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Chiral expansion for lattice computations of $B^+ \to D^0 K^+(\pi^+)$ and $B^+ \to \overline{D}^0 K^+(\pi^+)$ amplitudes

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In this work, we suggest that hard-pion chiral perturbation theory may be applicable to the real parts of nonleptonic $B^+ \to D^0 P^+$ and $B^+ \to \overline{D}^0 P^+$ $(P = K, \pi)$ decay amplitudes. These amplitudes play an important role in the extraction of the angle γ in the $b-d$ unitarity triangle of the CKM matrix, and their real parts can be computed using lattice QCD. We construct the leading-order operator in the chiral expansion for these nonleptonic decays, and discuss the generic features of the next-to-leading-order terms.

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1. Introduction

Nonleptonic B decays have played an important role in CKM physics. In particular, the angle γ in the $b-d$ unitarity triangle can in principle be determined with high precision from the study of charged B meson decays processes,

$$B^+ \to D^0 P^+ \to f P^+ \text{ and } B^+ \to \overline{D}^0 P^+ \to f P^+, \tag{1.1}$$

with the same final state fP^+ [1, 2, 3, 4]. In all the methods for extracting γ from the above decay channels, the knowledge of the branching ratios, $Br[B^+ \to D^0 P^+]$ and $Br[B^+ \to \overline{D}^0 P^+]$, is a key ingredient. While $Br[B^+ \to D^0 P^+]$ has been experimentally measured with good accuracy, $Br[B^+ \to \overline{D}^0 P^+]$ is very difficult to obtain. For this reason, despite the large statistics of the two B-factories [$\sim \mathcal{O}(10^9)$ charged B meson samples], γ is presently determined to only $\sim \mathcal{O}(25\%)$. This is compared with about 3% for β, and about 5% for α. To further improve the precision on γ, inputs from lattice QCD (LQCD) to these branching ratios will be very helpful

The study of hadronic weak decays on the lattice is very challenging, because of the Maiani-Testa no-go theorem (MTNGT) [5]. This theorem states that by using Euclidean four-point correlators to study nonleptonic two-body decays, it is impossible to obtain information about the strong phases. Therefore one can only compute the real parts of nonleptonic decay amplitudes from such correlators. For the calculation of $K \to \pi\pi$ on the lattice, one can avoid the MTNGT using the Lellouch-Lüscher (LL) method [6]. On the other hand, the lattice computation of nonleptonic B decays remains challenging, because the LL method is only applicable to processes involving elastic final-state scatterings. Nevertheless, lattice results for the real part of these amplitudes could provide valuable information of $Br[B^+ \to D^0 P^+]$ and $Br[B^+ \to \overline{D}^0 P^+]$, and help in the extraction of γ.

In the near future, lattice computations for such four-point functions are unlikely to be preformed exactly at physical pion mass. Thus, it is important to rely on the chiral expansion to perform extrapolations to the physical point. In Ref. [15], we examine the possibility of the chiral expansion of the real parts of $B^+ \to D^0 P^+$ and $B^+ \to \overline{D}^0 P^+$ amplitudes, in the framework of heavy-meson chiral perturbation theory (HMχPT). [7, 8, 9, 10, 11]. The straightforward application of the chiral expansion for $B \to DP$ amplitudes near the physical kinematics is questionable, since the final-state hadrons carry momenta ~ 2 GeV. From the structure of subleading terms in $1/M_D$ (M_D is the D meson mass) in the effective theory [12, 13, 14], it is clear that the large momentum carried by the D meson will only lead to (significant) dependence on M_D in the low-energy constants (LEC's). What remains to be considered is the convergence of the chiral expansion for hard final-state hadrons from lattice studies.

In Ref. [15], we address this issue in nonleptonic B decays, in a framework that is similar to the hard-pion chiral perturbation theory (HPχPT) which was first proposed and applied to the analysis of $K_{\ell 3}$ decays in Ref. [16]. This approach was also used to carried out investigations of $K \to 2\pi$ amplitudes [17][1] , as well as semi-leptonic B-decays [18].

[1] As pointed out in Ref. [17], hard-pion χPT may not be used to estimate the strong phases in $K \to \pi\pi$ processes. We will comment on this issue for nonleptonic B decays in Sec. 4.

