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Muon capture rates within the projected QRPA

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The conservation of the number of particles within the QRPA plays an important role in the evaluation muon capture rates in all light nuclei with $A \lesssim 30$. The violation of the CVC by the Coulomb field in this mass region is of minor importance, but this effect could be quite relevant for medium and heavy nuclei studied previously. The extreme sensitivity of the muon capture rates on the pp coupling strength in nuclei with large neutron excess when described within the QRPA is pointed out. We reckon that the comparison between theory and data for the inclusive muon capture is not a fully satisfactory test on the nuclear model that is used. The exclusive muon transitions are much more robust for such a purpose.

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1. Introduction

Among different semileptonic processes, the muon capture is one of the weak observables that, together with the β-decay, has available a fruitful set of experimental data that were collected in the last fifty years. Several works were focused to establish the universal $V-A$ character of nuclear muon capture, the role of induced currents, second-class currents, and nonexistence of $V+A$ interactions. It is known that the experimental value of the induced pseudoscalar coupling g_P is the least known of the four constants (g_V, g_A, g_M, g_P) defining the weak nucleon current.

Its size is dictated by chiral symmetry arguments, and its measurement represents an important test of quantum chromodynamics at low energies [1]. During the past two decades a large body of new data relevant to the coupling g_P has been accumulated from measurements of radiative and non radiative muon capture on targets ranging from ^{3}He to complex nuclei. Only transitions to unnatural parity states depend on g_P, as can be seen from Eq. (2.5). A summary of references on these issues are cited in review papers Ref. [2, 3, 4].

Simultaneously, the muon capture processes have been used to scrutinize the nuclear structure models, since they provide a testing ground for wave functions and, indirectly, for the interactions that generate them. Being the momentum transfer of the order of the muon mass $m_\mu = 105.6$ MeV, the phase space and the nuclear response favor lower nuclear excitation energies, and thus the transitions to nuclear states in the giant resonance region are the dominate ones. We will cite only a few of them. Most of these works were done within the shell model (SM) framework [4, 5, 6, 7, 8]. Several studies were performed by employing the random phase approximation (RPA) [7, 9, 10, 11]. In the last work, where the total muon capture rates for a large number of nuclei with $6 < Z < 94$ have been evaluated, the authors claimed that an important benchmark was obtained by introducing the pairing correlations. They have done this ad-hoc by multiplying the one-body transition matrix elements by the BCS occupation probabilities. However, we know that the quasiparticle RPA (QRPA) formalism is a full self-consistent procedure to describe consistently both i) short-range particle-particle (pp) pairing correlations, and ii) long-range particle-hole (ph), correlations handled with RPA. Quite recently, the relativistic QRPA (RQRPA) was applied in the calculation of total muon capture rates on a large set of nuclei from ^{12}C to ^{244}Pu, for which experimental values are available [13].

In the present work we do a systematic study of the muon capture rates of nuclei with $12 \leq A \leq 56$ masses (^{12}C, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{40}Ar, ^{52}Cr, ^{54}Cr, ^{56}Fe) within the number projected QRPA (PQRPA). The motivation for this investigation comes from the successful description of weak observables in the triad $\{^{12}\mathrm{B}, ^{12}\mathrm{C}, ^{12}\mathrm{N}\}$ within this model [12, 14]. There, it was shown that the projection procedure played an essential role in properly accounting for the configuration mixing in the ground state wave function of ^{12}N. The employment of PQRPA for the inclusive $^{12}\mathrm{C}(\nu_e, e^-)^{12}\mathrm{N}$ cross section, instead of the continuum RPA (CRPA) used by the LSND collaboration in the analysis of $\nu_\mu \to \nu_e$ oscillations of the 1993-1995 data sample, leads to an increased oscillation probability [15]. The charge-exchange PQRPA, derived from the time-dependent variational principle, was used to study the two-neutrino $\beta\beta$-decay amplitude $\mathscr{M}_{2\nu}$ in ^{76}Ge [16]. In that work, the projection procedure was less important and the QRPA and PQRPA yield qualitatively similar results for $\mathscr{M}_{2\nu}$. The PQRPA was recently used to calculate the $^{56}\mathrm{Fe}(\nu_e, e^-)^{56}\mathrm{Co}$ cross section [17].

