

## Form factors for $B$ to $Kl$ semileptonic decay from three-flavor lattice QCD

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We study the  $B \rightarrow Kl^{+}l^{-}$  semileptonic decay process in three-flavor lattice QCD. We analyze several ensembles generated by the MILC collaboration at different lattice spacings and sea-quark masses. We use the asqtad improved staggered action for the light quarks and the clover action with the Fermilab interpretation for the heavy  $b$  quark. We present preliminary results for the vector current induced form factors for a range of kaon energies. Our analysis includes chiral and continuum extrapolations based on SU(2) staggered  $\chi$ PT.

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## 1. Introduction

Rare decays of  $B$  or  $D$  mesons may play an important role in the discovery of new physics in the flavor sector. Transitions in which a  $b$  quark decays to an  $s$  quark proceed through a flavor changing neutral current. As these currents only occur at higher order within the Standard Model (SM), such decays are rare, and a small contribution from new physics beyond the Standard Model may be comparable to the SM contribution and hence observable. In this work, we focus on the  $B \rightarrow Kl^{+}l^{-}$  semileptonic decay process, which occurs via the transition of  $b \rightarrow sl^{+}l^{-}$  at the quark level. References [1] and [2] summarize both the experimental and theoretical status of  $B$ -meson decays. The BABAR Collaboration studied both  $B \rightarrow Kl^{+}l^{-}$  and  $B \rightarrow K^{*}l^{+}l^{-}$  semileptonic decays in Ref. [3]. The Belle Collaboration published their results on the same process in Ref. [4]. The CDF Collaboration studied the  $B \rightarrow K^{*}\mu^{+}\mu^{-}$  decay in Ref. [5].

The  $B \rightarrow Kl^{+}l^{-}$  form factors are calculable from first principles using lattice QCD as there is only one hadron in the initial state and one in the final state. Recently, calculations using the MILC 2+1 flavor dynamical quark ensembles have been done for  $B \rightarrow K^{*}l^{+}l^{-}$  by Liu *et al.* [6] and a study of  $B \rightarrow K^{*}\gamma$  form factors was done by Becirevic *et al.* [7]. The Fermilab Lattice and MILC Collaborations presented some preliminary results for the  $B \rightarrow Kl^{+}l^{-}$  form factors in Ref. [8]. Additional ensembles covering a wider range of lattice spacings have been analyzed since then. In this brief report, we show a more comprehensive (but still preliminary) analysis of the meson masses, form factors and chiral and continuum extrapolations.

## 2. Theoretical Background

An operator production expansion (OPE) analysis of  $B \rightarrow Kl^{+}l^{-}$  shows that two currents, a vector current  $\bar{s}\gamma^{\mu}b$  and a tensor current  $\bar{s}\sigma^{\mu\nu}q_{\nu}b$  contribute to this process at lowest order [1]. We study the vector current here and defer study of the tensor current to later work. The matrix element of the vector current can be expressed in terms of two form factors  $f_{+}$  and  $f_{0}$  as:

$$\langle K|\bar{s}\gamma^{\mu}b|B\rangle = f_{+}(q^2) \left( p_B^{\mu} + p_K^{\mu} - \frac{m_B^2 - m_K^2}{q^2} q^{\mu} \right) + f_0(q^2) \frac{m_B^2 - m_K^2}{q^2} q^{\mu}, \quad (2.1)$$

where  $q^{\mu} = p_B^{\mu} - p_K^{\mu}$ . We study the form factors in the the  $B$ -meson rest frame, so only the kaon has non-zero momentum. The form factors  $f_{\parallel}$  and  $f_{\perp}$  are defined as:

$$f_{\parallel} = \frac{\langle K|\bar{s}\gamma^0 b|B\rangle}{\sqrt{2m_B}}, \quad (2.2)$$

$$f_{\perp} = \frac{\langle K|\bar{s}\gamma^1 b|B\rangle}{\sqrt{2m_B p_K^1}}. \quad (2.3)$$

They are more convenient for our lattice calculation, and are related to  $f_{+}$  and  $f_0$  by:

$$f_{+} = \frac{1}{\sqrt{2m_B}} [f_{\parallel} + (m_B - E_K)f_{\perp}], \quad (2.4)$$

$$f_0 = \frac{\sqrt{2m_B}}{m_B^2 - m_K^2} [(m_B - E_K)f_{\parallel} + (E_K^2 - m_K^2)f_{\perp}]. \quad (2.5)$$

The form factor  $f_{\perp}$ , as compared to  $f_{\parallel}$ , gives the dominant contribution to  $f_{+}$ , and hence to the experimental decay rate.