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The relevant current-current, $\Delta b = 1$, operators (α, β are colour indices) in our work are,

$$Q_1^{b\to c,i} = (\overline{q}^i_\alpha \gamma^\mu (1-\gamma_5) b_\alpha)(\overline{c}_\beta \gamma_\mu (1-\gamma_5) u_\beta) \,, \tag{1.2}$$
$$Q_2^{b\to c,i} = (\overline{q}^i_\alpha \gamma^\mu (1-\gamma_5) b_\beta)(\overline{c}_\beta \gamma_\mu (1-\gamma_5) u_\alpha) \,, \tag{1.3}$$
$$Q_1^{b\to \overline{c},i} = (\overline{q}^i_\alpha \gamma^\mu (1-\gamma_5) b_\alpha)(\overline{u}_\beta \gamma_\mu (1-\gamma_5) c_\beta) \,, \tag{1.4}$$
$$Q_2^{b\to \overline{c},i} = (\overline{q}^i_\alpha \gamma^\mu (1-\gamma_5) b_\beta)(\overline{u}_\beta \gamma_\mu (1-\gamma_5) c_\alpha) \,. \tag{1.5}$$

We will focus on the nonleptonic decays with the underlying processes $b \to c\overline{u}d$, $b \to c\overline{u}s$, $b \to u\overline{c}d$, and $b \to u\overline{c}s$. The first two will be mapped onto different operators in the chiral effective theory from the last two, as explained in the next section.

2. The leading-order chiral expansion for $B \to DK(\pi)$ amplitudes

In weak nonleptonic B decays, the final-state hadrons carry large momenta. This makes it difficult to apply the chiral expansion to such processes. On the other hand, it has been recently proposed that chiral perturbation theory (χPT) can be valid for amplitudes containing hard final-state particles [16, 17, 18, 19]. One crucial point is to allow the low-energy constants (LECs) to depend on the hard momentum scales which result from either the kinematics or the mass of the external particles. That is, the LECs in the chiral expansion are no longer universal quantities. Another important ingredient in such chiral expansions is the separation of the hard and soft scales. This separation of scales is made possible because of the structure of derivative couplings in χPT, leading to the absorption of the hard, external, momenta into the LECs. We will discuss this procedure explicitly for $B \to DP$ amplitudes with an example diagram in Sec. 4.

In this section, we construct the χPT weak operators corresponding to those in Eqs. (1.2)–(1.5). Neglecting the colour indices which do not play a role in χPT, these operators can be written as

$$Q^{b\to c,i} = \left(\overline{q}^i_L \,\Gamma_1\, b\right)\left(\overline{c}\,\Gamma_2\, u_L\right),$$

$$Q^{b\to \overline{c},i} = \left(\overline{q}^i_L \,\overline{\Gamma}_1\, b\right)\left(\overline{u}_L\,\overline{\Gamma}_2\, c\right), \tag{2.1}$$

where $q^i = d$ or s, and

$$q_L = \left(\frac{1-\gamma_5}{2}\right) q;\;\; \Gamma_1 = \Gamma_2 = \overline{\Gamma}_1 = \overline{\Gamma}_2 = \gamma_\mu(1-\gamma_5). \tag{2.2}$$

Under the $\mathrm{SU}(3)_\mathrm{L} \otimes \mathrm{SU}(3)_\mathrm{R}$ chiral symmetry group, $Q^{b\to c,i}$ is in the $(\mathbf{8_L}, \mathbf{1_R})$ representation, while $Q^{b\to \overline{c},i}$ is in the $(\overline{\mathbf{6}}_\mathbf{L}, \mathbf{1_R})$ representation. To bosonise these operators, we promote $\Gamma_{1,2}$ and $\overline{\Gamma}_{1,2}$ to be spurion fields which transform as

$$\Gamma_1 \to L\,\Gamma_1\, S^\dagger,\;\; \Gamma_2 \to S\,\Gamma_2\, L^\dagger,\;\; \overline{\Gamma}_1 \to L\,\overline{\Gamma}_1\, S^\dagger,\;\; \overline{\Gamma}_2 \to L\,\overline{\Gamma}_2\, S^\dagger, \tag{2.3}$$

