η and η' meson masses from $N_f = 2+1+1$ twisted mass lattice QCD

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We determine mass and flavour content of η and η' states using $N_f = 2+1+1$ Wilson twisted mass lattice QCD. We describe how those flavour singlet states need to be treated in this lattice formulation. Results are presented for two values of the lattice spacing, $a \approx 0.08$ fm and $a \approx 0.09$ fm, with a range of light quark masses corresponding to values of the pion mass from 270 to 500 MeV and fixed bare strange and charm quark mass values.

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ensemble	β	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	L/a
A40.24	1.90	0.0040	0.150	0.190	24
A60.24	1.90	0.0060	0.150	0.190	24
A80.24	1.90	0.0080	0.150	0.190	24
A80.24s	1.90	0.0080	0.150	0.197	24
B25.32	1.95	0.0025	0.135	0.170	32
B35.32	1.95	0.0035	0.135	0.170	32
B85.24	1.95	0.0085	0.135	0.170	24

Table 1: The ensembles used in this investigation. The notation of ref. [3] is used for labeling the ensembles.

1. Introduction

From experiment it is known that the masses of the nine light pseudo-scalar mesons show an interesting pattern. Taking the quark model point of view, the three lightest mesons, the pions, contain only the two lightest quark flavours, the *up-* and *down-*quarks. The pion triplet has a mass of $M_\pi \approx 140$ MeV. For the other six, the *strange* quark contributes also, and hence they are heavier. In contrast to what one might expect five of them, the four kaons and the η meson, have roughly equal mass around 500 to 600 MeV, while the last one, the η' meson, is much heavier, with mass of about 1 GeV. On the QCD level, the reason for this pattern is thought to be the breaking of the $U_A(1)$ symmetry by quantum effects. The η' meson is, even in a world with three massless quarks, not a Goldstone boson. In this proceeding contribution, we discuss the determination of η and η' meson masses using twisted mass lattice QCD (tmLQCD) with $N_f = 2+1+1$ dynamical quark flavours. This will not only allow a study of the dependence of the η, η' masses on the light quark mass value, but also an investigation of the charm quark contribution to both of these states. Moreover, the η_c meson mass can be studied in principle. For recent lattice studies in $N_f = 2+1$ flavour QCD see [1, 2].

2. Lattice Action

We use gauge configurations as produced by the European Twisted Mass Collaboration (ETMC) with $N_f = 2+1+1$ flavours of Wilson twisted mass quarks and Iwasaki gauge action [3, 4]. The details are described in ref. [3] and the ensembles used in this investigation are summarised in table 1. The twisted mass Dirac operator in the light – i.e. up/down – sector reads [5]

$$D_\ell = D_W + m_0 + i\mu_\ell \gamma_5 \tau^3 \tag{2.1}$$

and in the strange/charm sector [6]

$$D_h = D_W + m_0 + i\mu_\sigma \gamma_5 \tau^1 + \mu_\delta \tau^3, \tag{2.2}$$

where D_W is the Wilson Dirac operator. The value of m_0 was tuned to its critical value as discussed in refs. [7, 3] in order to realise automatic $\mathcal{O}(a)$ improvement at maximal twist [8]. Note that the bare twisted masses $\mu_{\sigma,\delta}$ are related to the bare strange and charm quark masses via the relation

$$m_{c,s} = \mu_\sigma \pm (Z_P/Z_S)\,\mu_\delta \tag{2.3}$$

2

with pseudo-scalar and scalar renormalisation constants Z_P and Z_S. Quark fields in the twisted basis are denoted by $\chi_{\ell,h}$ and in the physical basis by $\psi_{\ell,h}$. They are related via the axial rotations

$$\chi_\ell = e^{i\pi\gamma_5\tau^3/4}\psi_\ell, \quad \bar{\chi}_\ell = \bar{\psi}_\ell\, e^{i\pi\gamma_5\tau^3/4}, \qquad \chi_h = e^{i\pi\gamma_5\tau^1/4}\psi_h, \quad \bar{\chi}_h = \bar{\psi}_h\, e^{i\pi\gamma_5\tau^1/4}. \tag{2.4}$$

With automatic $\mathscr{O}(a)$ improvement being the biggest advantage of tmLQCD at maximal twist, the downside is that flavour symmetry is broken at finite values of the lattice spacing. This was shown to affect mainly the mass value of the neutral pion mass [9, 10, 11], however, in the case of $N_f = 2 + 1 + 1$ dynamical quarks, it implies the complication of mixing between strange and charm quarks.