We will also give a glance on the violation of the CVC by the Coulomb field, which was

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worked out recently [14], and appears in the first operator (2.4) for natural parity states [1]. This effect is expected to be tiny for the nuclei studied here, since $\Delta E_{\rm Coul}$ is relatively small in comparison with m_μ; it goes from 3.8 MeV in ^{12}C to 9.8 MeV in ^{56}Fe.

2. μ-capture rates formalism

When negative muons pass through matter, they can be captured into high-lying atomic orbitals. From there they then quickly cascade down into the $1S$ orbit with binding energy E_B^μ, where two competing processes occur: one is ordinary decay $\mu^- \to e^- + \nu_\mu + \tilde{\nu}_e$ with characteristic free lifetime 2.197×10^6 sec, and the other is (weak) capture by the nucleus $\mu^- + (Z,A) \to (Z-1,A) + \nu_\mu$. The latter, naively expected to scale with Z, is drastically enhanced by an additional factor of Z^3, originating from the square of the atomic wave function ϕ_{1S} evaluated at the origin [2]. Thus, its rate is roughly proportional to Z^4 and dominates decay at large Z. This dominance is however significantly diminished by the gradual decrease of the effective-charge correction factor $\mathscr{R}(Z)$ [13, 18].

Then the muon capture rate from the ground state in the initial nucleus (Z,A) to the state J_n^π in the final nucleus $(Z-1,A)$ reads

$$\Lambda(\mathsf{J}_n^\pi) = \frac{E_\nu^2}{2\pi}|\phi_{1S}|^2\mathscr{R}(Z)\mathscr{T}_\Lambda(E_\nu,\mathsf{J}_n^\pi), \tag{2.1}$$

where

$$E_\nu \equiv \kappa = m_\mu - (M_n - M_p) - E_B^\mu - \omega_{\mathsf{J}_n^\pi} \tag{2.2}$$

is the neutrino energy, and

$$\mathscr{T}_\Lambda(E_\nu,\mathsf{J}_n^\pi) = 4\pi G^2\left[|\langle \mathsf{J}_n^\pi||\mathsf{O}_{\emptyset\mathsf{J}}(E_\nu) - \mathsf{O}_{0\mathsf{J}}(E_\nu)||0^+\rangle|^2 + 2|\langle \mathsf{J}_n^\pi||\mathsf{O}_{-1\mathsf{J}}(E_\nu)||0^+\rangle|^2\right], \tag{2.3}$$

is the transition probability, being the Fermi coupling constant $G = (3.04545\pm 0.00006)\times 10^{-12}$ natural units. The nuclear operators are:

$$\begin{aligned}\mathsf{O}_{\emptyset\mathsf{J}} - \mathsf{O}_{0,\mathsf{J}} &= g_{\rm V}\frac{m_\mu - \Delta E_{\rm Coul} - E_B^\mu}{E_\nu}\mathscr{M}_\mathsf{J}^{\rm V},\\ \mathsf{O}_{-1\mathsf{J}} &= -(g_{\rm A}+\overline{g}_{\rm W})\mathscr{M}_{-1\mathsf{J}}^{A,I} + g_{\rm V}\mathscr{M}_{-1\mathsf{J}}^{V,R},\end{aligned} \tag{2.4}$$

for *natural parity states* ($\pi = (-)^J$, i.e., $J^\pi = 0^+, 1^-, 2^+, 3^-, \cdots$), and

