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Renormalization of the NN interaction with multiple subtractions and power counting: the 3P0 channel

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We investigate the nucleon-nucleon scattering in the 3P0 partial-wave channel within the multiple subtractions renormalization approach known as subtracted kernel method (SKM). We consider the chiral expansion up to next-to-next-to-leading-order (NNLO) using two different power counting schemes and present a systematic analysis of the predicted phase-shifts and the corresponding relative errors for several values of the renormalization scale. The results for the modified power counting scheme, in which some higher-order contact interactions are promoted to lower-order, exhibit a considerable improvement over Weinberg's power counting scheme, which is based on naive dimensional analysis.

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1.Introduction

The renormalization of nuclear forces has been the subject of great interest in the last decade and several works have approached this problem using different techniques [1-10]. The standard method for the non-perturbative renormalization of the nucleon-nucleon (NN) interaction in the context of chiral effective field theory (ChEFT) consists of two steps. The first is to solve the Lippman-Schwinger (LS) equation with the NN potential truncated at a given order in the chiral expansion, which consists of pion-exchange and contact interaction terms. This requires the use of a regularization scheme in order to overcome the ultraviolet divergences generated in the momentum integrals when potentials of these types are iterated. The second step is to determine the renormalized strengths of the contact interactions, the so called low-energy constants (LEC's), by fitting a set of low-energy scattering data.

The most common approach used to regularize the LS equation is to introduce a sharp or smooth regularizing function that suppresses the contributions from the potential matrix elements for momenta larger than a certain cutoff scale Λ, thus eliminating the divergences in the momentum integrals. The renormalization of the NN interaction can also be performed in configuration space. An alternative approach is the subtractive renormalization or subtracted kernel method (SKM) [11-15] in which, instead of using a regularizing function, the LS equation is regularized by performing subtractions in its kernel at a given subtraction point (renormalization scale) while keeping the original interaction divergent, with no cutoff regularization. A similar approach based on subtractive renormalization of the LS equation is described in Refs. [16-18], but in that case the momentum cutoff is still important.

2. Power counting schemes

We consider two diffrent power counting schemes in this work. The first is Weinberg's power counting (WPC), based on naive dimensional analysis, in which the NN potential for the 3P0 partial-wave channel is given by

$$V_{WPC}^{3P0} = V_{OPE} + V_{TPE-NLO} + C_1^{NLO}(p \times p') + V_{TPE-NNLO} \quad , \qquad (1)$$

where V_{OPE}, $V_{TPE-NLO}$ and $V_{TPE-NNLO}$ are the one-pion and two-pion exchange potentials projected in the 3P0 partial-wave channel. The second is a modified power counting (MPC), in which the potential is written as

$$\begin{aligned} V_{MPC}^{3P0} = V_{OPE} + C_1^{LO}(p \times p') + V_{TPE-NLO} + C_3^{NLO}(p \times p')(p^2 + p'^2) \\ + V_{TPE-NNLO} + C_5^{NNLO}(p^3 \times p'^3) \quad . \qquad (2) \end{aligned}$$

Note that in the WPC scheme there is only one contact interaction at next-to-leading-order (NLO), while in the MPC one contact term is included at each order by promoting higher-order contact interactions to lower-order. The lack of contact terms at leading-order (LO) and next-to-next-to-leading-order (NNLO) in Weinberg's power counting scheme has consequences that will be discussed in section 4, where the numerical results are presented.

3. Renormalized K-matrix

The conventional approach to obtain finite results from the scattering equation with the divergent potentials of Section 2 is to introduce a momentum cutoff Λ and fix the values of the low-energy constants C_i to fit the results from the Nijmegen partial-wave analysis (PWA) [19]. Here we use a renormalization procedure based on multiple subtractions, which is also efficient to renormalize the NN interaction and provides reliable results [11-15].

