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Shell model formalism for the vector proton asymmetry in the nonmesonic weak hypernuclear decay

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I present an independent-particle shell-model formalism for the calculation of the primary proton asymmetry of nonmesonic weak hypernuclear decay in terms of relative-space transition amplitudes. In previous work the asymmetry has been usually defined in terms of the single-proton angular distribution, having in mind the measurement of this quantity by single-proton detection, as done in the older experiments. More recently one strives for better determinations of this quantity through proton-neutron coincidence measurements, which are expected to suffer less contamination from rescattering inside the nuclear medium. Therefore I start here from a definition of the asymmetry based on the angular distribution of a proton in coincidence with a neutron in back-to-back kinematics. The two calculated quantities are not identical, but should have similar numerical values due to the fact that the opening-angle distribution for the primary nucleons is strongly peaked at this momentum orientation.

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1. Introduction

The free decay of a Λ hyperon occurs almost exclusively through the mesonic mode, $\Lambda \to \pi N$, but inside hypernuclei a new channel opens up and rapidly dominates the decay for $A \geq 5$. This is the nonmesonic mode, $\Lambda N \to nN$, whose large momentum transfers ($\approx$ 400 MeV/c) are enough to overcome the Pauli blocking. At present, this decay mode is the only way available to probe the strangeness-changing weak interaction between hadrons. To this end, the existing experimental data of greatest theoretical interest are the full (single-nucleon induced) nonmesonic decay rate, $\Gamma = \Gamma_n + \Gamma_p = \Gamma(\Lambda n \to nn) + \Gamma(\Lambda p \to np)$, and the n/p branching ratio, Γ_n/Γ_p, of several hypernuclei, and the vector proton asymmetry, A_V, in the decay of polarized s- and p- shell hypernuclei. These are not raw data, however, since it is necessary to disentangle from the measurements the effect of rescattering of the primary nucleons inside the nuclear medium due to final state interactions. To help in this task there have recently become available some good quality single-nucleon and two-nucleon-coincidence spectra and opening angle distributions of the decay products of a few light hypernuclei. For a review and updates on nonmesonic hypernuclear decay, see Refs. [1, 2, 3], and, for experimental determinations of the proton asymmetry, see Refs. [4, 5, 6].

Most of the theoretical work on nonmesonic decay constructs the transition potential by means of one-meson-exchange models with coupling constants fixed through unitary symmetry arguments, as explained, for instance, in Ref. [7]. All such calculations, including those by the present author and collaborators [8, 9, 10], reproduce quite well the full nonmesonic decay rate, but, until recently, seemed to strongly disagree with the experimental values for the other two observables. In the last few years, however, progress in both the theoretical and the experimental sides has solved this discrepancy for the branching ratio. But the question is not yet quite settled for the proton asymmetry.

2. Back-to-back proton asymmetry

The rate for the proton-induced ($\Lambda p \to np$) nonmesonic decay of a hypernucleus in state $|\nu_I J_I M_I\rangle$ with the emission of a neutron and a proton having momenta and third components of spin and isospin ($\mathbf{p}_n$, s_n, $t_n \equiv -1/2$) and ($\mathbf{p}_p$, s_p, $t_p \equiv +1/2$), respectively, and leaving behind a residual nucleus in state $|\nu_F J_F M_F\rangle$ is given by Fermi's golden rule ($\hbar = c = 1$) as $d\Gamma_p = 2\pi \frac{d^3\mathbf{p}_n}{(2\pi)^3} \frac{d^3\mathbf{p}_p}{(2\pi)^3}\, \delta(\text{E.C.})\, |\langle \mathbf{p}_n s_n t_n\, \mathbf{p}_p s_p t_p\, \nu_F J_F M_F |V| \nu_I J_I M_I\rangle|^2$, where $\delta(\text{E.C.})$ is the energy-conserving delta function and V is the transition potential. An incoherent mixture having vector polarization $\mathscr{P}$ along the direction $\hat{\mathbf{n}}$ is described by the density matrix [11] $\rho_I = \frac{1}{2J_I+1}\left[1_I + \frac{3}{J_I+1}\mathscr{P}\mathbf{J}_I \cdot \hat{\mathbf{n}}\right]$. Then, taking the trace of $\rho_I\, d\Gamma_p$ and summing/integrating over the free final states/variables, one gets for the angular distribution of the emitted proton in coincidence with a neutron in back-to-back kinematics ($\hat{\mathbf{p}}_n = -\hat{\mathbf{p}}_p$) in the decay of this incoherent but polarized initial state

