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2ν Double Beta Decay within the Relativistic QRPA

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We perform a self-consistent relativistic QRPA (RQRPA) calculation of $2\nu\beta\beta$-decay based on relativistic BCS (RBCS) mean field theory results for odd-odd intermediate nuclei ^{48}Sc, ^{76}As, ^{82}Br, ^{100}Tc, ^{128}I, and ^{130}I. The RBCS equations that resemble the non-relativistic ones are constructed from a Dirac-Gorkov variational functional. We use the parameter set $NL1$ for the σ, ω and ρ mesons. The RQRPA equations are solved for the residual $\pi+\rho$ interaction by employing the same parameters used in the RBCS for the latter meson, and experimental values for the pion and nucleon. The RQRPA results for the $2\nu\beta\beta$ matrix elements are similar to those obtained within the QRPA and the shell model.

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1. Introduction

In nature there are about 50 nuclear systems in which the single β decay is energetically forbidden, and $\beta\beta$ decay turns out to be the only possible mode of disintegration. It is the nuclear pairing force which causes such an "anomaly", by making the mass of the odd-odd isobar, $(N-1, Z+1)$, greater than the masses of its even-even neighbors, (N,Z) and $(N-2,Z-2)$. The modes by which this decay can take place are connected with the neutrino (ν)-antineutrino ($\tilde{\nu}$) distinction. In fact, they are defined by the transitions:

$$\begin{aligned} n &\rightarrow p + e^- + \tilde{\nu}_{RH}, \\ \nu_{LH} + n &\rightarrow p + e^-, \end{aligned} \tag{1.1}$$

the neutrino ν being left-handed (LH) and the antineutrino $\tilde{\nu}$ right-handed (RH) because of parity non-conservation in weak interactions. Therefore, regardless of the Dirac ($\nu \neq \tilde{\nu}$) or Majorana ($\nu = \tilde{\nu}$) nature of the neutrino and independently of conservation of helicity, the two-neutrino mode ($2\nu\beta\beta$) decay can occur by two successive single β-decays:

$$\begin{aligned} (N,Z) &\xrightarrow{\beta^-} (N-1,Z+1) + e^- + \tilde{\nu} \\ &\xrightarrow{\beta^-} (N-2,Z+2) + 2e^- + 2\tilde{\nu} \end{aligned} \tag{1.2}$$

passing through the intermediate virtual states of the $(N-1,Z+1)$ nucleus. Yet, the occurrence of the neutrinoless $\beta\beta$ decay ($0\nu\beta\beta$):

$$(N,Z) \xrightarrow{\beta\beta^-} (N-2,Z+2) + 2e^- \tag{1.3}$$

is much more convoluted since the right-handed neutrino emitted in the first step of (1.2) has the wrong helicity to be reabsorbed in a second step and to give rise to (1.3). For massless neutrinos there is no mixture of left and right handedness, and the $0\nu\beta\beta$-decay cannot occur, regardless of the Dirac or Majorana nature of the neutrino. Yet, experiments with solar, atmospheric and reactor neutrinos have provided remarkable evidence in recent years for the existence of neutrino oscillations driven by nonzero neutrino masses and neutrino mixing [1, 2]. Once the neutrino becomes massive, the helicity is no longer a good quantum number. Then, if the neutrino is in addition a Majorana particle with an effective mass $\langle m_\nu \rangle$, the mixture of ν_{LH} in ν_{RH} is proportional to $\langle m_\nu \rangle / E_\nu$, and $0\nu\beta\beta$-decay is allowed. [1] This fact inspired experimental searches in many nuclei, not only for the $0\nu\beta\beta$-decay but also for the $2\nu\beta\beta$-decay, since these two modes of disintegration are related through nuclear structure effects. In fact, their half-lives can be cast in the form:

$$T_{2\nu}^{-1} = \mathscr{G}_{2\nu} \mathscr{M}_{2\nu}^2, \qquad T_{0\nu}^{-1} = \mathscr{G}_{0\nu} \mathscr{M}_{0\nu}^2 \langle m_\nu \rangle^2, \tag{1.4}$$

where $\mathscr{G}'s$ are geometrical phase space factors, and the $\mathscr{M}'s$ are nuclear matrix elements (NME's). $\mathscr{M}_{2\nu}$ and $\mathscr{M}_{0\nu}$ present many similar features, to the extent that it is frequently stated that we shall not understand $0\nu\beta\beta$-decay until we understand $2\nu\beta\beta$-decay.