$a$ (fm)	$am_l^{\text{sea}}/am_s^{\text{sea}}$	$am_l^{\text{val}}/am_s^{\text{val}}$	$\kappa_b$	$N_{\text{measure}}$
0.12	0.02/0.05	0.02/(0.0415, 0.05)	0.0918	2052
0.12	0.01/0.05	0.01/(0.0415, 0.05)	0.0901	2259
0.12	0.007/0.05	0.005/(0.0415, 0.05)	0.0901	2110
0.12	0.005/0.05	0.005/(0.0415, 0.05)	0.0901	2099
0.09	0.0124/0.031	0.0124/(0.0261, 0.0310)	0.0982	1996
0.09	0.0062/0.031	0.0062/(0.0261, 0.0310)	0.0979	1931
0.09	0.00465/0.031	0.0047/(0.0261, 0.0310)	0.0997	984
0.09	0.0031/0.031	0.0031/(0.0261, 0.0310)	0.0976	1015
0.09	0.00155/0.031	0.00155/(0.0261, 0.0310)	0.0976	791
0.06	0.0072/0.018	(0.0072)/(0.0188)	0.1048	593
0.06	0.0018/0.018	(0.0018)/(0.0188)	0.1052	827

**Table 1:** Ensembles of QCD gauge field configurations used in the current  $B \rightarrow Kl^+l^-$  work.  $am_l^{\text{sea}}$  and  $am_s^{\text{sea}}$  denote the light and strange sea quark masses.  $am_l^{\text{val}}$  and  $am_s^{\text{val}}$  denote spectator and daughter quark masses in the  $b \rightarrow s$  transition. Both unitary and partially quenched kaon mass points are included in the chiral-continuum extrapolations.

The lattice form factors  $f_{\parallel}$  and  $f_{\perp}$  are computed numerically at several values of the lattice spacing and of the average up-down and strange quark masses. These results must then be extrapolated to the physical quark masses and the continuum using chiral perturbation theory ( $\chi$ PT).  $SU(3)$  staggered chiral perturbation theory for heavy-light semileptonic form factors [9] was successfully applied to the case of  $B \rightarrow \pi\ell\nu$  decay in Ref. [10]. More recently,  $SU(2)$   $\chi$ PT was applied to the extrapolation of  $D \rightarrow \pi\ell\nu$  form factors on  $N_f = 2$  lattices [11]. Some studies purport that  $SU(2)$   $\chi$ PT may be a better effective theory for heavy-light physics projects [12]. Here we test both  $SU(3)$  and  $SU(2)$  formalisms in our  $B \rightarrow Kl^+l^-$  chiral-continuum extrapolations.

### 3. Numerical Simulation

Our lattice calculations are done on MILC's  $N_f=2+1$  flavor gauge configurations with asqtad improved quarks [13]. The clover action with the Fermilab interpretation is used for the heavy quark [14]. MILC's ensembles cover many lattice spacings, light quark masses and volumes, which allows good control on the form factors' chiral and continuum extrapolations. The  $b$  quark mass is tuned close to its physical value as in Ref. [15]. In this report, we include results from the coarse ( $a \approx 0.12$  fm), fine ( $a \approx 0.09$  fm), and superfine ( $a \approx 0.06$  fm) ensembles.

### 4. Numerical Results

The first step of our analysis is to determine the  $B$  meson masses and kaon masses and energies from fits to the two-point correlators on every ensemble. States of both positive and negative parity contribute to the two-point correlators [10]. We vary the number of states in the fits and select the fit range from  $t_{\min}$  to  $t_{\max}$  carefully to control the excited state contribution and obtain a good  $p$ -value (confidence level) of the fit. Finally, the error on the mass is estimated via the standard jackknife method.