$$\begin{aligned}\mathsf{O}_{\emptyset\mathsf{J}} - \mathsf{O}_{0\mathsf{J}} &= g_{\rm A}\mathscr{M}_\mathsf{J}^{\rm A} + (g_{\rm A}+\overline{g}_{\rm A}-\overline{g}_{\rm P})\mathscr{M}_{0\mathsf{J}}^{\rm A},\\ \mathsf{O}_{-1\mathsf{J}} &= -(g_{\rm A}+\overline{g}_{\rm W})\mathscr{M}_{-1\mathsf{J}}^{A,R} - g_{\rm V}\mathscr{M}_{-1\mathsf{J}}^{V,I},\end{aligned} \tag{2.5}$$

for *unnatural parity states* ($\pi = (-)^{J+1}$, i.e., $J^\pi = 0^-, 1^+, 2^-, 3^+, \cdots$). The elementary operators are:

$$\begin{aligned}&\mathscr{M}_\mathsf{J}^{\rm V} = j_\mathsf{J}(\rho)Y_\mathsf{J}(\hat{\mathbf{r}})\ ,\ \mathscr{M}_{m\mathsf{J}}^{\rm V} = M^{-1}\sum_{\mathsf{L}\geq 0} i^{\mathsf{J}-\mathsf{L}-1}F_{\mathsf{LJ}m}j_\mathsf{L}(\rho)[Y_\mathsf{L}(\hat{\mathbf{r}})\otimes\nabla]_\mathsf{J},\\ &\mathscr{M}_\mathsf{J}^{\rm A} = M^{-1}j_\mathsf{J}(\rho)Y_\mathsf{J}(\hat{\mathbf{r}})(\sigma\cdot\nabla)\ ,\ \mathscr{M}_{m\mathsf{J}}^{\rm A} = \sum_{\mathsf{L}\geq 0} i^{\mathsf{J}-\mathsf{L}-1}F_{\mathsf{LJ}m}j_\mathsf{L}(\rho)\,[Y_\mathsf{L}(\hat{\mathbf{r}})\otimes\sigma]_\mathsf{J}\end{aligned} \tag{2.6}$$

[1]When the consequences of the CVC are not considered, as in Ref. [3], the factor $(m_\mu - \Delta E_{\rm Coul} - E_B^\mu)/E_\nu$ in this relation goes to unity.

where $F_{\mathsf{LJ}m} = (-)^{1+m}(1,-m\mathsf{J}m|\mathsf{L}0)$ is a Clebsch-Gordon coefficient, $\rho = |\mathbf{k}|r$, and the superscripts *R*, and *I* in (2.4) and (2.5) stand for real and imaginary pieces of the operators (2.6). Moreover,

$$\Delta E_{\rm Coul} \cong \frac{6e^2 Z}{5R} \cong 1.45 Z A^{-1/3}\ \ \text{MeV}, \quad E_B^{\mu} = (eZ)^2 \frac{\mathrm{m}_\mu}{2} \cong 2.66\times 10^{-5} Z^2 \mathrm{m}_\mu, \tag{2.7}$$

and

$$\overline{g}_{\rm A} = g_{\rm A}\frac{E_\nu}{2\mathrm{M}},\ \ \overline{g}_{\rm W} = (g_{\rm V}+g_{\rm M})\frac{E_\nu}{2\mathrm{M}};\ \ \overline{g}_{\rm P} = g_{\rm P}\frac{E_\nu}{2\mathrm{M}}, \tag{2.8}$$

with g_V, g_A, g_M, and g_P, being the effective vector, axial-vector, weak magnetism, and pseudo-scalar coupling constants, respectively. We adopt

$$g_V\ =\ 1,\quad g_A = 1.135,\ g_M = 3.70,\ g_P = g_A \frac{2\mathrm{M}\mathrm{m}_\mu}{k^2+\mathrm{m}_\pi^2} \cong 6.7, \tag{2.9}$$

where the value for g_P comes from the PCAC, pion-pole dominance and the Goldberger-Trieman relation [19], and for g_A we use the same value as in Ref. [13].