3. Flavour Singlet Pseudo-Scalar Mesons in $N_f = 2 + 1 + 1$ tmLQCD

In order to compute masses of pseudo-scalar flavour singlet mesons we have to include light, strange and charm contributions to build the appropriate correlation functions. In the light sector, one appropriate operator is given by [12]

$$\frac{1}{\sqrt{2}}(\psi_u i\gamma_5 \psi_u + \psi_d i\gamma_5 \psi_d) \quad \rightarrow \quad \frac{1}{\sqrt{2}}(\bar{\chi}_u \chi_u - \bar{\chi}_d \chi_d) \equiv \ell. \tag{3.1}$$

In the strange and charm sector, the corresponding operator reads

$$\begin{pmatrix} \bar{\psi}_c \\ \bar{\psi}_s \end{pmatrix} i\gamma_5 \frac{1 \pm \tau^3}{2} \begin{pmatrix} \psi_c \\ \psi_s \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} \bar{\chi}_c \\ \bar{\chi}_s \end{pmatrix} \frac{-\tau^1 \pm i\gamma_5\tau^3}{2} \begin{pmatrix} \chi_c \\ \chi_s \end{pmatrix}. \tag{3.2}$$

In practice we need to compute correlation functions of the following interpolating operators

$$\begin{aligned} P_{ss} &\equiv (\bar{\psi}_s i\gamma_5 \psi_s) = (\bar{\chi}_c i\gamma_5 \chi_c - \bar{\chi}_s i\gamma_5 \chi_s)/2 - \frac{Z_S}{Z_P}(\bar{\chi}_s \chi_c + \bar{\chi}_c \chi_s)/2, \\ P_{cc} &\equiv (\bar{\psi}_c i\gamma_5 \psi_c) = (\bar{\chi}_s i\gamma_5 \chi_s - \bar{\chi}_c i\gamma_5 \chi_c)/2 - \frac{Z_S}{Z_P}(\bar{\chi}_s \chi_c + \bar{\chi}_c \chi_s)/2. \end{aligned} \tag{3.3}$$

Note that the sum of pseudo-scalar and scalar contributions appears with the ratio of renormalisation factors $Z \equiv Z_S/Z_P$, which needs to be taken into account properly. Z has not yet been determined for all values of β non-perturbatively.

However, for the mass determination, we can avoid this complication by changing the basis and compute the real and positive definite correlation matrix

$$\mathscr{C} = \begin{pmatrix} \eta_{\ell\ell} & \eta_{\ell P_h} & \eta_{\ell S_h} \\ \eta_{P_h\ell} & \eta_{P_h P_h} & \eta_{P_h S_h} \\ \eta_{S_h\ell} & \eta_{S_h P_h} & \eta_{S_h S_h} \end{pmatrix}, \tag{3.4}$$

with the notation

$$P_h \equiv (\bar{\chi}_c i\gamma_5 \chi_c - \bar{\chi}_s i\gamma_5 \chi_s)/2, \qquad S_h \equiv (\bar{\chi}_s \chi_c + \bar{\chi}_c \chi_s)/2 \tag{3.5}$$

and η_{XY} denoting the corresponding correlation function. Masses can determined by solving the generalised eigenvalue problem [13, 14]

$$\mathscr{C}(t)\, \eta^{(n)}(t,t_0) = \lambda^{(n)}(t,t_0)\, \mathscr{C}(t_0)\, \eta^{(n)}(t,t_0). \tag{3.6}$$

3

Taking into account the periodic boundary conditions for a meson, we can determine the effective masses by solving

$$\frac{\lambda^{(n)}(t,t_0)}{\lambda^{(n)}(t+1,t_0)} = \frac{e^{-m^{(n)}t} + e^{-m^{(n)}(T-t)}}{e^{-m^{(n)}(t+1)} + e^{-m^{(n)}(T-(t+1))}} \tag{3.7}$$

for $m^{(n)}$, where n counts the eigenvalues. The state with the lowest mass should correspond to the η and the second state to the η' meson.

From the components $\eta_{0,1,2}^{(n)}$ of the eigenvectors, we can reconstruct the physical flavour contents $c_{\ell,s,c}^{(n)}$ from

$$\begin{aligned}
c_\ell^{(n)} &= \frac{1}{\mathcal{N}^{(n)}}(\eta_0^{(n)}) \\
c_s^{(n)} &= \frac{1}{\mathcal{N}^{(n)}}(-Z\eta_1^{(n)} + \eta_2^{(n)})/\sqrt{2} \\
c_c^{(n)} &= \frac{1}{\mathcal{N}^{(n)}}(-Z\eta_1^{(n)} - \eta_2^{(n)})/\sqrt{2}
\end{aligned} \tag{3.8}$$

with normalisation

$$\mathcal{N}^{(n)} = \sqrt{(\eta_0^{(n)})^2 + (Z\eta_1^{(n)})^2 + (\eta_2^{(n)})^2} \, .$$

At this point the ratio $Z \equiv Z_S/Z_P$ is needed again. Assuming for a moment that charm does not contribute significantly to the η and η' states, one can extract the η-η' mixing angle ϕ from

$$\cos(\phi) = c_\ell^{(0)} \approx c_s^{(1)}, \quad \sin(\phi) = -c_s^{(0)} = c_\ell^{(1)} \tag{3.9}$$

with $^{(0)}$ ($^{(1)}$) denoting the η (η') state.

