

## Hybrid stars with the Nambu-Jona-Lasinio model

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**César H. Lenzi**<sup>\*†</sup>

*Universidade Federal do ABC, Centro de Ciências Naturais e Humanas*  
E-mail: cesar.lenzi@ufabc.edu.br

**César V. Flores**

*Universidade Federal do ABC, Centro de Ciências Naturais e Humanas*  
E-mail: cesarovfsky@gmail.com

**Germán Lugones**

*Universidade Federal do ABC, Centro de Ciências Naturais e Humanas*  
E-mail: german.lugones@ufabc.edu.br

We investigate the stability of compact stars with a quark core using an equation of state (EOS) in the framework of the Nambu-Jona-Lasinio model in the SU(3) version (NJL) with repulsive vector coupling. The EOS for the quark phase using the NJL model without coupling vector is too soft and the maximum mass of hybrid stars is less than recent observational data. In order to remedy this problem we suggest the introduction of the vector coupling constant in the NJL model, which turns the EOS stiffer, increasing the maximum mass values. Assuming a vector coupling constant,  $g_v = 0.15g_s$ , (where  $g_s$  is the scalar coupling constant) the maximum mass calculated reaches values greater than  $2M_\odot$ , which agrees with recent observational data.

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<sup>\*</sup>Speaker.

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## 1. Introduction

Compact stars are complex systems which may contain exotic matter such as hyperons, kaon condensation, a nonhomogenous mixed quark-hadron phase, or, in their core, a pure quark phase [1].

The quark phase has frequently been described by the schematic MIT bag model [2, 3] or by the Nambu-Jona-Lasinio (NJL) model [4, 5]. The NJL model contains some of the basic symmetries of QCD, namely chiral symmetry. It has been very successful in describing the vacuum properties of low lying mesons and predicts at sufficiently high densities/temperatures a phase transition to a chiral symmetric state [6]. However, it is just an effective theory that does not take into account quark confinement.

The authors of reference [7] have studied the possible existence of deconfined quarkmatter in the interior of neutron stars using the NJL model. They could show that is possible to obtain a stable compact stars with quark core by introducing a momentum cutoff dependent on the chemical potential. However, even this approach, the maximum mass is less than recent observational data [8]. In order to remedy this problem we suggest the introduction of the vector coupling in the NJL model [9, 10], which makes the EOS stiffer, increasing the maximum mass values.

After a brief introduction about the models of hadronic and deconfined phases, in Sec. 3 we discuss the star stability and the dependence of the maximum mass configuration on the vector coupling constant. In the last section we draw some conclusions.

## 2. Equations of State

We start with the low-density part of the EOS, where the degrees of freedom are the nucleons and hyperons. We work with the relativistic mean-field model which are fitted to the bulk properties of nuclear matter and hypernuclear data. We adopt the following Walecka Lagrangian, which includes self-interacting  $\sigma$ -field:

$$\begin{aligned} \mathcal{L}_H = & \sum_B \bar{\psi}_B \left[ \gamma_\mu \left( i\partial^\mu - g_{\omega B}\omega^\mu - \frac{1}{2}g_{\rho B}\vec{\tau}\cdot\vec{\rho}^\mu \right) - (m_B - g_{\sigma B}\sigma) \right] \psi_B \\ & + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} \\ & + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu - \frac{1}{3}bm_n(g_\sigma)^3 - \frac{1}{4}c(g_\sigma\sigma)^4, \end{aligned} \quad (2.1)$$

where the  $B$ -sum is over the baryonic octet,  $\psi_B$  are the corresponding Dirac fields, whose interactions are mediated by the  $\sigma$  scalar,  $\omega_\mu$  isoscalar-vector and  $\rho_\mu$  isovector-vector meson fields. We have used one of choices discussed in literature [11], in which the hyperons are neglected. The value of nucleon-meson couplings are  $(g_\sigma/m_\sigma)^2 = 11.79 \text{ fm}^2$ ,  $(g_\omega/m_\omega)^2 = 7.149 \text{ fm}^2$  and  $(g_\rho/m_\rho)^2 = 4.411 \text{ fm}^2$ . The couplings in the self-interaction terms of the  $\sigma$ -field are given by  $b = 0.002947$  and  $c = -0.002651$ . These parameters correspond to the GM1 parametrization.

To describe the high-density quark matter we use the NJL model with scalar-pseudoscalar and 't Hooft six fermion interaction. The Lagrangian density of NJL model is defined by:

$$\begin{aligned}
\mathcal{L}_Q = & \bar{\psi} (i\gamma^\mu \partial_\mu - \hat{m}) \psi + g_s \sum_a \left[ (\bar{\psi} \lambda^a \psi)^2 + (\bar{\psi} i \gamma_5 \lambda^a \psi)^2 \right] \\
& - g_v \sum_a \left[ (\bar{\psi} \gamma^\mu \psi_j)^2 + (\bar{\psi} i \gamma_5 \gamma_\mu \lambda^a \psi_j) \right] \\
& + g_t \left\{ \det [\bar{\psi}_i (1 + \gamma_5) \psi_j] + \det [\bar{\psi}_i (1 - \gamma_5) \psi_j] \right\},
\end{aligned} \tag{2.2}$$

where, in flavor space,  $\psi = (u; d; s)$  denotes the quark fields and the  $\lambda^a$  matrices are generators of the U(3) algebra. The term  $\hat{m} = \text{diag}(m_u, m_d, m_s)$  is the quark current mass, which explicitly breaks the chiral symmetry of the Lagrangian, and  $g_s$ ,  $g_v$  and  $g_t$  are coupling constants.