The multiple subtractions formalism developded for the T-matrix equation can also be applied to obtain the K-matrix by using a principal value prescription. The LS equation for the K-matrix projected in a given partial-wave channel with a generic number of subtractions, n, is given by

$$K^{\rm SJ}_{l_1 l_2}(p,p';k^2) = V^{\rm SJ(n)}_{l_1 l_2}(p,p';-\mu^2) + \frac{2}{\pi}\sum_{l_3}\mathcal{P}\int_0^\infty dq\, q^2 \left(\frac{\mu^2+k^2}{\mu^2+q^2}\right)^{\rm n} \frac{V^{\rm SJ(n)}_{l_1 l_3}(p,q;-\mu^2)}{k^2-q^2} K^{\rm SJ}_{l_3 l_2}(q,p';k^2) \quad , \qquad (3)$$

where $V^{SJ(n)}_{l_1 l_2}(p,p';-\mu^2)$ corresponds to the driving term with n subtractions, projected in a partial-wave channel with total angular momentum J and total spin S and is determined recursively through the equation

$$V^{(n)}(-\mu^2;E) \equiv \left[1-(-\mu^2-E)^{n-1}V^{(n-1)}(-\mu^2;E)G_0^n(-\mu^2)\right]^{-1} V^{(n-1)}(-\mu^2;E) \; , \; (4)$$

where $G_n^{(+)}(E;-\mu^2)$ is the Green function with multiple subtractions, given by

$$G_n^{(+)}(E;-\mu^2) \equiv \left[(-\mu^2-E)G_0(-\mu^2)\right]^n G_0^{(+)}(E). \qquad (5)$$

$G_0^{(+)}$ is the free Green function, given in terms of the free hamiltonian H_0:

$$G_0^{(+)}(E) = [E - H_0 + i\epsilon]^{-1} \quad . \qquad (6)$$

4. Numerical Results

In order to compute the phase-shifts for several values of the renormalization scale we need to find the strengths of the contact interactions for each of these scales in both WPC and MPC schemes. The fits are performed by follwing the prescription of Ref. [20], which consists of matching the inverse effective K-matrix with the inverse K-matrix evaluated from the Nijmegen potential at very low momenta.

We start our discussion by showing the results for the phase-shifts in the 3P0 partial-wave channel at selected energies as a function of the renormalization scale for the LO interaction in the WPC scheme. As one can see in Fig. 1, phase-shifts exhibit the same limit-cycle behaviour with the renormalization scale as they do with the cutoff for the LO interaction when the conventional renormalization approach is used.

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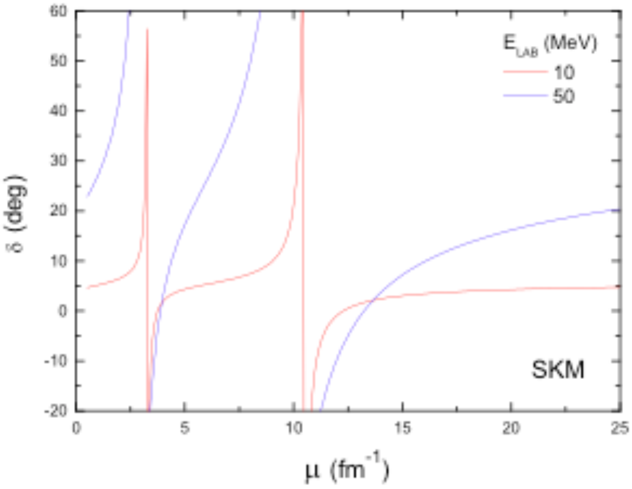

Figure 1: Phase-shifts in the 3P0 partial-wave channel at E_{LAB} = 10 MeV and E_{LAB} = 50 MeV as a function of the renormalization scale for the LO interaction in the WPC scheme.

Now we turn to a comparison between the two power counting schemes described in Section 2. The phase-shifts and the corresponding relative errors (with respect to the results from the Nijmegen PWA) computed using the WPC scheme for several values of the renormalization scale are displayed in Fig. 2. At LO, the flat behaviour of the relative error is a consequence of the fact that a contact interaction is lacking at this order order in the WPC scheme. At NLO, with the inclusion of the contact term, the phase-shifts present a systematic variation with the renormalization scale and the relative errors present a power-law behavior with the energy. At NNLO, since there is no new contact interaction, even though the phase-shifts show a small improvement due to the inclusion of the two-pion exchange contribution, there is no power-law improvement in the relative errors.