4. Results

We have computed all contractions needed for building the correlation matrix of eq. (3.4). For the connected contributions, we used stochastic time-slice sources (the so called "one-end-trick" [15]). For the disconnected contributions, we used stochastic volume sources with complex Gaussian noise [15]. As discussed in ref. [12] one can estimate the light disconnected contributions very efficiently using the identity

$$D_u^{-1} - D_d^{-1} = -2i\mu_\ell D_d^{-1} \, \gamma_5 \, D_u^{-1} \, .$$

For the heavy sector such a simple relation does not exist, but we can use the so called hopping parameter variance reduction, which relies on the same equality as in the mass degenerate two flavour case (see ref. [15] and references therein)

$$D_h^{-1} = B - BHB + B(HB)^2 - B(HB)^3 + D_h^{-1}(HB)^4$$

with $D_h = (1+HB)A$, $B = 1/A$ and H the two flavour hopping matrix. We use 24 stochastic volume sources per gauge configuration in both the heavy and the light sector.

We use both local and fuzzed sources to enlarge our correlation matrix by a factor two. In addition to the interpolating operator quoted in eqs. (3.1) and (3.2), we also plan to consider the
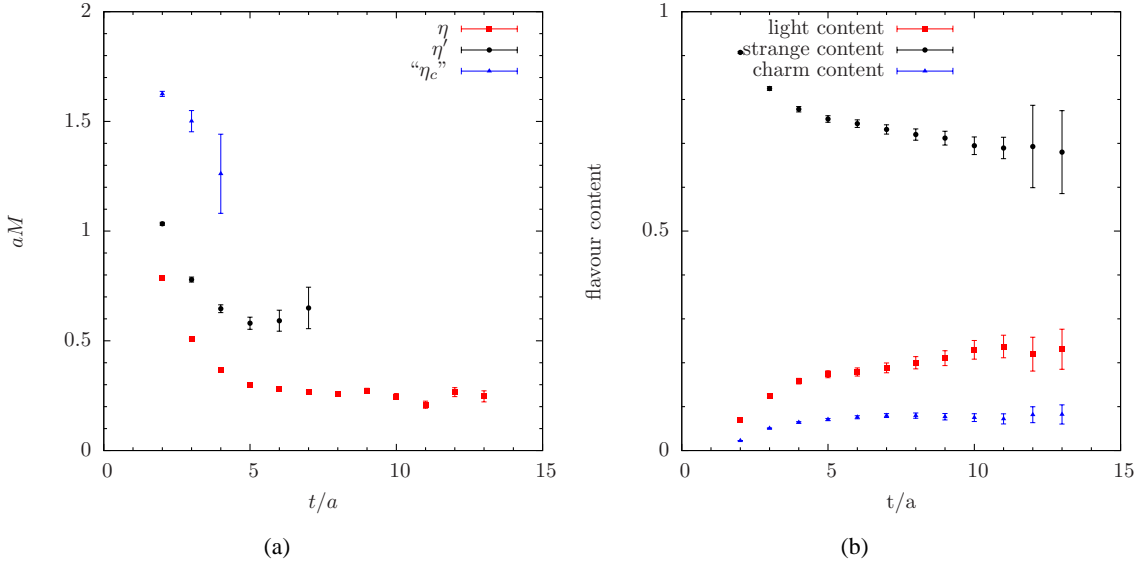
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Figure 1: (a) Effective masses in lattice units determined from solving the generalised eigenvalue problem with $t_0/a = 1$ for ensemble B25.32. We show the results extracted from a 3×3 matrix. (b) squared flavour content of η for B25.32.

γ-matrix combination $i\gamma_0\gamma_5$, which will increase the correlation matrix by another factor of two. The number of gauge configurations investigated per ensemble is in most cases around 1200, and for ensemble B25.32 is 1500. Statistical errors are computed using the bootstrap method with 1000 samples.

In figure 1 we show the effective masses determined from solving the generalised eigenvalue problem for ensemble B25.32 from a 3×3 matrix with local operators only. We kept $t_0/a = 1$ fixed. One observes that the ground state is very well determined and it can be extracted from a plateau fit. The second state, i.e. the η', is much more noisy and a mass determination is questionable, at least from a 3×3 matrix. Enlarging the matrix size significantly reduces the contributions of excited states to the lowest states and, due to smaller statistical errors at smaller t values, a determination becomes possible. The third state appears to be in the region where one would expect the η_c mass value, however, the signal is lost at $t/a = 5$ already, which makes a reliable determination not feasible.