$$I_{pn}(\hat{\mathbf{p}}_p) = \frac{1}{2J_I+1}\left[\sum_{M_I} \sigma(\pi;\nu_I J_I M_I) + \frac{3}{J_I+1}\mathscr{P}\,\hat{\mathbf{p}}_p \cdot \hat{\mathbf{n}} \sum_{M_I} M_I\, \sigma(\pi;\nu_I J_I M_I)\right], \qquad (2.1)$$

where the back-to-back strengths are given by

$$\sigma(\pi;\nu_I J_I M_I) = \frac{2\pi}{(4\pi)^4}\sum_{\nu_F J_F}\int d\,\tilde{f}_{\nu_F J_F} \sum_{s_n s_p M_F} |\langle \tilde{\mathbf{p}}_n s_n t_n\, \tilde{\mathbf{p}}_p s_p t_p\, \nu_F J_F M_F |V| \nu_I J_I M_I\rangle|^2 . \qquad (2.2)$$

We have introduced the compact notation

$$\int d\,\tilde{f}_{\nu_F J_F}\ldots=\left(\frac{2}{\pi}\right)^2\int p_p^2\,dp_p\int p_n^2\,dp_n\,\delta\left(\frac{p_n^2}{2\mathrm{M}}+\frac{p_p^2}{2\mathrm{M}}+\frac{(p_p-p_n)^2}{2\mathrm{M}_R}-\Delta_{\nu_F J_F}\right)\ldots \tag{2.3}$$

where M is the nucleon mass, M_R is that of the residual nucleus, and $\Delta_{\nu_F J_F}$ is the liberated energy. The matrix element in (2.2) must be computed in the proton helicity frame, where the z-axis points along the proton momentum. Therefore, $\tilde{\mathbf{p}}_p=p_p\,\mathbf{e}_z$ and $\tilde{\mathbf{p}}_n=-p_n\,\mathbf{e}_z$.

The back-to-back vector proton asymmetry, A_V, for this decay is operationally defined by

$$\mathscr{P}A_V=\frac{I_{pn}(\hat{\mathbf{p}}_p\!=\!\hat{\mathbf{n}}))-I_{pn}(\hat{\mathbf{p}}_p\!=\!-\hat{\mathbf{n}}))}{I_{pn}(\hat{\mathbf{p}}_p\!=\!\hat{\mathbf{n}}))+I_{pn}(\hat{\mathbf{p}}_p\!=\!-\hat{\mathbf{n}}))}\,, \tag{2.4}$$

leading, through (2.1), to the following theoretical expression

$$A_V\;=\;\frac{3}{J_I+1}\,\frac{\displaystyle\sum_{M_I}M_I\,\sigma(\pi;\nu_I J_I M_I)}{\displaystyle\sum_{M_I}\sigma(\pi;\nu_I J_I M_I)}\,. \tag{2.5}$$

It is convenient to split the transition potential into its parity-conserving (PC) and a parity-violating (PV) parts, i.e., to write $V=V^{PC}+V^{PV}$, where V^{PC} is invariant under space inversion, while V^{PV} changes sign. Then, assuming that V is time-reversal invariant, one gets