In Fig. 3, we show the phase-shifts and the corresponding relative errors computed using the MPC scheme. At LO, we can see that by promoting the NLO contact interaction to LO leads to a power-law behavior in the relative errors qualitatively similar to that obtained at NLO in WPC scheme (note that at LO there is no contribution from two-pion exchange). As we move to NLO and NNLO, we observe a systematic power-law improvement in the relative errors which is due to the promotion of higher-order contact interactions, although the phase-shifts agree to the results from the Nijmegen PWA only at very low energies.

5. Final Remarks

Taking the NN interaction in the 3P0 channel as an example and using the subtracted kernel method (SKM) approach to renormalize the interaction up to next-to-next-to-leading order (NNLO), we have shown that a modified power counting scheme in which one new contact interaction is included at each order leads to a systematic order-by-order power-law improvement in the relative errors for the phase-shifts. Weinberg's power counting scheme fails at leading order (LO) and does not provide such a systematic improvement. The modified power counting scheme may be implemented to perform calculations in any partial-wave channel.

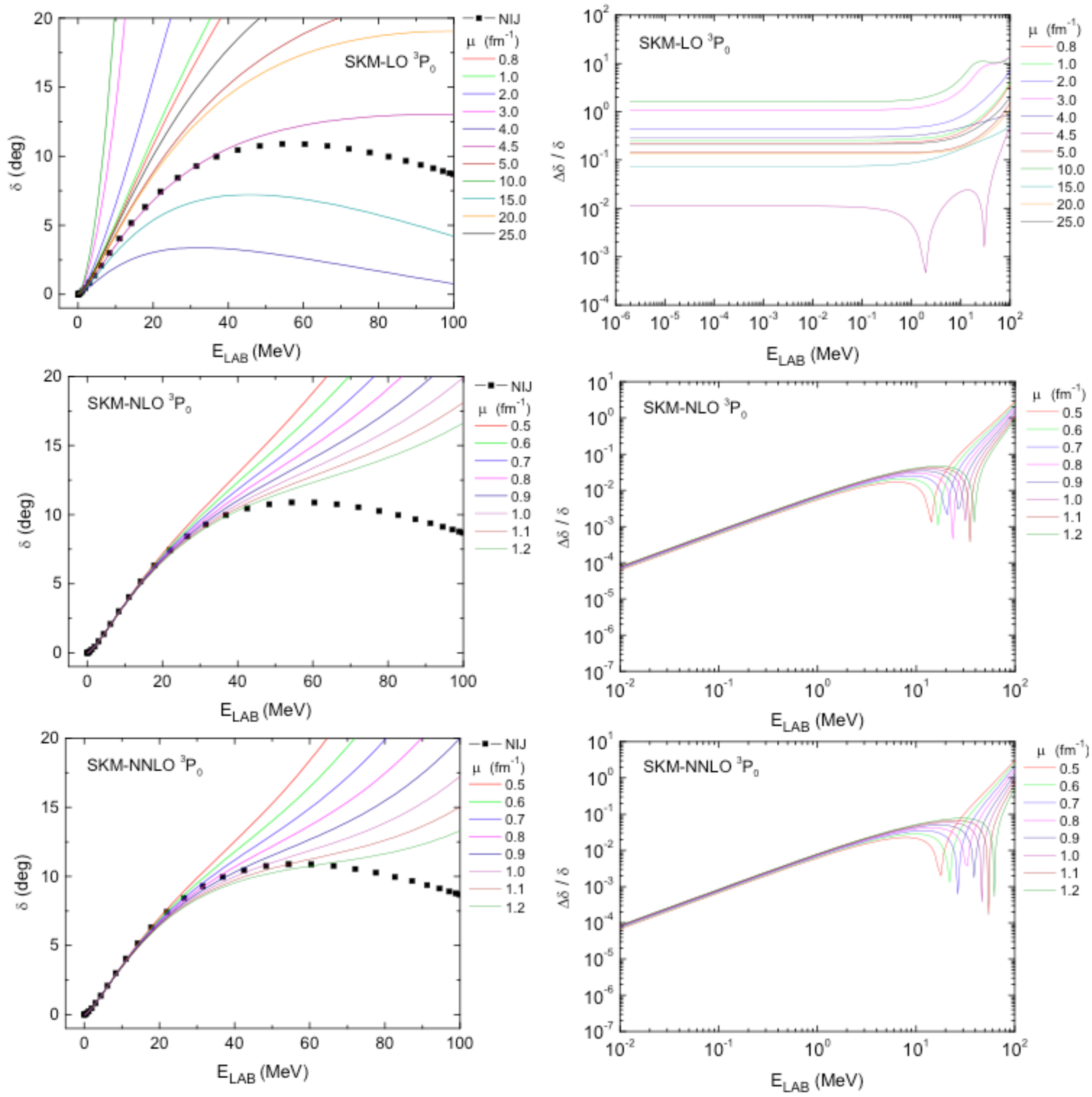

Figure 2: Phase-shifts in the 3P0 partial-wave channel as a function of the laboratory energy E_{LAB} (left panels) and the corresponding relative errors (right panels) for several values of the renormalization scale for the LO, NLO and NNLO interactions in the WPC scheme.

