

# Higgs boson resonance parameters and the finite temperature phase transition in a chirally invariant Higgs-Yukawa model

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We study a chirally invariant Higgs-Yukawa model regulated on a space-time lattice. We calculate Higgs boson resonance parameters and mass bounds for various values of the mass of the degenerate fermion doublet. Also, first results on the phase transition temperature are presented. In general, this model may be relevant for BSM scenarios with a heavy fourth generation of quarks.

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The development of a lattice regularization of the Dirac fermion bilinear which respects a chiral symmetry at finite lattice spacing [1] suggests that lattice studies of the Higgs-Yukawa sector be revisited [2,3]. To this end, we examine a model which contains a single complex scalar doublet and two fermion fields, the left-handed components of which are associated into a doublet. The continuum Lagrangian density of our model is therefore

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \bar{t}\not{\partial}t + \bar{b}\not{\partial}b + y\begin{pmatrix} \bar{t} \\ \bar{b} \end{pmatrix}_L^T \phi b_R + y\begin{pmatrix} \bar{t} \\ \bar{b} \end{pmatrix}_L^T i\sigma_2\phi^* t_R + h.c., \quad (1)$$

where  $t$  and  $b$  denote the fermion fields and  $\sigma_2$  is the second Pauli matrix. Note that we have set the Yukawa couplings of both fermion fields to be equal. Additionally, we shall find it convenient to define the parameters  $\kappa$  and  $\hat{\lambda}$  via

$$\lambda = \frac{\hat{\lambda}}{4\kappa^2}, \quad m^2 = \frac{1 - 2\hat{\lambda} - 8\kappa}{\kappa}. \quad (2)$$

At finite lattice spacing, the overlap discretization is used, which provides well-defined chiral projectors to isolate the left and right handed components of the fermion fields. For details on the discretization, see Ref. [4]. Furthermore, it is assumed that we are in the broken phase where the scalar field acquires a non-zero vacuum expectation value  $v$ . Although there is no explicit fermion mass term in the Lagrangian, in the broken phase the fermions acquire a mass ( $m_f$ ) which at leading order in the quartic and Yukawa couplings is proportional to  $y \times v$ . Large Yukawa couplings are therefore required to study the model at large  $m_f$ , where the perturbative expansion may break down.

Due to the lack of gauge fields (the effects of which presumably can be described perturbatively), the complicated Neuberger-Dirac operator is diagonal in momentum space and may therefore be constructed exactly, up to machine precision. The model can then be evaluated efficiently using the Fast Fourier Transform (FFT) algorithm.

To simulate the model, we employ the pHMC algorithm [5], with many improvements [4]. Specifically, Fourier acceleration [6, 7] is necessary to reduce the autocorrelation times of low-momentum modes of the Higgs and fermion propagators, and several types of preconditioning have been used to reduce the condition number of the Dirac operator.

We set the scale in our theory via the renormalized vacuum expectation value  $v_R$  of the scalar field, which is non-zero in the broken phase. In the standard model, this can be expressed as

$$v_R = \frac{v}{\sqrt{Z_G}} = \frac{2M_W}{g} = 246\text{GeV}, \quad (3)$$

where  $g$  is the weak gauge coupling and  $Z_G$  is the renormalization constant for the Goldstone boson propagator determined by the condition

$$Z_G^{-1} = \frac{d}{dp^2} \text{Re} (G_G^{-1}(p^2)) \Big|_{p^2=m_G^2}. \quad (4)$$

In practice  $Z_G$  is determined from fits to the Goldstone propagator.

The lattice cutoff in our theory cannot be removed while maintaining non-zero interactions. This is the well known triviality (i.e. existence of a Landau pole in perturbation theory) of the  $\phi^4$

theory. There is some evidence from the large- $N_F$  expansion [2] that the theory with fermions is also trivial.