$$\begin{aligned}
&\sum_{M_I}\sum_{s_n s_p M_F}|\langle\tilde{\mathbf{p}}_n s_n t_n\,\tilde{\mathbf{p}}_p s_p t_p\,\nu_F J_F M_F|V|\nu_I J_I M_I\rangle|^2\;=\\
&\quad\frac{1}{2}\sum_{M_I}\sum_{SM_SM_F}\Big(\big|\sum_T(-)^T\langle\tilde{\mathbf{p}}\tilde{\mathbf{P}}\,SM_S\,T0\,\nu_F J_F M_F|V^{PC}|\nu_I J_I M_I\rangle\big|^2\\
&\quad+\big|\sum_T(-)^T\langle\tilde{\mathbf{p}}\tilde{\mathbf{P}}\,SM_S\,T0\,\nu_F J_F M_F|V^{PV}|\nu_I J_I M_I\rangle\big|^2\Big)\,,
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
&\sum_{M_I}M_I\sum_{s_n s_p M_F}|\langle\tilde{\mathbf{p}}_n s_n t_n\,\tilde{\mathbf{p}}_p s_p t_p\,\nu_F J_F M_F|V|\nu_I J_I M_I\rangle|^2\;=\\
&\quad\Re\sum_{M_I}M_I\sum_{SM_SM_F}\Big[\Big(\sum_T(-)^T\langle\tilde{\mathbf{p}}\tilde{\mathbf{P}}\,SM_S\,T0\,\nu_F J_F M_F|V^{PC}|\nu_I J_I M_I\rangle\Big)\\
&\quad\times\Big(\sum_{T'}(-)^{T'}\langle\tilde{\mathbf{p}}\tilde{\mathbf{P}}\,SM_S\,T'0\,\nu_F J_F M_F|V^{PV}|\nu_I J_I M_I\rangle\Big)^*\Big]\,.
\end{aligned} \tag{2.7}$$

On the right hand sides, we have performed the change of basis $|\tilde{\mathbf{p}}_n s_n t_n\,\tilde{\mathbf{p}}_p s_p t_p\rangle\rightarrow|\tilde{\mathbf{p}}\tilde{\mathbf{P}}\,SM_S\,T0\rangle$, where S and T are the total spin and isospin, respectively, $\tilde{\mathbf{p}}\equiv\frac{1}{2}(\tilde{\mathbf{p}}_p-\tilde{\mathbf{p}}_n)=\frac{1}{2}(p_p+p_n)\mathbf{e}_z$, $\tilde{\mathbf{P}}\equiv\tilde{\mathbf{p}}_p+\tilde{\mathbf{p}}_n=(p_p-p_n)\mathbf{e}_z$ and we have noticed that $\langle\,\frac{1}{2}t_n\frac{1}{2}t_p\,|\,TM_T\,\rangle=\frac{1}{\sqrt{2}}(\delta_{T1}-\delta_{T0})\delta_{M_T\,0}$.

3. Shell-model formalism

From what we have shown in Ref. [12] and from $Y_{lm}(\pm\mathbf{e}_z)=(\pm)^l\sqrt{\frac{2l+1}{4\pi}}\,\delta_{m0}$, it follows that

$$\begin{aligned}
&\langle\tilde{\mathbf{p}}\tilde{\mathbf{P}}\,SM_S\,T0\;\nu_F J_F M_F|V|\nu_I J_I M_I\rangle\;=\;\frac{4\pi}{\sqrt{2J_I+1}}\sum_{lL}i^{-L}\,i^{-l}\,\mathrm{sign}(p_p-p_n)^L[Ll]^{1/2}\\
&\quad\times\sum_{JK}\langle\,J_F M_F\,JM_S\,|\,J_I M_I\,\rangle\,C(JLKlSM_S)\\
&\quad\times\sum_{n\bar{l}\bar{S}N}\langle\nu_I J_I||\left(a_\Lambda^\dagger a_{t_p}^\dagger\right)_{n\bar{l}\bar{S}KNLJ}||\nu_F J_F\rangle^*(\tilde{P}L\,|\,NL\,)\mathscr{M}(\tilde{p}lSK\,T0;n\bar{l}\bar{S}K\,\Lambda t_p)\,.
\end{aligned} \tag{3.1}$$