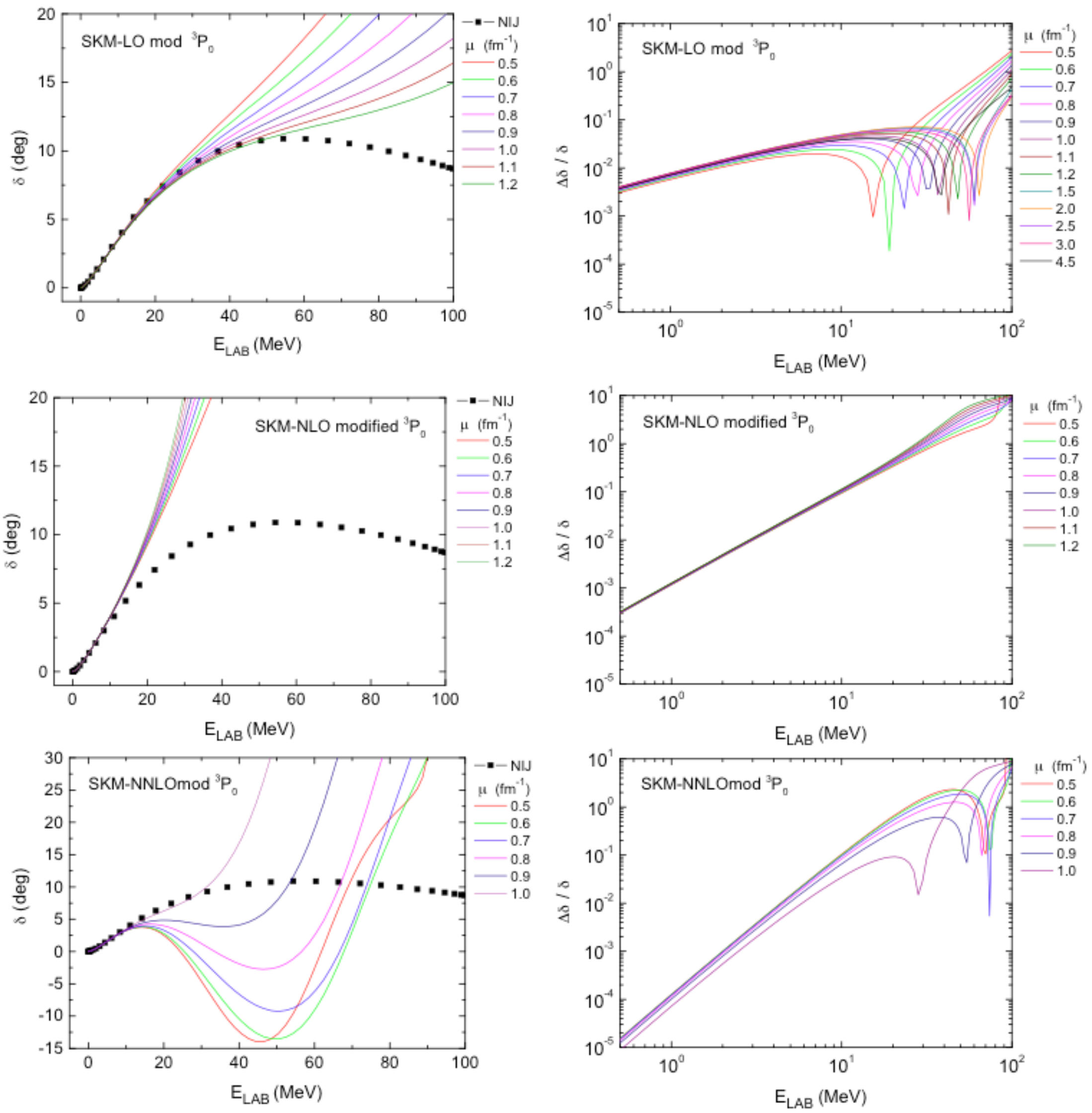

Figure 3: Phase-shifts in the 3P0 partial-wave channel as a function of the laboratory energy E_{LAB} (left panels) and the corresponding relative errors (right panels) for several values of the renormalization scale for the LO, NLO and NNLO interactions in the MPC scheme.

Acknowledgements

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References

[1] S. Weinberg, Phys. Lett. **B 251**, 288 (1990).

[2] E. Epelbaum, H.-W. Hammer and U.-G. Meissner, nucl-th/0811.1338.

[3] C. Ordóñez, L. Ray and U. van Kolck, Phys. Rev. Lett. **72**, 1982 (1994).

[4] E. Epelbaum W. Glockle e U.-G. Meissner, Nucl. Phys. **A 637**, 107 (1998); **A 671**, 295 (2000).

[5] P. F. Bedaque and U. van Kolck, Ann. Rev. Nucl. Part. Sci. **52**, 339 (2002).

[6] D. R. Entem and R. Machleidt, Phys. Rev. **C68**, 041001 (2003).

[7] R. Higa, M. R. Robilotta and C. A. Rocha, Phys. Rev. **C69**, 03400 (2004).

[8] E. Epelbaum, W. Glockle and U.- G. Meissner, Nucl. Phys. **A 747**, 362 (2005).