Quantum hadrodynamics (QHD) aims to describe the nuclear many-body system in terms of nucleons and mesons [3]. Proposed initially as a fully renormalizable quantum field theory, at

[1] For simplicity we assume that right-handed weak currents do not play an essential role in the neutrinoless decay.

present it is seen as an *effective field theory*, derivable, in principle, from quantum chromodynamics [4]. Relativistic mean field theory (RMFT), which can be thought as a mean field (Hartree) approximation to QHD, has been applied with great success during the last decades to account for nuclear matter saturation and the ground state properties of finite nuclei along the whole periodic table. Through a relativistic version of the random phase approximation (RRPA), various excited states and the Gamow-Teller (GT) and Fermi (F) resonances have been studied in this context as well [5, 6, 7, 8, 9, 10, 11, 12, 13]. When this approximation is based on the Hartree-Bogoliubov (HB) or BCS approximation, it is called the relativistic quasiparticle RPA (RQRPA). This approach has been used to evaluate several weak interaction processes, such as beta-decays, neutrino-nucleus reactions, and muon captures [14, 15, 16]. Here the first application of the RQRPA to $2\nu\beta\beta$-decay is made.

2. $2\nu\beta\beta$ matrix element

Independently of the nuclear model used and when only allowed transitions are considered, the $2\nu\beta\beta$ matrix element for the $|0^+_f\rangle$ final state reads [17]

$$\mathscr{M}_{2\nu}(f) = \sum_{\lambda=0,1}(-)^{\lambda}\sum_{\alpha}\left[\frac{\langle 0^+_f||\mathscr{O}^{\beta^-}_{\lambda}||\lambda^+_{\alpha}\rangle\langle\lambda^+_{\alpha}||\mathscr{O}^{\beta^-}_{\lambda}||0^+\rangle}{E_{\lambda^+_{\alpha}}-E_0+E_{0^+_f}/2}\right] \equiv \mathscr{M}^F_{2\nu}(f)+\mathscr{M}^{GT}_{2\nu}(f) \quad (2.1)$$

where E_0 and $E_{0^+_f}$ are, respectively, the energy of the initial state $|0^+\rangle$ and of the final states $|0^+_f\rangle$. The summation goes over all intermediate virtual states $|\lambda^+_{\alpha}\rangle$, and

$$\mathscr{O}^{\beta^-}_{\lambda} = (2\lambda+1)^{-1/2}\sum_{pn}\langle p||\mathrm{O}_{\lambda}||n\rangle\left(c^{\dagger}_{p}c_{\bar{n}}\right)_{\lambda}, \qquad \text{with} \qquad \begin{cases}\mathrm{O}_0 = 1 & \text{for F}\\ \mathrm{O}_1 = \sigma & \text{for GT}\end{cases} \quad (2.2)$$

are operators for the β^--decay. The corresponding β^+-decay operators are $\mathscr{O}^{\beta^+}_{\lambda} = \left(\mathscr{O}^{\beta^-}_{\lambda}\right)^{\dagger}$. The total $\beta^{\pm}$ strengths

$$S^{\beta^{\pm}}_{\lambda} = (2\lambda+1)^{-1}\sum_{\alpha}|\langle\lambda^+_{\alpha}||\mathscr{O}^{\beta^{\pm}}_{\lambda}||0^+\rangle|^2, \quad (2.3)$$

obey the well-known single-charge-exchange Ikeda sum rule (ISR) [18] for both the F and the GT transitions:

$$S^{\beta}_{\lambda} \equiv S^{\beta^-}_{\lambda} - S^{\beta^+}_{\lambda} = (-)^{\lambda}(2\lambda+1)^{-1}\langle 0^+|[\mathscr{O}^{\beta^+}_{\lambda},\mathscr{O}^{\beta^-}_{\lambda}]_0|0^+\rangle = N-Z. \quad (2.4)$$

Similarly, the $\beta\beta$-decay strengths

$$S^{\beta\beta^{\pm}}_{\lambda} = (2\lambda+1)^{-1}\sum_{f}|\langle 0^+_f|\mathscr{O}^{\beta^{\pm}}_{\lambda}\cdot\mathscr{O}^{\beta^{\pm}}_{\lambda}|0^+\rangle|^2 \quad (2.5)$$

obey the double-charge-exchange sum rules (DSR):

$$S^{\beta\beta}_{\lambda} = S^{\beta\beta^-}_{\lambda} - S^{\beta\beta^+}_{\lambda} = (2\lambda+1)^{-1}\langle 0^+|[\mathscr{O}^{\beta^+}_{\lambda}\cdot\mathscr{O}^{\beta^+}_{\lambda},\mathscr{O}^{\beta^-}_{\lambda}\cdot\mathscr{O}^{\beta^-}_{\lambda}]|0^+\rangle, \quad (2.6)$$

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which when evaluated give [19]:

$$
\begin{aligned}
S_F^{\beta\beta} \equiv S_0^{\beta\beta} &= 2(N-Z)(N-Z-1), \\
S_{GT}^{\beta\beta} \equiv S_1^{\beta\beta} &= 2(N-Z)\left(N-Z-1+2S_1^{\beta^+}\right) - \frac{2}{3}C,
\end{aligned}
\tag{2.7}
$$

where C is a relatively small quantity and is given by [19, Eq. (5)]. The DSR are as important to $\beta\beta$-decay as the ISR is for simple β-decay.