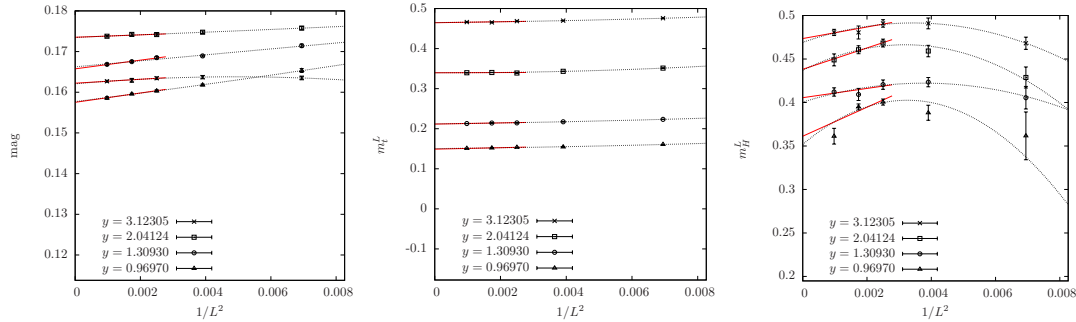
This model has been used to study the triviality and vacuum instability bounds of the Higgs boson at  $m_f = m_t = 175\text{GeV}$  [8, 9] as well as the phase structure of the theory [3, 10]. It has been shown that in the  $\phi^4$  theory the Higgs boson mass is an increasing function of the bare quartic coupling  $\lambda$  at a fixed value of the cutoff [11]. Furthermore, the quartic coupling must be  $\geq 0$  to preserve the stability of the theory. It was found for the model considered here [8] that the maximum value of the Higgs boson mass occurs at  $\lambda = \infty$  while the minimum occurs at  $\lambda = 0$ .

The Higgs mass is determined by the pole position of the Higgs boson propagator

$$\text{Re} (G_H^{-1}(p^2)) \Big|_{p^2 = -M_H^2} = 0. \quad (5)$$

We determine the pole position by fitting the propagator to an ansatz which neglects the finite width of the Higgs to decay into Goldstone bosons. The Higgs mass can also be determined from the temporal correlation function of two suitable interpolating fields.

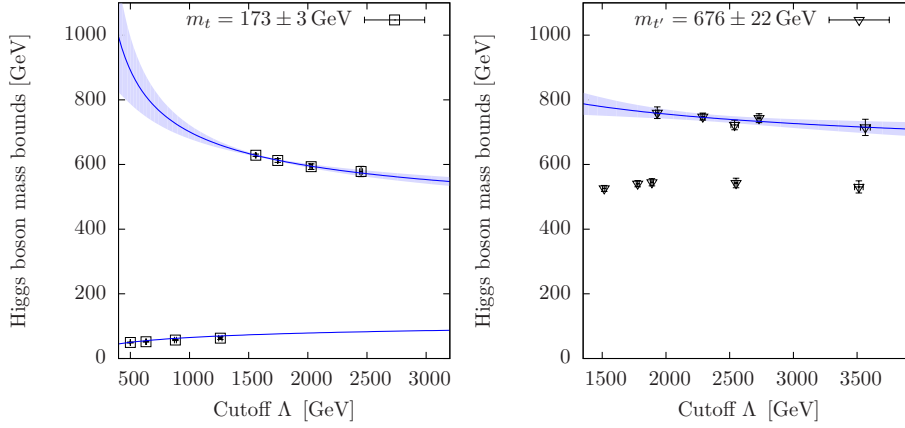
Particular attention must be paid to finite volume effects in the model. Due to the lack of an explicit symmetry breaking parameter in our Lagrangian, we are in the so-called  $\varepsilon$ -regime, rather than the  $p$ -regime where most lattice QCD simulations are performed. In the  $\varepsilon$ -regime finite volume effects decrease with  $L^{-2}$  rather than  $e^{-ML}$  for the  $p$ -regime [12, 13]. Due to these algebraic finite size effects, an infinite volume extrapolation of the data is necessary. An example of such an extrapolation is shown in Fig. 1.



