

## One-Proton Radioactivity from Spherical Nuclei

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The proton emission process from nuclear ground state of a spherical nuclei is studied in the quantum-mechanical tunneling approach through a potential barrier, which is generated by a Woods-Saxon potential (including the spin-orbit component) added to the Coulomb and centrifugal potential energy contributions. We have taken the values for the nuclear potential parameters from the nucleon-nucleus scattering analysis performed by Becchetti and Greenlees [1]. Since we are using a spherical mean field potential form in the calculation, we have applied the present model to determine the half-life of the most spherical proton emitters, as it is the case for  $^{155-157}\text{Ta}$  isotopes. The result is very satisfactory and encouraging as well, and the present approach may be used to other proton emitter nuclides.

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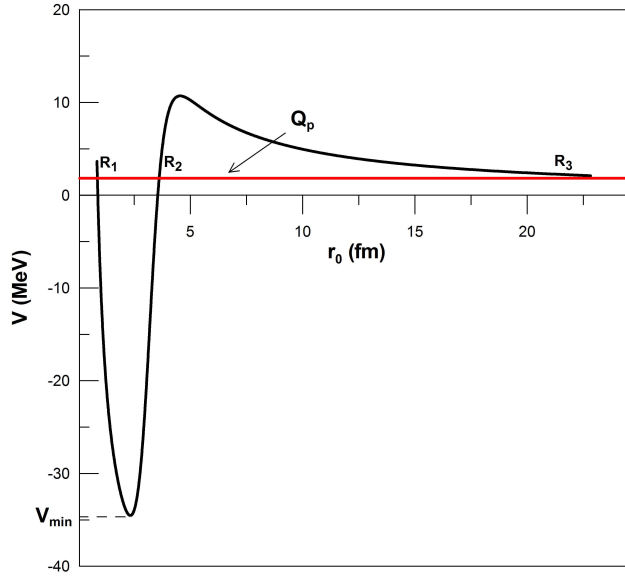
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## 1. The potential barrier

Many different forms of potential barrier have been used to calculate the half-life of one-proton emission process from ground state of exotic nuclei. Some of the potentials are taken as schematic effective approaches with adjustable parameters to reproduce the experimental half-lives. However, the connection of these potential barriers with the current models of nuclear structure is still very qualitative. In present work we are generating the barrier by a typical shell model potential and calculating the penetration barrier to determine the half-life,  $\tau_{1/2}$ , and the decay constant,  $\lambda$  of spherical proton emitter. These quantities are defined as in ref. [2],

$$\tau_{1/2} = \frac{\ln(2)}{\lambda} ; \quad \lambda = \lambda_0 e^{-G}, \quad (1.1)$$

where  $G$  is Gamow's factor through the barrier and  $\lambda_0$  is the frequency of proton assaults to the potential barrier.



**Figure 1:** Potential barrier showing the minimum value of the barrier  $V_{min}$ . The black horizontal line represents the decay energy. The radius values  $R_1$ ,  $R_2$  and  $R_3$  are the classical turning points.

In order to determine  $\lambda_0$  an harmonic approximation for the potential well is used in the region of the pocket in the case for a non-null decay angular momentum. With this approximation we have

$$\lambda_0 = \sqrt{\frac{Q_p - V_{min}}{4\mu(R_2 - R_1)^2}}. \quad (1.2)$$

The nuclear decay with  $\ell = 0$  has the pocket better approximated by a square well form to determine the assault rate,

$$\lambda_0 = \sqrt{\frac{Q_p - V_{min}}{2\mu(R_2 - R_1)^2}}, \quad (1.3)$$

where  $V_{min}$  corresponds to the minimum value of the potential. The radius values  $R_1$  and  $R_2$  are the classical turning points in the region inside the potential well, and  $\mu$  is the reduced mass of the system p+core. The  $Q_p$ -value is the decay energy as illustrated in figure 1. Gamow's penetrability factor in Eq. (1.1) is calculated as

$$G = \frac{2}{\hbar} \int_{R_2}^{R_3} \sqrt{2\mu [V(r) - Q_p]} dr, \quad (1.4)$$

where  $V(r)$  is the total shell model potential form [3], accounting for the superposition of the nuclear Woods-Saxon potential, the spin-orbit, and Coulomb and centrifugal potential energies. Thus the total potential form is given by