under the heavy-quark spin/flavour and chiral rotations. This renders the operators in Eq. (2.1) invariant with respect to such transformations. Using this property, and taking into account all the possible insertions of Dirac structures [20], we obtain the leading-order (LO) operators,

$$\begin{aligned}
\mathscr{O}_{\chi,i} = & \quad \left[\beta_1 + (\beta_1+\beta_2)\,(v'\cdot v)\right]\left[\left(\sigma_{1k}\mathscr{P}_k^{(c)\dagger}\right)\left(\mathscr{P}_l^{(b)}\sigma_{li}^\dagger\right)\right] \\
& + \left[(\beta_1-\beta_2)\,v'^\mu - \beta_1 v^\mu\right]\left[\left(\sigma_{1k}\mathscr{P}_k^{(c)\dagger}\right)\left(\mathscr{V}_{\mu,l}^{*(b)}\sigma_{li}^\dagger\right)\right] \\
& + \left[\beta_1 v'^\mu - (\beta_1+\beta_2)\,v^\mu\right]\left[\left(\sigma_{1k}\mathscr{V}_{\mu,k}^{*(c)\dagger}\right)\left(\mathscr{P}_l^{(b)}\sigma_{li}^\dagger\right)\right] \\
& - 4\left[(\beta_1-\beta_2)+\beta_1(v'\cdot v)\right]\left[\left(\sigma_{1k}\mathscr{V}_{\mu,k}^{*(c)\dagger}\right)\left(\mathscr{V}_l^{*(b)\mu}\sigma_{li}^\dagger\right)\right], \\
\overline{\mathscr{O}}_{\chi,i} = & \quad \left[\overline{\beta}_1 + \overline{\beta}_2(v'\cdot v)\right]\left[\left(\mathscr{P}_k^{(\overline{c})\dagger}\sigma_{k1}^\dagger\right)\left(\mathscr{P}_l^{(b)}\sigma_{li}^\dagger\right)\right] \\
& - \left[\overline{\beta}_2 v'^\mu - \left(\overline{\beta}_1+\overline{\beta}_5\right)v^\mu - \overline{\beta}_3(v'\cdot v)v^\mu\right]\left[\left(\mathscr{P}_k^{(\overline{c})\dagger}\sigma_{k1}^\dagger\right)\left(\mathscr{V}_{\mu,l}^{*(b)}\sigma_{li}^\dagger\right)\right] \\
& + \left[\overline{\beta}_1 v'^\mu - \overline{\beta}_2 v^\mu\right]\left[\left(\mathscr{V}_{\mu,k}^{*(\overline{c})\dagger}\sigma_{k1}^\dagger\right)\left(\mathscr{P}_l^{(b)}\sigma_{li}^\dagger\right)\right] \\
& + \left[4\overline{\beta}_2 - \overline{\beta}_3 - 2\left(\overline{\beta}_1+\overline{\beta}_4+\overline{\beta}_5\right)(v'\cdot v)\right]\left[\left(\mathscr{V}_{\mu,k}^{*(\overline{c})\dagger}\sigma_{k1}^\dagger\right)\left(\mathscr{V}_l^{*(b)\mu}\sigma_{li}^\dagger\right)\right],
\end{aligned} \tag{2.4}$$

where β_i and $\overline{\beta}_i$ are LECs, v and v' are the velocities of B and D mesons, $\sigma = \sqrt{\Sigma} = \exp(i\Phi/f)$ with Φ and Σ being the standard linear and nonlinear Goldstone fields, $\mathscr{P}^{(h)}$ and $\mathscr{V}^{*(h)}$ are the annihilation fields for the pseudoscalar and vector mesons containing the heavy quark h. At the LO,

$$\begin{aligned}
\langle D^0 K^-|\mathscr{O}_{\chi,s}|B^-\rangle &= \langle D^0\pi^-|\mathscr{O}_{\chi,d}|B^-\rangle = \frac{i}{f}\langle D^-|\mathscr{O}_{\chi,s}|B^-\rangle, \\
\langle \overline{D}^0 K^-|\overline{\mathscr{O}}_{\chi,s}|B^-\rangle &= \langle \overline{D}^0\pi^-|\overline{\mathscr{O}}_{\chi,d}|B^-\rangle = \frac{i}{f}\langle D^-|\overline{\mathscr{O}}_{\chi,s}|B^-\rangle\ .
\end{aligned} \tag{2.5}$$

This is not valid at the next-to-leading-order (NLO) chiral expansion.