From Eqs. (A6) and (A7) in Ref. [14] one sees that g_P is contained in axial-vector pieces of both operators $\mathsf{O}_{\emptyset\mathsf{J}}$ (temporal), and $\mathsf{O}_{0,\mathsf{J}}$ (spacial). They contribute destructively, being dominant the second one. In Ref. [13] g_P appears only in the temporal operator. However, after making use of the energy conservation condition (2.2), *i.e.,* $\kappa \cong \mathrm{m}_\mu + k_\emptyset$ ($k_\emptyset = -\omega_{\mathsf{J}^\pi_n}$) one ends up with the same result for $\mathsf{O}_{\emptyset\mathsf{J}} - \mathsf{O}_{0\mathsf{J}}$.

The $0^+ \leftrightarrow 0^-$ transitions are determined by two nuclear matrix elements only: $\mathscr{M}^{\rm A}_0$ and $\mathscr{M}^{\rm A}_{00}$, as can be seen from the first relation in (2.5). As such they are the most appropriate to extract the magnitude of g_P from the muon capture experiments. In fact, studies of the $^{16}\mathrm{O}(0^+_1) \rightarrow {}^{16}\mathrm{N}(0^-_1)$ transition within large-basis SM calculations have yielded values of $g_P = 6-9$ [5], and $g_P = 7.5\pm 0.5$ [6] that are consistent with the estimate (2.9) as well as with theoretical prediction $g_P = 8.2$ from chiral symmetry arguments [1]. More recently, Gorringe [4] reported from the SM study of muon capture $^{40}\mathrm{Ca}(0^+_1) \rightarrow {}^{40}\mathrm{K}(0^-_1)$ have extracted from the experimental result of Λ the values $g_P = 14.3^{+1.8}_{-1.6}$, and $g_P = 10.3^{+2.1}_{-1.9}$.

3. Numerical results

For the set of nuclei discussed here we have adopted the single-particle energies (s.p.e.) from the self-consistent calculation performed by Marketin *et al.* [13] within the relativistic Hartree-Bogoliubov model (RHB), using effective Lagrangians with density-dependent meson-nucleon couplings and DD-ME2 parametrization. The residual interaction is approximated by the δ-force (in MeV fm^3)

$$V = -4\pi\,(v_s P_s + v_t P_t)\,\delta(r),$$

with singlet (v_s), and triplet (v_t) coupling constants different *ph*, *pp*, and pairing channels. The proton and neutron pairing parameters $v_s^{pair}(p)$ and $v_s^{pair}(n)$, used in solving the BCS and PBCS equations, were determined from the experimental data by the adjusting procedure described in Ref. [20]. For the parameters in the *ph* channel we employe the values $v_s^{ph} = 27$ and $v_t^{ph} = 64$, which were obtained in a systematic study of the GT resonances [21]. The ratio $t = v_t^{pp}/v_s^{pair}$ was considered as free parameter within the *pp* channel. It was found [22] that the muon capture, just

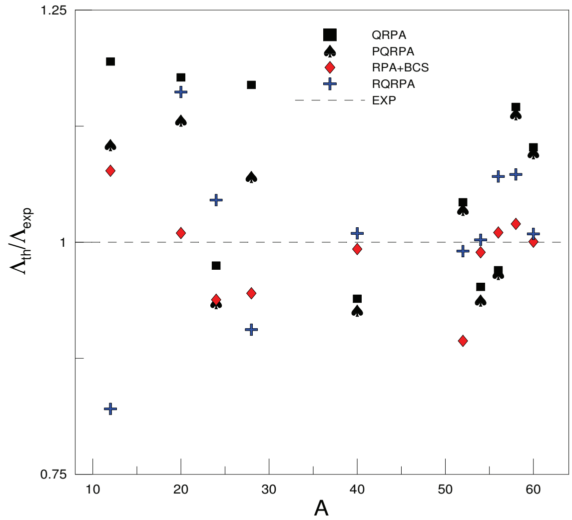

Figure 1: (Color online) Ratios of theoretical to experimental inclusive muon capture rates for different nuclear models, as function of the mass number *A*. The present QRPA and PQRPA results, as well as the RQRPA calculation [13] were done with $g_A = 1.135$, while in the RPA+BCS model [11] was used the unquenched value $g_A = 1.26$ for all multipole operators, except for the GT ones where it was reduced to $g_A \sim 1$.