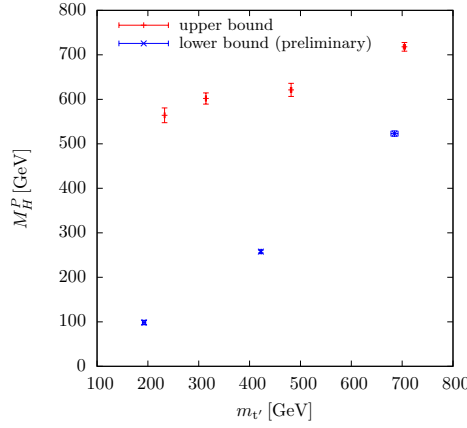
**Figure 1:** Finite size effects in (from left to right) the renormalized vacuum expectation value, fermion mass, and Higgs Boson mass at  $\Lambda = 1/a = 1.5\text{TeV}$  and  $\lambda = \infty$ . Data are shown for several values of the bare Yukawa coupling corresponding to fermion masses in the range  $m_f = 200\dots 700\text{GeV}$ . While the renormalized vacuum expectation value exhibits asymptotic behavior already at rather small lattice volumes, for a reliable infinite volume extrapolation of the Higgs boson mass, larger volumes are required.

This procedure was first carried out for the situation  $m_f = m_t = 175\text{GeV}$  [8] and repeated for  $m_f = 676\text{GeV}$  [14] for various values of the lattice cutoff. Results from these two calculations are shown in Fig. 2. We now report on additional results from a scan in  $m_f$  at fixed cutoff  $\Lambda = 1/a = 1.5\text{TeV}$ . These results are shown in Fig. 3 and should be regarded as preliminary, as an infinite volume extrapolation for the lower bound has not yet been performed.

As we discussed above, the method for determining the Higgs boson mass neglects its finite decay width into Goldstone bosons. Here we report on a calculation of the resonance parameters



**Figure 2:** The cutoff dependence of the upper and lower Higgs boson mass bounds for the physical top quark mass  $m_f = m_t = 175\text{GeV}$  (left) and for  $m_f = m_{t'} = 676\text{GeV}$  (right). All data have been extrapolated to infinite volume.



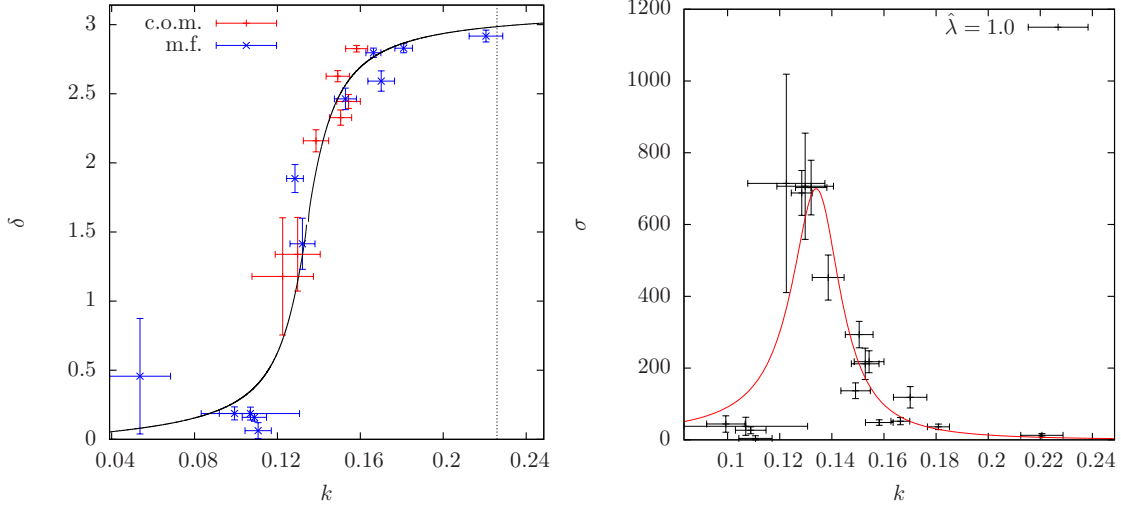
**Figure 3:** Higgs boson mass bounds at  $\Lambda = 1/a = 1.5\text{TeV}$  as a function of  $m_f = m_{t'}$ . The data for the upper bound has been extrapolated to infinite volume, while for the lower bound the results were computed on a single  $(24^3 \times 48)$  volume and are therefore preliminary.

of the Higgs boson [15], demonstrating that at  $m_f = m_t$  the mass obtained by a resonance fit to the scattering phase shift is statistically equivalent to the pole mass and correlator mass.