3. Resonance contributions

In this section we investigate resonance contribution to $B \to DP$ correlators (in the time-momentum representation) and amplitudes[2]. The vector heavy-light resonances are already incorporated in HMχPT. The inclusion of heavier resonances in the effective theory is beyond the scope of this work. Because of the complications in formulating HMχPT in Euclidean space, and here we are only studying the generic feature of the resonance contribution, we work in Minkowski space.

[2]The conclusion presented in this section is also valid for $B \to \overline{D}P$ decays.

Figure 1: Tree-level correlators contributing to $B \to DP$. The box is the weak operator, with (a) being the direct $B \to DP$ term and (b) being the term with an intermediate resonance (here a D^*).

First, we calculate the LO correlator in Fig. 1(a). We perform a Fourier transform for the spatial directions for each of the external-source points, while fixing the location of the weak operator (the square in the diagram) to be at the origin. As in Sec. 2, we denote the velocity of B and B^* by v and that of D and D^* by v'. For simplicity, we choose the frame in which $v = \left(1, \vec{0}\right)$, and implement the time-ordering as $t_B < 0 < t_D \leq t_P$, where $t_{B,D,P}$ are the temporal locations of the sources for B, D and P mesons. Using the the weak operators in Eq. (2.4), the result of this diagram is

$$C_{\rm LO} = \frac{g_{BDP}}{f}\left(\frac{1}{2}\right)\left(\frac{\mathrm{e}^{-i\overline{\delta}_D t_D}}{2v_0'}\right)\left(\frac{\mathrm{e}^{-i\omega_P t_P}}{2\omega_P}\right), \tag{3.1}$$

where $\overline{\delta}_D = \vec{v}' \cdot \vec{p}_D$, and $\omega_P = \sqrt{M_P^2 + \vec{p}_P^{\,2}}$, with $\vec{p}_D$ and $\vec{p}_P$ denoting the spatial momenta of the D and the P mesons. The coupling g_{BDP} is one of the linear combinations of the LEC's β_i in Eq. (2.4). Next, we compute the diagram in Fig. 1(b), which leads to the result

$$C_{\rm res} = \frac{g_{BD^*} g_\pi}{f^2}\left(\frac{1}{2}\right)\left(\frac{\mathrm{e}^{-i\overline{\delta}_D t_D}}{2v_0'}\right)\left(\frac{\mathrm{e}^{-i\omega_P t_P}}{2\omega_P}\right)\left[\frac{\mathrm{e}^{i(\omega_P + \overline{\delta}_D - \overline{\Delta}_{DP})t_D} - 1}{2v_0'(\omega_P + \overline{\delta}_D - \overline{\Delta}_{DP})}\right], \tag{3.2}$$

where $\overline{\Delta}_{DP} = \vec{v}' \cdot (\vec{p}_D + \vec{p}_\pi) + \Delta_D / v_0'$, with Δ_D denoting the $D^* - D$ mass splitting resulting from the heavy-quark spin symmetry breaking effects. The coupling g_π is the LO $B^* - B - \pi$ axial coupling in the HMχPT Lagrangian, and g_{BD^*} is a linear combination of the LEC's β_i in Eq. (2.4). Notice that g_{BD^*} is different from g_{BDP} and thus the resonance contribution results in general in an additional unknown parameter for $B \to DP$ amplitude at the tree level. When the final-state momenta are tuned such that the resonance is on-shell, the factor in the square brackets in Eq. (3.2) becomes linear in t_D. This can be interpreted as an energy shift of the final state. In any case, the t_D dependence in these square brackets will be cancelled when taking the ration between $C_{\rm LO} + C_{\rm res}$ and the square root of the $DP \to DP$ correlator to obtain the $B \to DK$ matrix element.