The second step of the analysis is to extract the form factors  $f_{\parallel}$  and  $f_{\perp}$  from fits of the ratio of three-point and two-point functions. The three-point function is defined as:

$$C_{3,\mu}^{B \rightarrow K}(t, T; \vec{p}_K) = \sum_{\vec{x}, \vec{y}} e^{i\vec{p}_K \cdot \vec{y}} \langle \mathcal{O}_K(0, \vec{0}) V_{\mu}(t, \vec{y}) \mathcal{O}_B^{\dagger}(T, \vec{x}) \rangle, \quad (4.1)$$

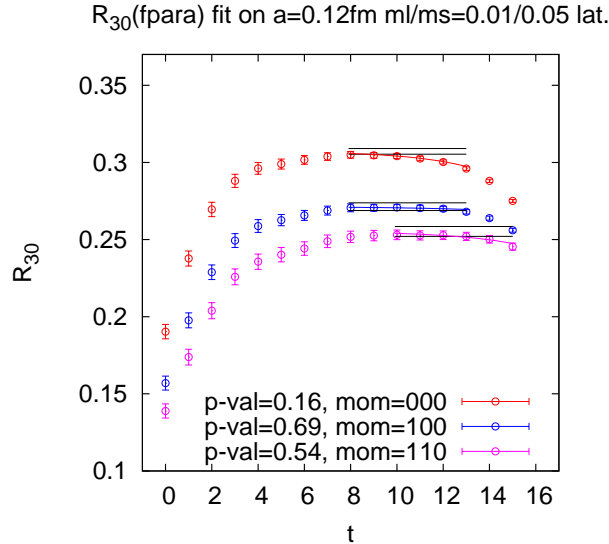
where  $V_{\mu} = i\bar{s}\gamma_{\mu}b$  and  $T$  is the location of the sink operator. Because we study the form factors in  $B$ -meson rest frame, only the kaon has non-zero momentum ( $p_K$ ). In a finite volume, the kaon's momentum is discrete. We choose  $p = (0, 0, 0)2\pi/L$ ,  $(1, 0, 0)2\pi/L$ , and  $(1, 1, 0)2\pi/L$ , where  $L$  is the box size. Higher momentum data are omitted due to their large statistical fluctuations. In addition, an iterative averaging trick is used to suppress the contribution of the states that have an alternating sign [10]. The ratio of two and three-point functions is defined as:

$$\bar{R}_{3,\mu}^{B \rightarrow K}(t, T) \equiv \frac{\bar{C}_{3,\mu}^{B \rightarrow K}(t, T)}{\sqrt{\bar{C}_2^K(t) \bar{C}_2^B(T-t)}} \sqrt{\frac{2E_K}{e^{-E_K^{(0)}t} e^{-m_B^{(0)}(T-t)}}}. \quad (4.2)$$

where  $\bar{C}_2$  and  $\bar{C}_3$  are averaged correlation functions [10]. After multiplying  $\bar{R}$  by the required renormalization constant, we obtain the continuum form factors:<sup>1</sup>

$$f_{\parallel}^{\text{cont}} = \rho \sqrt{Z_V^{hh} Z_V^{ll}} \bar{R}_{3,0}^{B \rightarrow K}(t, T), \quad (4.3)$$

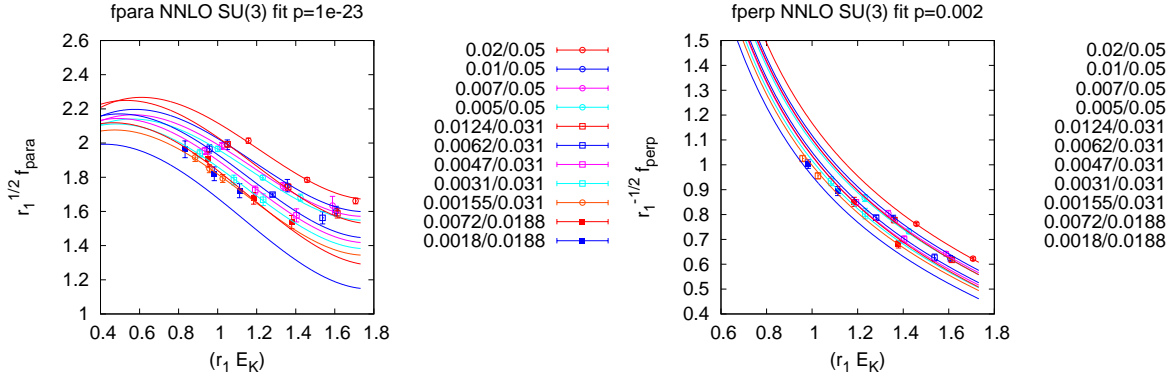
$$f_{\perp}^{\text{cont}} = \rho \sqrt{Z_V^{hh} Z_V^{ll}} \frac{1}{p_{\pi}^i} \bar{R}_{3,i}^{B \rightarrow K}(t, T). \quad (4.4)$$