Infinite volume elastic scattering phase shifts may be extracted from finite volume lattice data by examining the dependence of the energy eigenvalues near elastic thresholds [16]. As this technique is valid only for elastic scattering, we must add a small explicit symmetry breaking term to the Lagrangian, which generates a finite Goldstone boson mass  $M_G$ . The magnitude of the symmetry breaking parameter is chosen such that  $M_G \approx M_H/3$ . Accordingly, our results for the scattering phase shift are confined to the region  $k < 2M_G$ .

To extract energy eigenvalues of the lattice Hamiltonian, a matrix of correlation functions must be constructed and the corresponding generalized eigenvalue problem (GEVP) must be solved [16, 17]. The momentum resolution of the scattering phase shifts is increased by examining the system at rest and in a moving frame [18]. Results are shown in Fig. 4 and Tab. 1. From this analysis we can see that at least for  $m_f = m_t$  and  $M_G \approx M_H/3$ , the Higgs boson has a relatively narrow width

to decay into two Goldstones and determinations of the Higgs boson mass that neglect this decay width are consistent with the full resonance analysis.



**Figure 4:** The scattering phases (left) and the total cross section (right) for  $\Lambda \approx 1.5\text{TeV}$  at  $m_f = m_t = 175\text{GeV}$  and  $\hat{\lambda} = 1.0$ . These results are from various lattice volumes:  $L_s/a \times 40$  with  $L_s/a = 12, 16, 18, 20, 24, 32, 40$ . Points obtained using both the center of mass frame (c.o.m) and moving frame (m.f.) are shown. Adding the moving frame is crucial to obtain a reasonable momentum resolution. The vertical line in the left plot denotes the inelastic threshold.

$\hat{\lambda}$	Cutoff $\Lambda=1/a$	$aM_H^R$	$a\Gamma_H^R$	$aM_H^P$
0.01	883(1) GeV	0.2811(6)	0.007(1)	0.278(2)
1.0	1503(5) GeV	0.374(4)	0.033(4)	0.386(28)
$\infty$	1598(2) GeV	0.411(3)	0.040(4)	0.405(4)

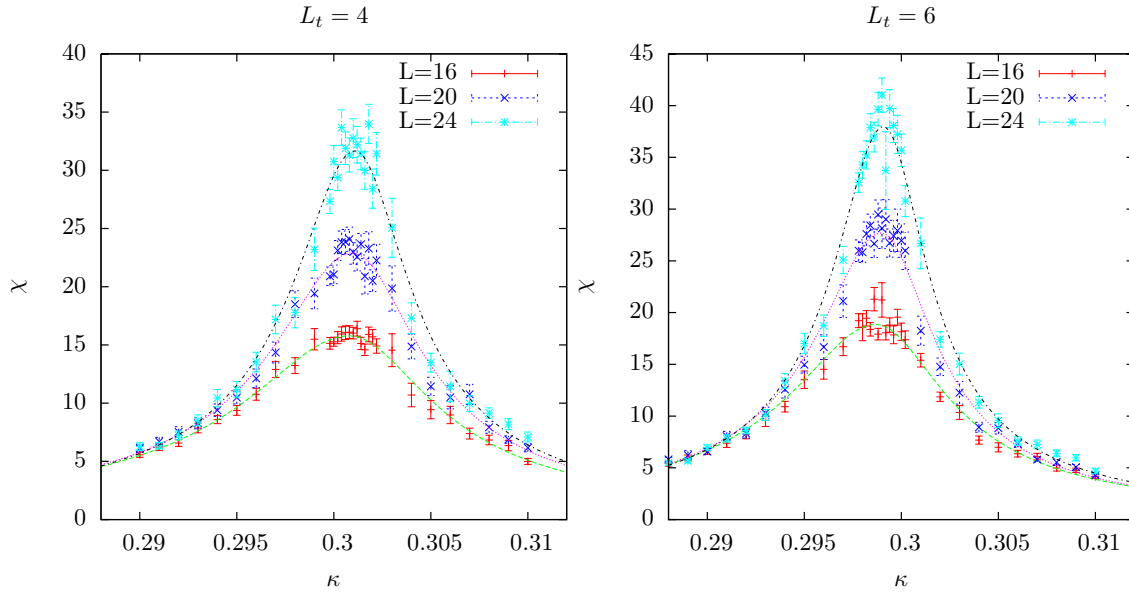
**Table 1:** The resonance mass  $M_H^R$  of the Higgs boson together with the resonance width and the mass extracted from the propagator  $M_H^P$  at  $m_f = m_t = 175\text{GeV}$ . For all three values of the bare quartic coupling, the width is less than 10% of the resonance mass.