like the $\beta\beta$-decay, probes the final leg of a $\beta\beta$-transition and as such strongly depends on the strength of the *pp* interaction. Even worse, the QRPA model collapses as whole in the physical region of *t* [16, 21, 23]. Yet, the distinction between the initial and final legs in the $\beta\beta$-decay only makes sense in nuclei that possess an appreciable neutron excess, which doesn't happen in nuclei under discussion where $N \cong Z$. Moreover, the results of the PQRPA calculations in ^{12}C, displayed in Fig. 5 of Ref. [24] suggest that the choice $t = 0$ could be appropriate for the description of $N \cong Z$ nuclei. Therefore, this value of the *pp* coupling strength is adopted here.

Ratios of theoretical to experimental inclusive muon capture rates for different nuclear models are exhibited in Fig. 1. It is self evident that the number projection plays an important role in light nuclei with $A \lesssim 30$, making that the PQRPA agrees better with data than the plain QRPA. On the other hand it is difficult to judge whether our estimates are better or worse than the previous ones [11, 13].

We have found that the consequences of the violation of the CVC by the Coulomb potential [14] for the nuclei considered here is very tiny. In fact, the major effect appears in ^{56}Fe, where the total muon capture is reduced from $\Lambda = 4260 \times 10^3$ s^{-1} to $\Lambda = 4056 \times 10^3$ s^{-1}.

In the case of ^{12}C we have at our disposal also the experimental data for exclusive muon capture rates to bound excited states $J^\pi_n = 1^+_1, 2^+_1, 2^-_1$, and 1^-_1 in ^{12}B [2, 26]. They have been discussed previously in the framework of the PQRPA [12, 24], but for the sake of completeness we show them again in Table 1. The most relevant to highlight in this table is that, while both PQRPA calculations of the inclusive muon capture rates agree fairly well with the experiment, the corresponding exclusive reactions are very different in the two calculations. In other words, the agreement between theory and data for the inclusive muon capture does not guarantee the goodness of the model that is used.

Table 1: Energies (in units of MeV) and exclusive muon capture rates (in units of 10^3 s^{-1}) for the bound excited states in ^{12}B. Besides the present PQRPA result, we also show a previous one [12], as well as those evaluated within the SM [8], and the RPA [9, 10].

Model	J^π_n	1^+_1	2^+_1	2^-_1	1^-_1	Λ_{inc}
PQRPA	E	0.00	0.43	6.33	6.83	
	Λ	8.80	0.20	0.60	0.85	37
PQRPA [12]	E	0.00	0.50	2.82	3.31	
	Λ	6.50	0.16	0.18	0.51	40
SM [8]	E	0.00	0.76	1.49	1.99	
	Λ	6.0	0.25	0.22	1.86	
RPA [9, 10]	Λ	25.4 (22.8)	$\leq 10^{-3}$	0.04 (0.02)	0.22 (0.74)	
Exp. [2, 26]	E	0.00	0.95	1.67	2.62	
	Λ	6.00 ± 0.40	0.21 ± 0.10	0.18 ± 0.10	0.62 ± 0.20	38 ± 1

4. Final remarks

We have shown that, when the capture of muons is evaluated in the context of the QRPA, the conservation of the number of particles is very important not only for carbon but in all light nuclei with $A < 30$. The consequence of this is the superiority of the PQRPA on the QRPA in this nuclear mass region, as can be seen from Fig. 1.

The violation of the CVC by the Coulomb field in this mass region is of minor importance, since in (2.4) is $\Delta E_{\rm Coul} + E^\mu_B$ is $\cong 11.7$ MeV, which is small in comparison with m_μ. However, this effect could be quite relevant for medium and heavy nuclei studied in Refs. [4, 11]. For instance, for ^{208}Pb is $\Delta E_{\rm Coul} + E^\mu_B \cong 39.0$ MeV, which implies a reduction of the operator $O_{\emptyset J} - O_{0,J}$ for natural parity states by a factor 0.37, or equivalently that its contribution is only $\sim 13\%$ of that when the Coulomb field is not considered.