Here $C(JLKlSM_S) = \langle KM_S\, L0\,|\,JM_S\,\rangle\,\langle\, l0\,SM_S\,|\,KM_S\,\rangle$,

$$\mathscr{M}(\tilde{p}lSK\,T0; n\bar{l}\bar{S}K\,\Lambda t_p) \;=\; \frac{1}{\sqrt{2}}\left[1-(-)^{l+S+T}\right]((\tilde{p}l\,S)K\,T0|V|(n\bar{l}\bar{S})K\,\Lambda t_p) \tag{3.2}$$

are the relative-space transition matrix elements (Jastrow-like correlation functions are implied), and we have introduced the generalized spectroscopic amplitudes

$$\begin{aligned}&\langle \nu_I J_I||\left(a^\dagger_\Lambda a^\dagger_{t_p}\right)_{n\bar{l}\bar{S}KNLJ}||\nu_F J_F\rangle^* \;=\\ &\quad(\,n\bar{l}\bar{S}K\,NL\,J\,|\,n_\Lambda l_\Lambda j_\Lambda\, n_p l_p j_p\, J\,)\;\langle \nu_I J_I||\left(a^\dagger_{n_\Lambda l_\Lambda j_\Lambda \Lambda}\, a^\dagger_{n_p l_p j_p t_p}\right)_J||\nu_F J_F\rangle^*\,,\end{aligned} \tag{3.3}$$

where the second factor is a standard spectroscopic amplitude and the first one is a recoupled Moshinsky coefficient, i.e.,

$$\begin{aligned}&(\,n\bar{l}\bar{S}K\,NL\,J\,|\,n_\Lambda l_\Lambda j_\Lambda\, n_p l_p j_p\, J\,) \;=\\ &\sum_\lambda [\lambda\bar{S}KL]^{1/2}\left\{\begin{array}{ccc}\bar{l} & L & \lambda\\ \bar{S} & 0 & \bar{S}\\ K & L & J\end{array}\right\}[j_\Lambda j_p\lambda\bar{S}]^{1/2}\left\{\begin{array}{ccc}l_\Lambda & \frac{1}{2} & j_\Lambda\\ l_p & \frac{1}{2} & j_p\\ \lambda & \bar{S} & J\end{array}\right\}(\,n\bar{l}\,NL\,\lambda\,|\,n_\Lambda l_\Lambda\, n_p l_p\,\lambda\,)\,.\end{aligned} \tag{3.4}$$

The last factor above is the usual Moshinsky coefficient [13][1] and we are using the notation $[ab\ldots] = (2a+1)(2b+1)\ldots$. (Notice that, for the simple version of the shell model we are adopting,[2] the appropriate values of $n_\Lambda l_\Lambda j_\Lambda$ and $n_p l_p j_p$ in Eq. (3.3) are dictated by ν_F and ν_I.) Finally $(\,\tilde{P}L\,|\,NL\,) = \int_0^\infty R^2\, j_L(\tilde{P}R)\,\mathscr{R}_{NL}\left(b/\sqrt{2},\,R\right)\,dR$, where b is the oscillator size parameter, is the center-of-mass radial overlap, for which explicit expressions are available [8, 14].

To compute the quantities (2.6) and (2.7), we will need the intermediate summations

$$\begin{aligned}\left\{\begin{array}{c}\mathrm{SUM}(2.6)\\ \mathrm{SUM}(2.7)\end{array}\right\} &= \sum_{M_S} C(JLKlSM_S)C(J'L'K'l'SM_S)\\ &\times\left\{\begin{array}{c}\sum_{M_F M_I}\langle\, J_F M_F JM_S\,|\,J_I M_I\,\rangle\langle\, J_F M_F J'M_S\,|\,J_I M_I\,\rangle\\ \sum_{M_F M_I} M_I\langle\, J_F M_F\, JM_S\,|\,J_I M_I\,\rangle\,\langle\, J_F M_F\, J'M_S\,|\,J_I M_I\,\rangle\end{array}\right\}.\end{aligned} \tag{3.5}$$

Well known identities [15] lead to

$$\mathrm{SUM}(2.6) \;=\; \delta_{JJ'}\,\delta(J_F,J,J_I)\,\left[\frac{J_I}{J}\right]\sum_{M_S} C(JLKlSM_S)C(JL'K'l'SM_S) \tag{3.6}$$

and

$$\begin{aligned}\mathrm{SUM}(2.7) \;=\; &(-)^{J_I+J_F}[J_I]^{3/2}\sqrt{J_I(J_I+1)}\left\{\begin{array}{ccc}1 & J & J'\\ J_F & J_I & J_I\end{array}\right\}\\ &\times\sum_{M_S}(-)^{M_S}\begin{pmatrix}1 & J & J'\\ 0 & -M_S & M_S\end{pmatrix} C(JLKlSM_S)C(J'L'K'l'SM_S)\,,\end{aligned} \tag{3.7}$$

[1]With its phase adapted to our convention for the relative coordinate as explained in Ref [8].