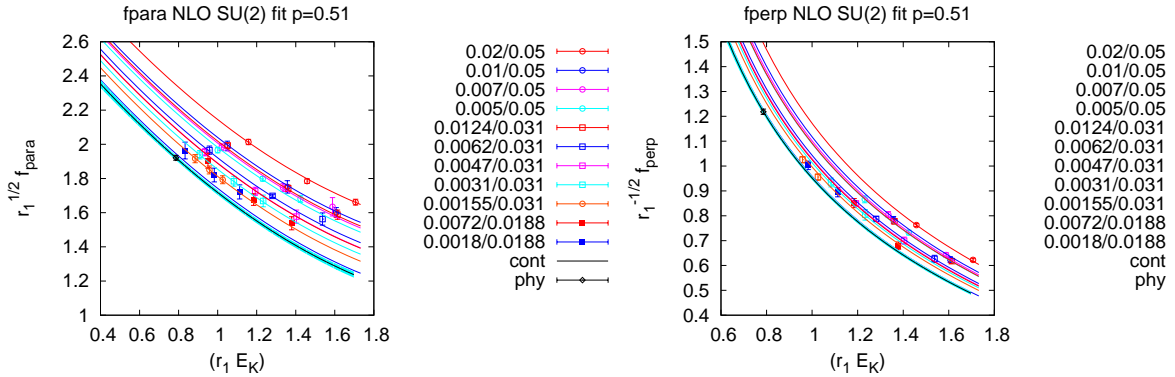
**Figure 1:** Example  $f_{\parallel}$  fit on the coarse ( $a = 0.12$  fm),  $a m_l / a m_s = 0.01 / 0.05$  ensemble. The y-axis is the ratio  $R_{3,0}$  [c.f. Eq. (4.1)] without any renormalization factors.

<sup>1</sup>The calculation of the renormalization constants on the superfine ensembles has not been finished yet. We use the values from the coarse and fine ensembles to estimate these factors on the superfine ensembles. We also set  $\rho$  as 1 for this current preliminary analysis.

Figure 1 shows an example  $f_{\parallel}$  fit on the coarse,  $am_l/am_s=0.01/0.05$  ensemble with  $p = (0,0,0)2\pi/L$ ,  $(1,0,0)2\pi/L$ , and  $(1,1,0)2\pi/L$ . The y-axis is  $R_{3,0}$  without multiplication by the renormalization constants. We fit  $R_{3,0}$  with a constant term plus an exponential decay term. We choose the preferred fit range by fixing the size the fit interval, *i.e.*,  $t_{\max} - t_{\min}$ , but shifting the location of the minimum time slice  $t_{\min}$  to obtain a stable central value and errors and a good  $p$ -value. We plot the the result of the constant term and its error, which is consistent with the single plateau fit method used in Ref. [10].



**Figure 2:**  $f_{\parallel}$  (left panel) and  $f_{\perp}$  (right panel) chiral-continuum extrapolations with NNLO  $SU(3)$   $S\chi$ PT. Partially-quenched points are included in the fits, but are not shown in the figures for clarity. Open circles denote coarse data points, open squares denote fine data, and filled squares denote superfine data. Fit lines should pass through the data points of the corresponding color.