In figure 2 we show the masses of the η and η' mesons for the various ensembles we used as a function of the squared pion mass. In addition we show the corresponding physical values. The scale was set from f_π and m_π using the results of ref. [3]. It is clear that the η meson mass can be extracted with high precision, while the η' meson mass requires a larger correlation matrix, which is work in progress. The comparison with the corresponding physical values seems to point towards good agreement.

We also determine the flavour content of the two states as explained above. It turns out that the η has a dominant strange quark content (see right panel of figure 1), while the η' is dominated by light quarks. For both the charm contribution is rather small, however, for the η it turns out to be significantly non-zero. A preliminary determination of the mixing angle eq. (3.9) yields a very stable value of about $60°$. Note that this is the mixing angle to the flavour eigenstates. The
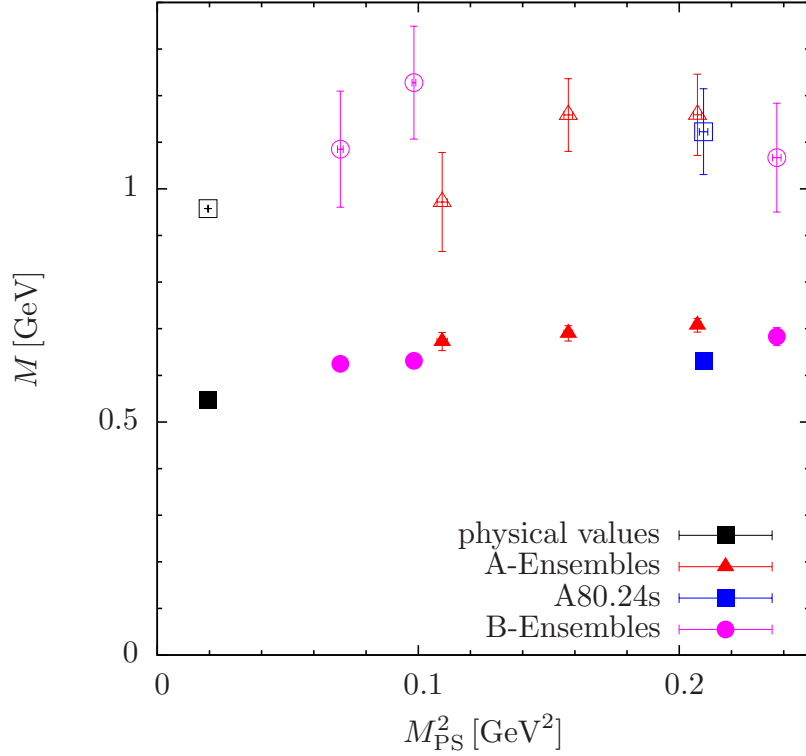
Figure 2: Preliminary values for m_η and $m_{\eta'}$ for two β-values in physical units as a function of the squared pion mass. Filled symbols represent m_η, open ones $m_{\eta'}$.

computation of the mixing angle with respect to the η_0 and η_8 states is in progress.

4.1 Bare Strange and Charm Quark Mass Dependence

The results displayed in figure 2 have been obtained using the bare values of μ_σ and μ_δ as used for the production of the ensembles. Those values, however, did not lead to the correct values of, for e.g., the kaon and D-meson masses, see e.g. ref. [4]. Moreover, the physical strange and charm quark mass values differ between the A and B ensembles. Hence, figure 2 is not yet conclusive with regards to the size of lattice artifacts and extrapolation to the physical point. What we can learn is that the light quark mass dependence in both states appears to be rather weak.

We also have two ensembles A80.24 and A80.24s with different bare values for $\mu_{\sigma,\delta}$ and a retuned value for κ but identical parameters otherwise. Ensemble A80.24s is significantly better tuned with respect to the physical kaon mass value [4]. As seen in figure 2, our results seem to indicate that this change in the bare parameters also has a significant impact on the η meson mass value, while the η' mass is unaffected within the (rather large) errors.

5. Summary and Outlook

We presented a computation of η and η' meson masses from $N_f = 2 + 1 + 1$ Wilson twisted mass lattice QCD. The results we obtained so far are rather encouraging, the η meson mass can

be determined with high precision. Also for the η' meson mass, we hope to be able to give more precise results by increasing the correlation matrix under investigation.

As it is not so easy to tune bare strange and charm quark masses exactly, we shall in the future use a mixed action approach for strange and charm quarks. This will not only avoid the complication induced by flavour symmetry breaking in the twisted mass formulation, but it will also allow to use the efficient noise reduction techniques in the heavy sector. However, this method requires a careful investigation of unitarity breaking effects.

Acknowledgments

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