$$V(r) = \frac{V_0}{1 + e^{(r-R)/a}} + \left(\frac{\lambda_\pi}{2\pi}\right)^2 \frac{V_{ls}}{a_{ls} r} \frac{e^{(r-R_{ls})/a_{ls}}}{(1 + e^{(r-R_{ls})/a_{ls}})^2} l \cdot s + V_C + \frac{l(l+1)\hbar^2}{2\mu r^2} \quad (1.5)$$

where  $s$  is the proton spin number. The potential depth is given by Becchetti's systematics relation Ref. [1],

$$V_0 = 54.0 - 0.32Q_p + 0.4 \frac{Z}{A^{1/3}} + 24.0 \frac{N-Z}{A} \text{ MeV} \quad (1.6)$$

where  $Z$  and  $N$  are the atomic and neutron numbers of parent nuclei; for the spin-orbit potential we have used  $V_{ls} = -6.2$  MeV and the pion Compton wavelength square was specified as  $\lambda_\pi^2 = 2.0$  fm<sup>2</sup>. The diffuseness parameters are  $a = a_{ls} = 0.75$  fm, as used in Ref. [1], and the radius  $R$  and  $R_{ls}$  are defined as usual for this range of nuclear mass by

$$R = R_{ls} = r_0 A^{1/3} \quad (1.7)$$

where  $r_0$  is the reduced radius parameter and  $A$  the mass number of parent nuclei. The Coulomb potential corresponds to an uniform spherical charge distribution,

$$V_{coul}(r) = \left[ \frac{Z_d e^2}{r} \right] \quad r \geq R, \quad (1.8)$$

$$V_{coul}(r) = \frac{Z_d e^2}{r} \left(3 - \frac{r}{R}\right)^2 \quad r < R, \quad (1.9)$$

where  $e$  is the electron charge for which  $e^2 = 1.4399652$  MeV.fm.

## 2. Results and Conclusion

To apply the present potential barrier approach to spherical nuclei we determine the half-life for the less deformed proton emitter nuclei <sup>155</sup>Ta, <sup>156</sup>Ta and <sup>157</sup>Ta among many other identified as proton emitters. The radius parameter  $r_0$  was adjusted to reproduce the value of the experimental half-life within an accuracy smaller than 0.5%. In table 1 we display our results obtained for

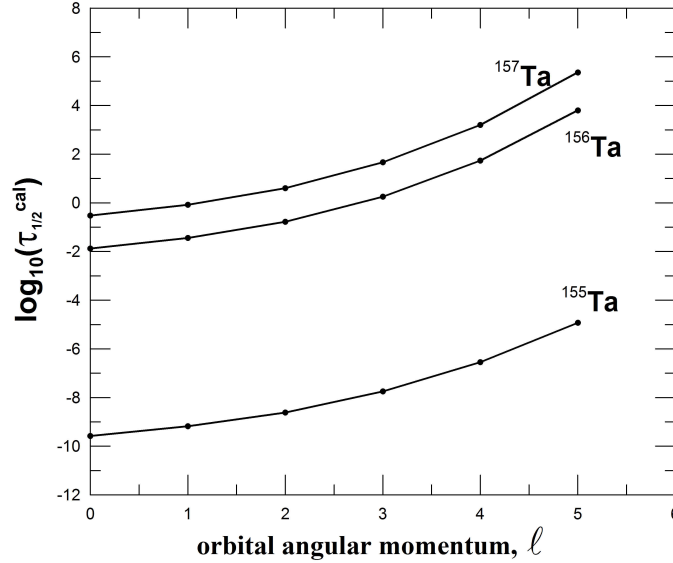
the half live and adjusted parameter, comparing with results of other recent works. Since each process has a different angular orbital momentum, we decided to investigate the sensivity of half life calculation to the angular momentum variation.

**Table 1:** Calculated and experimental half-lives and adjusted nuclear radius parameter  $r_0$ . The first column lists the emitter nucleus and in the second the  $\ell$ -values. Mass number of parent nuclei,  $A$ , is in the subsequent column. The decay energy,  $Q$ , taken from Ref. [4], is shown in the fourth column;  $V_0$ -values and the quadrupole deformation parameter  $\beta_2$  are included as well.