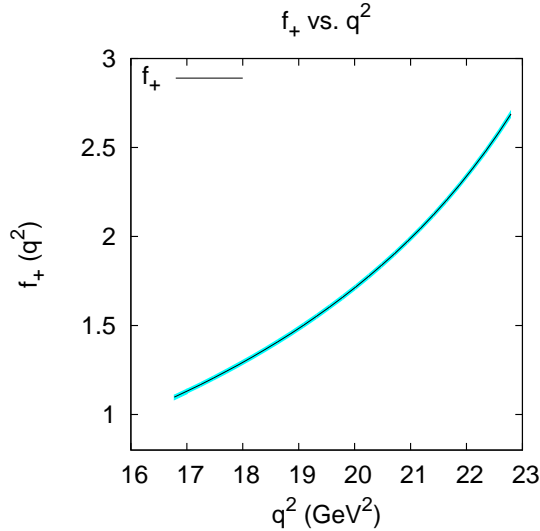
**Figure 3:**  $f_{\parallel}$  (left panel) and  $f_{\perp}$  (right panel) chiral-continuum extrapolations with NLO  $SU(2)$   $S\chi$ PT. Partially-quenched points are included in the fits, but are not shown in the figures for clarity. Open circles denote coarse data points, open squares denote fine data, and filled squares denote superfine data. Fit lines should pass through the data points of the corresponding color. The cyan band shows the continuum-extrapolated form factor at the physical light-quark mass with statistical errors.

To determine the form factors defined in the continuum with physical quark masses, a combined chiral and continuum extrapolations is applied. We use Staggered Chiral Perturbation Theory

( $S\chi$ PT) as the low energy effective theory.  $S\chi$ PT accounts for taste symmetry breaking effects in the staggered quark action. NLO  $SU(3)$   $S\chi$ PT supplemented by NNLO analytic terms was used successfully in  $B \rightarrow \pi$  and  $D \rightarrow \pi$  decays [10, 16], but we find that it fails in the  $f_{\parallel}$  fit for  $B \rightarrow Kl^+l^-$  process. Figure 2 shows the result of the NNLO  $SU(3)$   $S\chi$ PT fit for  $f_{\parallel}$ . The  $SU(3)$  fit gives a low  $p$ -value and incorrect behavior in the small  $E_K$  region, which may indicate the fact that the kaon is too heavy for  $SU(3)$   $S\chi$ PT. On the other hand,  $SU(3)$   $\chi$ PT gives a reasonable extrapolation for  $f_{\perp}$ , since the shape of  $f_{\perp}$  is dominated by the  $B^*$  pole and  $SU(3)$   $S\chi$ PT accounts for it correctly. However, the  $p$ -value is still poor.

Given these difficulties with  $SU(3)$   $S\chi$ PT, we switch to  $SU(2)$   $S\chi$ PT for the chiral-continuum extrapolations. Our  $SU(2)$  formula is inspired by the work of Ref. [11]. We take the heavy  $m_s$  limit of the  $SU(3)$   $S\chi$ PT result [9] to obtain the  $SU(2)$  reduction. Figure 3 shows that  $SU(2)$   $S\chi$ PT works well with our data even at NLO and provides better control in the low  $E_K$  region. The results here are still preliminary. The detailed comparisons between  $SU(3)$  and  $SU(2)$  fits are still under investigation.

After we obtain the continuum extrapolated  $f_{\parallel}$  and  $f_{\perp}$ , we can construct  $f_+$ , which is crucial to the  $B \rightarrow Kl^+l^-$  differential decay rate. The plot of  $f_+$  is given in Fig. 4. Our simulation corresponds to the momentum transfer  $q^2$  from  $16\text{GeV}^2$  to  $23\text{GeV}^2$ . More study of the  $q^2$  dependence of  $f_+$  in the full kinematic range using the  $z$ -expansion will be considered later [17].



**Figure 4:** Continuum extrapolated  $f_+$  for  $B \rightarrow Kl^+l^-$  decay. The width of the band indicates the statistical error; in addition, we estimate that the systematic errors will be  $\sim 5\%$ , depending somewhat on  $q^2$ .

## 5. Summary and Future Plans

We report preliminary results from the study of the  $B \rightarrow Kl^+l^-$  semileptonic decay process with statistical error only. Current chiral-continuum extrapolations are done with NLO  $SU(2)$   $S\chi$ PT. More comparison of the  $SU(3)$  and  $SU(2)$   $S\chi$ PT will be studied. We will work on the systematic errors in the next step. These systematic errors come from the uncertainties in  $r_1$  (used

to set the scale) and physical quark masses, finite volume effect, chiral fits and so on. Finally, the  $B \rightarrow Kl^+l^-$  process contains an additional form factor from the tensor current, which we will analyze in the future.

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