Another physical observable that may be studied in this model is the finite temperature phase transition from the symmetric phase to the broken phase, which may have implications for electroweak baryogenesis [19]. This phase transition is known to be second order in pure  $\phi^4$  theory (see e.g. Ref. [20]). However, when  $SU(2)$  gauge fields are added, the transition becomes first order as in Ref. [21].

The temperature in lattice field theory is changed by varying the temporal extent of the lattice, with periodic (anti-periodic) temporal boundary conditions for the boson (fermion) fields. The phase transition temperature is determined by first choosing a temporal extent  $L_t/a$  in lattice units and performing a scan in  $\kappa$  at fixed  $\lambda = \infty$  to determine the peak in the susceptibility.

The value of  $\kappa$  at which the susceptibility peaks is then used to perform a zero temperature ( $L_t = \infty$ ) simulation to set the scale in the usual way, from the zero temperature vacuum expectation value. From this scale we can determine  $T = 1/L_t$  in physical units. Results for the susceptibility and  $L_t/a = 4, 6$  are shown in Fig. 5. The preliminary value of the phase transition temperature is

similar at these two values of the lattice spacing and is  $\approx 500\text{GeV}$  in comparison to  $\approx 350\text{GeV}$  in the pure  $\phi^4$  theory.



**Figure 5:** The susceptibility as a function of  $\kappa$  for  $m_f = m_t = 175\text{GeV}$ . The data shown is for  $\lambda = \infty$  and should be considered preliminary. The left plot corresponds to  $L_t/a = 4$  while the right to  $L_t/a = 6$ . The dotted lines are from fits to finite size and critical scaling ansatz  $\chi = A(L_s^{-2/\nu} + B(\kappa - \kappa_c)^2)^{-\gamma/2}$  where, due to limited (preliminary) statistics, we have fixed  $\gamma = 1.38$  and  $\nu = 0.68$ . The critical temperature determined from the  $L_t/a = 4$  lattices is  $T_c = 514(15)\text{GeV}$  while from the  $L_t/a = 6$  lattices it is  $T_c = 491(24)\text{GeV}$ .

In conclusion, studies of several physical quantities in our chirally-invariant Higgs-Yukawa model are underway. The dependence of the Higgs boson mass bounds on  $m_f$  and  $a$  has shown that the lower bound seems to be sensitive to the mass of the fermion doublet, but rather mildly sensitive to the value of the lattice cutoff. We will complete an  $m_f$ -scan of these bounds in the near future. We also plan to repeat the determination of the Higgs boson resonance parameters at several values of  $m_f$ . Additionally, it may be interesting to look for bound states of Goldstone bosons and quarks. The finite temperature phase transition in our model may have implications for electroweak baryogenesis. We plan to complete this analysis at several values of  $m_f$  and  $a$ .

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