A	$\ell$	$Q_p$ (MeV)	$V_0$ (MeV)	$\beta_2^*$	$r_0$ (fm)			$\tau_{1/2}$ (s)			
					This work	Ref. [4]	Ref. [5]	$\tau_{1/2}^{exp}$	This work	Ref. [4]	Ref. [5]
155	5	1.786	60.261	0.008	1.230	1.220	1.200	$1.20 \times 10^{-5}$	$1.20 \times 10^{-5}$	$3.90 \times 10^{-5}$	$2.70 \times 10^{-5}$
156	2	1.026	60.634	-0.053	1.219	1.220	1.200	$1.65 \times 10^{-1}$	$1.65 \times 10^{-1}$	$9.40 \times 10^{-2}$	$1.20 \times 10^{-1}$
157	0	0.946	60.792	0.045	1.203	1.220	1.200	$3.00 \times 10^{-1}$	$3.00 \times 10^{-1}$	$1.80 \times 10^{-1}$	$2.77 \times 10^{-1}$

(\*)From Ref. [6].

In figure 2 we show the half-life as the function of the angular momentum, disregarding the almost irrelevant spin-orbit term. We note that the isotopes with higher atomic number have half-lives difering by one order of magnitude, while the isotope with lower atomic number present a half-life with significantly higher difference in the order of magnitude from the other.

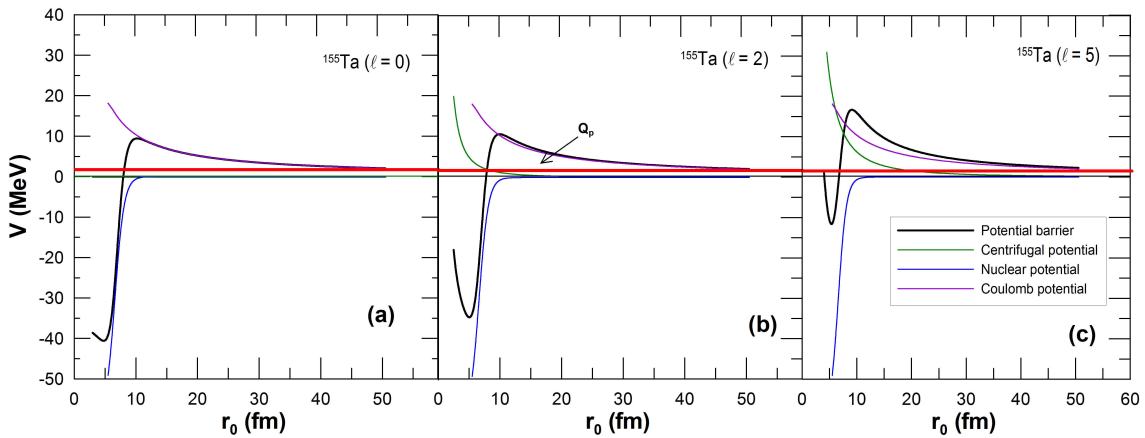


**Figure 2:** Half life dependence upon  $\ell$  for Tantalum isotopes using the adjusted radius parameter in table 1.

In order to illustrate the relevance of the centrifugal barrier to the total barrier, in figure 3 we show the different contributions to total potential barrier for  $^{155}\text{Ta}$  and  $\ell = 0, 2, 5$ .

From table 1 we can see that the proposed calculation model using explicitly a shell model potential barrier type reproduces the half-life of spherical proton emitter with values of the reduced radius parameter similar to currently used ones (the adjustable parameter of the model). The radius parameters,  $r_0$ , extracted from ref. [4] is obtained taking the nuclear radius expressions (eqs. 14-19 of that reference) and using  $R = r_0 A^{1/3}$ .

As a further step we intend to explore the effective character of the model to determine half-lives of proton emitters with small deformation and applying the calculation even to very deformed nuclei (a work in progress). Note that in spite of the spherical symmetry of the potential the purpose is to verify to what extent the spherical effective character of the calculation can reproduce half-life-values of deformed emitters by using the adjustable model parameters to compensate the deformation effect, not explicitly considered in the calculation. At this point it is important to call attention to the crucial role of the input parameter of the calculation, the Q-value, which is extracted from experimental masses. Consequently, this parameter carries implicitly a realistic contribution of the deformation to the barrier penetrability factor calculation. Finally, the ultimate motivation for this next step is to obtain a semianalytical calculation tool, offering satisfactory results and allowing a prompt prediction of new possible cases of proton emission to perform a scanning of a wide region in the nuclear chart, where can be found new proton emitters.



**Figure 3:** Potential barrier for  $^{155}\text{Ta}$  and  $\ell = 0, 2, 5$ , respectively. The total potential barrier (black line) includes the nuclear potential (blue line), centrifugal potential (green line) and Coulomb potential (purple line). The decay energy,  $Q_p$ , is marked by a red horizontal.

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