HPQCD is investigating nonperturbative matching strategies for NRQCD/HISQ currents which could reduce this error in the future [8].

In a completely different thrust, we are pursuing an alternate approach to  $B$  physics that uses the relativistic HISQ action for heavy quarks with masses  $m_H > m_{charm}$  on very fine lattices [9, 10]. One can then extrapolate up to the physical  $b$ -quark staying always within a relativistic framework. A recent result using this method gives  $f_{B_s} = 0.225(4)\text{GeV}$  [11] in excellent agreement with eq.(4.1), however with much reduced errors. Using relativistic heavy quarks enables us to work with absolutely normalized currents (based on Ward identities). The main source of the larger errors with NRQCD  $b$ -quarks, namely operator matching uncertainties, is thus removed. In order to carry out heavy HISQ calculations that can be extrapolated to the physical  $b$ -quark, very fine and hence large lattices are required. Repeating the  $f_{B_s}$  determination for  $f_B$  would be quite expensive. So for the next couple of years we believe the best strategy for precision  $B$  physics will be to work with HISQ heavy quarks for  $B_s$  physics and combine these results with ratios, such as  $f_{B_s}/f_B$  from NRQCD  $b$ -quark calculations. The calculations presented here are part of this comprehensive approach to precision  $B$  physics.

#### Acknowledgments

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**Figure 5:** Comparison of lattice spacing dependence between NRQCD/HISQ results for  $f_{B_s} \sqrt{M_{B_s}}$  from the current calculations (open circles and triangles) and previous results from reference [4] based on NRQCD/AsqTad quarks.

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