[2]Namely, an independent-particle harmonic-oscillator shell model [8, 14]. However the formalism can be easily extended to more sophisticated versions of the shell-model through an expansion in the harmonic-oscillator basis.

where $[a/b] = (2a+1)/(2b+1)$. Then, from Eqs. (2.2), (2.6), (2.7), (3.1), and (3.5), we arrive at

$$\begin{aligned}
\left\{\begin{array}{c} \sum_{M_I} \sigma(\pi; \nu_I J_I M_I) \\ \\ \sum_{M_I} M_I\, \sigma(\pi; \nu_I J_I M_I) \end{array}\right\} &= \frac{1}{8\pi[J_I]} \Re \sum_{\nu_F J_F} \int d\,\tilde{f}_{\nu_F J_F} \\
&\times \sum_{S} \sum_{TT'} \sum_{ll'} i^{-l+l'} [ll']^{1/2}\, \delta((-)^T, (-)^{l+S+1})\, \delta((-)^{T'}, (-)^{l'+S+1}) \\
&\times \sum_{KK'} \sum_{LL'} \delta((-)^L, (-)^{L'})\, i^{-L+L'} [LL']^{1/2} \sum_{JJ'} \left\{\begin{array}{c} \mathrm{SUM}(2.6) \\ \\ \mathrm{SUM}(2.7) \end{array}\right\} \sum_{nl\overline{S}} \sum_{n'\overline{l'}\overline{S'}} \delta((-)^{\overline{l}}, (-)^{\overline{l'}}) \\
&\times \sum_{N} \langle \nu_I J_I || \left(a_\Lambda^\dagger a_{t_p}^\dagger \right)_{nl\overline{S}KNLJ} || \nu_F J_F \rangle^* \, (\tilde{P}L\,|\,NL) \\
&\times \sum_{N'} \langle \nu_I J_I || \left(a_\Lambda^\dagger a_{t_p}^\dagger \right)_{n'\overline{l'}\overline{S'}K'N'L'J'} || \nu_F J_F \rangle (\tilde{P}L'\,|\,N'L')^* \\
&\times \left\{\begin{array}{c} \delta_{TT'}\, \delta((-)^l, (-)^{l'}) \\ \\ (\delta_{TT'} - 1)\, \delta((-)^l, (-)^{l'+1}) \end{array}\right\} \\
&\times \left\{\begin{array}{c} [\, \mathscr{M}^{PC}(\tilde{p}lSK\,T0; nl\overline{S}K\,\Lambda t_p) \mathscr{M}^{PC}(\tilde{p}l'SK'\,T'0; n'\overline{l'}\overline{S'}K'\,\Lambda t_p)^* \\ +\, \mathscr{M}^{PV}(\tilde{p}lSK\,T0; nl\overline{S}K\,\Lambda t_p) \mathscr{M}^{PV}(\tilde{p}l'SK'\,T'0; n'\overline{l'}\overline{S'}K'\,\Lambda t_p)^* \,] \\ \\ \mathscr{M}^{PC}(\tilde{p}lSK\,T0; nl\overline{S}K\,\Lambda t_p) \mathscr{M}^{PV}(\tilde{p}l'SK'\,T'0; n'\overline{l'}\overline{S'}K'\,\Lambda t_p)^* \end{array}\right\},
\end{aligned} \tag{3.8}$$

where we have explicitly indicated, through Kronecker deltas $\delta(a,b) \equiv \delta_{ab}$, the restrictions imposed on the summations by several selection rules. This allows us to write