In this work we consider the following set of parameters [12]:  $\Lambda_0 = 631.4$  MeV,  $g_s \Lambda^2 = 1.829$ ,  $g_t \Lambda_0^5 = -9.4$ ,  $m_u = m_d = 5.6$  MeV, and  $m_{0s} = 135.6$  MeV. The value of the vector coupling constant,  $g_v$ , is a free parameter because the masses of the vector mesons are not dictated by chiral symmetry.

For the composition of matter in the star interior we must impose the  $\beta$ -equilibrium and local electric charge neutrality [1]. We will consider cold matter, after the neutrinos have diffused out and the neutrino chemical potential is zero.

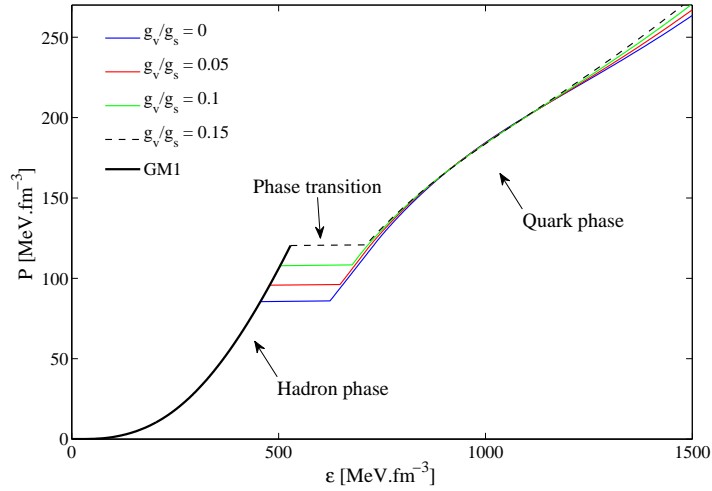
### 3. Star stability

We consider the Maxwell construction [13] for the phase transition from the hadronic phase to the deconfined phase. In this case the crossing point between the hadronic and the quark EOS in the pressure versus baryonic chemical potential plane identified the phase transition. At lower densities (below the transition point) an hadronic phase is favored and at higher densities (above the transition chemical potential) quark matter is favored. In the Figure 3 we plot the pressure as a function of the baryonic density for the complete EOS discussed above. The plateaus represent the deconfinement phase transition as a consequence of the first order Maxwell construction. As can be seen from Figure 3 the different values of the vector coupling constant  $g_v$  change the behavior of the plots. The quark EOS becomes softer with decreasing  $g_v$  and the phase transition occurs at lower densities.

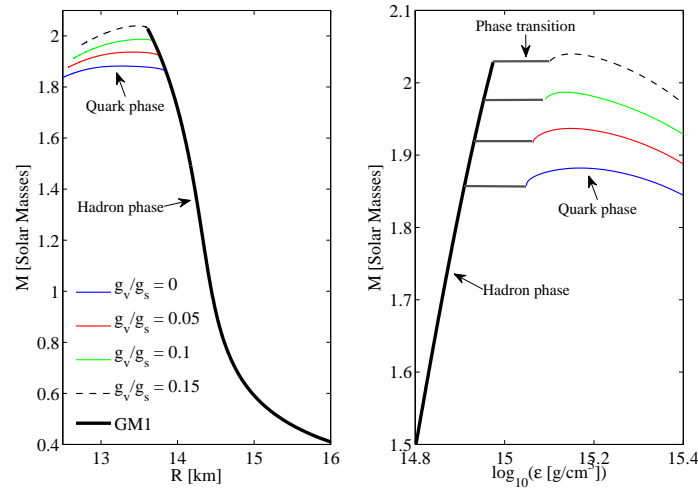
With the complete EOS, we calculate the neutron star configurations solving the Tolman-Oppenheimer-Volkoff (TOV) equations for a spherically symmetric and static star [14, 15]. The Figure 3 shows the gravitational mass of hybrid stars of the maximum mass configuration as a function of (a) the radius and of (b) the central energy for each value of vector coupling constants  $g_v$ .

The gravitational mass of the hybrid stars is characterized by a cusp in the mass versus radius plot and a plateau in the mass versus central density graph. The plateau is a consequence of the Maxwell construction and corresponds to the phase transition between a pure hadron and a pure quark phase. The cusp occurs at the onset of the quark phase in the interior of the star. As we can see in the Figure 3, in all cases the maximum mass appears after the plateau. Therefore, the star configurations with quark phase core are possibles using NJL model.

The influence of vector coupling constant,  $g_v$ , on stelar configuration is clear, the value of maximum mass increases if  $g_v$  increases. However, there is a limit of stability in  $g_v/g_s = 0.15$ . Both effects mentioned occur because the deconfined EOS becomes stiffer.



**Figure 1:** EOS of hybrid stellar matter: Maxwell construction for a first-order phase transition. Pressure as a function of the baryon density for different parametrizations of the cutoff.



**Figure 2:** The gravitational mass of the hybrid star is plotted as a function of (a) the star radius and (b) the central density for different values of  $g_v$ .

#### 4. Summary

We have studied the possibility of formation of stable compact stars with a quark core using NJL model including the repulsive vector interaction. The results show that quark core stable in a hybrid stars is possible if the hadronic and deconfined phase EOS are enough soft.

The results obtained including the repulsive vector interaction in this work agree with those described by Lenzi *et al* in [7]. However, they neglect the vector interaction term and, to obtain a

stable hybrid star, introduce a rather “ad hoc” dependence of the cutoff on the chemical potential. In this approach, the maximum mass is less than recent observational data.

The introduction of a vector coupling constant becomes deconfined EOS stiffer and leads the maximum mass to values greater than  $2\dot{M}_{\odot}$ , which agrees with recent observational data [8]. On the other hand if ratio  $g_v/g_s > 0.15$  the hybrid stars become unstable.

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