We agree with the finding of Kortelainen and Suhonen [22] on the extreme sensitivity of the muon capture rates on the *pp* coupling strength when described within the QRPA, as well as on a possible collapse of this approximation for the $J^\pi_n = 1^+_1$ state. Yet, in our opinion the QRPA behaves in this way dominantly in nuclei with a large neutron excess such as those analyzed in Refs. [11, 13]. It is clear that the RQRPA calculation [13] is sensitive to the *pp* coupling, while the RPA+BCS model [11] is not since it totally ignores the *pp* interaction.

Finally, we conclude that the comparison between theory and data for the inclusive muon capture is not a fully satisfactory test on a nuclear model. The exclusive muon transitions are much more robust with respect to such a comparison.

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References

[1] H. W. Fearing *et al.*, Phys. Rev. D **56**, 1783 (1997).

[2] D.F. Measday, Phys. Rep. **354**, 243 (2001).

[3] T.P. Gorringe and H. W. Fearing, Rev. Mod. Phys.**76**, 31 (2004).

[4] T.P. Gorringe, Phys. Rev. C **74**, 025503 (2006).

[5] W. Haxton and C. Johnson, Phys. Rev. Lett. **65**, 1325 (1990).

[6] E.K. Warburton, I.S. Towner and B.A. Brown Phys. Rev. **C 49**, 824 (1994).

[7] C. Volpe, N. Auerbach, G. Colò, T. Suzuki, N. Van Giai, Phys. Rev. C **62**, 015501 (2000).

[8] N. Auerbach and B.A. Brown, Phys. Rev. **C 65**, 024322 (2002).

[9] E. Kolbe, K. Langanke and S. Krewald, Phys. Rev. **C 49**, 1122 (1994).

[10] E. Kolbe, K. Langanke and P. Vogel, Phys. Rev. **C 50**, 2576 (1994).

[11] N.T. Zinner, K. Langanke and P. Vogel, Phys. Rev. C **74**, 024326 (2006).

[12] F. Krmpotić, A. Mariano and A. Samana, Phys. Lett. **B541**, 298 (2002).

[13] T. Marketin, N. Paar, T. Nikšić and D. Vretenar, Phys. Rev. C **79**, 054323 (2009).

[14] A.R. Samana, F. Krmpotić, N. Paar, and C.A. Bertulani, Phys. Rev. C **83**, 024303 (2011).

[15] A. Samana, F. Krmpotić, A. Mariano and R. Zukanovich Funchal, Phys. Lett. **B642**, 100 (2006).

[16] F. Krmpotić, A. Mariano, T.T.S. Kuo, and K. Nakayama, Phys. Lett. **B319**, 393 (1993),

[17] A.R. Samana and C.A. Bertulani, Phys. Rev. C **78**, 024312 (2008).

[18] J. D. Walecka, *Theoretical nuclear and subnuclear physics*, Imperial College Press and World Scientific, London (2004).

[19] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).

[20] J. Hirsch and F. Krmpotić, Phys. Rev. C **41**, 792 (1990).

[21] F. Krmpotić and Shelly Sharma, Nucl. Phys. **A572**, 329 (1994).

[22] M. Kortelainen and J. Suhonen, Europhys. Lett. **58**, 666 (2002).

[23] F. Krmpotić, Phys. Rev. C **48**, 1452 (1993).

[24] F. Krmpotić, A. Samana and A. Mariano, Phys. Rev. C **71**, 044319 (2005).

[25] A.R. Samana, F. Krmpotić and C. A. Bertulani, Comp. Phys. Comm. 181 (2010) 1123.

[26] T.J. Stocki, D.F. Maesday, E. Gete, M.A. Saliba and T.P. Gorrinde, Nucl. Phys. **A 697**, 55 (2002).

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