$$\begin{aligned}
\sum_{M_I} \sigma(\pi; \nu_I J_I M_I) \;&=\; \frac{1}{8\pi[J_I]} \Re \sum_{\nu_F J_F} \int d\,\tilde{f}_{\nu_F J_F} \sum_{S} \sum_{TT'} \delta_{TT'} \sum_{ll'} \delta((-)^l, (-)^{l'}) \\
&\times \delta((-)^T, (-)^{l+S+1})\, \delta((-)^{T'}, (-)^{l'+S+1}) \sum_{KK'} \sum_{nl\overline{S}} \sum_{n'\overline{l'}\overline{S'}} \delta((-)^{\overline{l}}, (-)^{\overline{l'}}) \\
&\times \big[C^{PC\,PC}_{\nu_F J_F}(lSKT, nl\overline{S}; l'K'T', n'\overline{l'}\overline{S'}; \tilde{P}) \\
&\times \mathscr{M}^{PC}(\tilde{p}lSK\,T0; nl\overline{S}K\,\Lambda t_p) \mathscr{M}^{PC}(\tilde{p}l'SK'\,T'0; n'\overline{l'}\overline{S'}K'\,\Lambda t_p)^* \\
&+ C^{PV\,PV}_{\nu_F J_F}(lSKT, nl\overline{S}; l'K'T', n'\overline{l'}\overline{S'}; \tilde{P}) \\
&\times \mathscr{M}^{PV}(\tilde{p}lSK\,T0; nl\overline{S}K\,\Lambda t_p) \mathscr{M}^{PV}(\tilde{p}l'SK'\,T'0; n'\overline{l'}\overline{S'}K'\,\Lambda t_p)^* \big]
\end{aligned} \tag{3.9}$$

and

$$\begin{aligned}
\sum_{M_I} M_I\, \sigma(\pi; \nu_I J_I M_I) \;&=\; \frac{1}{8\pi[J_I]} \Re \sum_{\nu_F J_F} \int d\,\tilde{f}_{\nu_F J_F} \sum_{S} \sum_{TT'} (\delta_{TT'} - 1) \sum_{ll'} \delta((-)^l, (-)^{l'+1}) \\
&\times \delta((-)^T, (-)^{l+S+1})\, \delta((-)^{T'}, (-)^{l'+S+1}) \sum_{KK'} \sum_{nl\overline{S}} \sum_{n'\overline{l'}\overline{S'}} \delta((-)^{\overline{l}}, (-)^{\overline{l'}}) \\
&\times C^{PC\,PV}_{\nu_F J_F}(lSKT, nl\overline{S}; l'K'T', n'\overline{l'}\overline{S'}; \tilde{P}) \\
&\times \mathscr{M}^{PC}(\tilde{p}lSK\,T0; nl\overline{S}K\,\Lambda t_p) \mathscr{M}^{PV}(\tilde{p}l'SK'\,T'0; n'\overline{l'}\overline{S'}K'\,\Lambda t_p)^*
\end{aligned} \tag{3.10}$$

The expressions for the coefficients $C^{PC\,PC}_{\nu_F J_F}$, $C^{PV\,PV}_{\nu_F J_F}$ and $C^{PC\,PV}_{\nu_F J_F}$ can be obtained by comparison with Eq. (3.8).

4. Summary

The vector proton asymmetry, A_V, or equivalently the intrinsic asymmetry parameter [11],

$$a_\Lambda = \begin{cases} A_V & \text{for } J_I = J_C + 1/2 \\ -\dfrac{J_I+1}{J_I} A_V & \text{for } J_I = J_C - 1/2\,, \end{cases} \tag{4.1}$$

where J_C is the spin of the hypernuclear core, thus defined in order to subdue its dependence on the hypernuclear spin, is an important observable of nonmesonic weak decay. Being related to interference terms between parity-conserving and parity-violating transition amplitudes, it not only contains further information when compared to the transition rate and the neutron-to-proton branching ratio, but also has a greater power to discriminate between different models of hypernuclear nonmesonic decay. Discrepancies between its measured and calculated values still remain.

It seems appropriate, therefore, to develop a general formalism for its theoretical computation within a shell-model approach. This we have done in the present contribution. The general expressions are rather involved, but they can certainly be simplified by adopting suitable approximations, such as limiting the initial state to an s-wave, as illustrated, for instance, in Ref. [10]. This, however, has been left for a future development.

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