[9] A. Nogga, R. G. E. Timmermans and U. van Kolck, Phys. Rev. **C 72**, 054006 (2005).

[10] M. P. Valderrama and E. R. Arriola, Phys. Rev. **C 74**, 064004 (2006).

[11] T. Frederico, V. S. Timóteo and L. Tomio, Nucl. Phys. **A 653**, 209 (1999).

[12] T. Frederico, A. Delfino and L. Tomio, Phys. Lett. **B 481**, 143 (2000).

[13] V. S. Timóteo, T. Frederico, A. Delfino and L. Tomio, Phys. Lett. **B 621**, 109 (2005).

[14] V. S. Timóteo, T. Frederico, A. Delfino and L. Tomio, Phys. Rev. **C 83**, 064005 (2011).

[15] S. Szpigel, V. S. Timóteo e F. O. Durães, Annals of Physics 326, 364-405 (2011).

[16] C.-J. Yang, Ch. Elster and D. R. Phillips, Phys. Rev. **C 77**, 014002 (2008).

[17] C.-J. Yang, Ch. Elster and D.R. Phillips, nucl-th/0901.2663.

[18] C.-J. Yang, Ch. Elster and D.R. Phillips, nucl-th/0905.4943.

[19] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen e J.J.de Swart, Phys. Rev. **C 49**, 2950 (1994).

[20] J. V. Steele and R. J. Furnstahl, Nucl. Phys. A 637, 46 (1998).