4. Hard-pion chiral perturbation theory at one-loop

To account for nonanalytic dependence on m_π in $B \to DK$ amplitudes, it is necessary to go beyond the tree-level in the chiral expansion. In order to treat these processes in the physical regime, we use the methods of Refs. [16, 17, 18]: Hard-pion χPT (HPχPT), in which one key point is the separation of the hard external momenta with the soft scales (the Goldstone masses) which appear

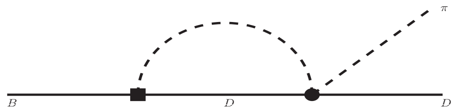

Figure 2: One of the many one-loop diagrams that contribute to $B \to D\pi$, specifically one which shows the essential features that arise in HPχPT.

in the loop. This is acheivable because of the structure of the derivative couplings. Focusing on the SU(2) χPT, the chiral logarithms for $B \to DK$ amplitudes in HPχPT takes the generic form,

$$\mathscr{M} = \mathscr{M}^{\text{tree}} \left[1 + a \frac{m_\pi^2}{16\pi^2 f^2} \ln \left(\frac{m_\pi^2}{\Lambda^2} \right) + L m_\pi^2 \right] , \tag{4.1}$$

where $\mathscr{M}^{\text{tree}}$ is its LO value. a is a coefficient that depends on the particular kinematics chosen for the diagram, and L is a linear combination of low-energy constants as well as terms arising from higher-order chiral-level weak operators. The coefficient a would be determined from evaluating the one-loop corrections to the amplitude. Both a and L depend on all of the hard quantities (such as the momenta of the external D meson and pion).

In order to understand the specific details, we examine the diagram in Fig. 2, through which we will demonstrate the dependence on the hard scales in the coefficient of the chiral logarithm. In this diagram, as in lattice calculations, momentum will be conserved at the strong vertex, but need not be at the weak operator. This momentum insertion, p_{wk}, is related to the momenta of the external mesons via $p_B + p_{\text{wk}} = p_D + p_\pi$. Denoting the residual momentum of the external D by k, and the internal pion momentum by l, the integral that needs to be computed for this diagram is

$$I = \int \frac{d^4\ell}{(2\pi)^4} \frac{i}{\ell^2 - m_\pi^2 + i\varepsilon} \frac{i v' \cdot (\ell - p_\pi)}{v' \cdot (\ell - k - p_\pi) - \Delta + i\varepsilon} , \tag{4.2}$$

where v' is the velocity of the D mesons, and $\Delta = m_D - m_B$ is the D-B meson mass splitting. This integral can be evaluated using dimensional regularisation,

$$I = \frac{1}{16\pi^2 f^2} \left[\frac{v' \cdot k + \Delta}{v' \cdot (k + p_\pi) + \Delta + i\varepsilon} I_2(m_\pi, v' \cdot (k + p_\pi) + \Delta + i\varepsilon) - m_\pi^2 \ln \left(\frac{m_\pi^2}{\Lambda^2} \right) \right] , \tag{4.3}$$

where I_2 takes the form in the hard-pion limit $v' \cdot k \gg m_\pi$,

$$I_2(m_\pi, v' \cdot (k + p_\pi) + \Delta) \approx -m_\pi^2 \ln \left(\frac{m_\pi^2}{\Lambda^2} \right) , \tag{4.4}$$

so that the full coefficient a [as defined in Eq. (4.1)] in the integral I is -2 when we insert momentum into the weak vertex such that $p_\pi \approx 0$, and is $-3/2$ when we choose p_{wk} such that $p_\pi \approx k$.

Here we comment that the imaginary part in this diagram is proportional to $\sqrt{[v' \cdot (p_\pi + k)]^2 - m_\pi^2}$, therefore grows with the final-state momenta, leading to the failure of the chiral expansion when p_D and p_π are large. This can be understood by noting that the imaginary part arises from the contribution in which both mesons in the loop are on-shell, and therefore cannot be soft.

5. Summary

We studied the chiral expansion for lattice computations of the real parts of the $B^+ \to D^0 P^+$ and $B^+ \to \overline{D}^0 P^+$, which are important inputs in the determination of the angle γ in the $b-d$ untarity triangle in the CKM matrix. The calculation of these real parts are not obstacled by the MTNGT. We derived the LO operators for these matrix elements in HMχPT, argue that HPχPT is applicable to these computations, and investigate some generic features of the NLO contributions.

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