Contributions from the first-forbidden operators appearing in the multipole expansion of the weak Hamiltonian, as well as those from the weak-magnetism term and other second order corrections on the allowed $2\nu\beta\beta$-decay, have been examined rather thoroughly [20, 21].

3. Charge-exchange QRPA

The pn-QRPA was formulated and applied to the allowed $\beta^{\pm}$-decays and to the collective GT resonance by Halbleib and Sorensen (HS) in 1967 [22]. They solved the QRPA equation

$$
\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega_\alpha \begin{pmatrix} X \\ -Y \end{pmatrix}, \tag{3.1}
$$

within the pn quasiparticle (qp) space for the BCS vacuum

$$
|0^+\rangle = \prod_p (u_p + v_p c_p^\dagger c_{\bar{p}}^\dagger) \prod_n (u_n + v_n c_n^\dagger c_{\bar{n}}^\dagger) |\rangle, \tag{3.2}
$$

of the initial nucleus (N,Z), where $|\rangle$ stands for the particle vacuum. The transition matrix elements are:

$$
\begin{aligned}
\langle 1_\alpha^+ || \mathscr{O}_1^{\beta^-} || 0^+ \rangle &= \sum_{pn} \left[u_p v_n X_{pn;1_\alpha^+} + v_p u_n Y_{pn;1_\alpha^+} \right] \langle p || \mathrm{O}_1 || n \rangle, \\
\langle 1_\alpha^+ || \mathscr{O}_1^{\beta^+} || 0^+ \rangle &= \sum_{pn} \left[v_p u_n X_{pn;1_\alpha^+} + u_p v_n Y_{pn;1_\alpha^+} \right] \langle p || \mathrm{O}_1 || n \rangle,
\end{aligned}
\tag{3.3}
$$

the ISR (2.3) yields $N-Z$, and the ground state correlations (GSC) in (3.3) play an essential role in suppressing β^+-decay.

Intensive applications of the QRPA to $\beta\beta$-decay began only about 20 years later when Vogel and Zirnbauer [23] discovered that the β^+-decay suppression mechanism could also be invoked to explain the quenching of the $2\nu\beta\beta$ decay rates. Their adaptation of the HS model in essence implies: 1) To use a second BCS vacuum for the final nucleus $(N-2, Z+2)$, and to solve a second QRPA equation for the intermediate $\overline{1}^+$ states with the ISR equal to $N-Z-4$, and 2) To substitute (2.1) by the ansatz [24]:

$$
\mathscr{M}_{2\nu} = 2 \sum_{\alpha\alpha'} \frac{\langle \overline{1}_{\alpha'}^+ || \mathscr{O}_1^{\beta^+} || \overline{0}^+ \rangle \langle \overline{1}_{\alpha'}^+ | 1_\alpha^+ \rangle \langle 1_\alpha^+ || \mathscr{O}_1^{\beta^-} || 0^+ \rangle}{\omega_{1_\alpha^+} + \omega_{\overline{1}_{\alpha'}^+}}. \tag{3.4}
$$

To circumvent the nonphysical averaging procedure implicit in the overlap $\langle \overline{1}_{\alpha'}^+ | 1_\alpha^+ \rangle$, a different recipe for the application of the QRPA to the $\beta\beta$-decay has been introduced [25, 26, 27, 28, 29, 30],

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which continues to involve two BCS ground states but deals with only one set of QRPA solutions. Finally, the procedure has been simplified even more by performing a straightforward adaptation to the $\beta\beta$-decay of Cha's prescription for the evaluation of single β-decay [31] within the HS model, which implies solving both the BCS and QRPA equations for the intermediate $(N-1,Z+1)$ nucleus. The ISR then gives $N-Z-2$, and the above expression becomes [32]

$$\mathscr{M}_{2\nu} = \sum_{\alpha} \frac{\langle 1^+_\alpha||\mathscr{O}_1^{\beta^+}||0^+\rangle\langle 1^+_\alpha||\mathscr{O}_1^{\beta^-}||0^+\rangle}{\omega_{1^+_\alpha}}. \tag{3.5}$$

Numerical tests show that all the approaches above furnish quite similar results for $\mathscr{M}_{2\nu}$ [25, 26, 28, 29, 32]. Thus, for the sake of simplicity, we make use of the last expression in the present work.

When applied to the $\beta\beta$-decay, the QRPA turns out to be an uncomplete model, since it deals with 0 and 2 qp states only, while to evaluate Eq. (1.2) it is necessary to consider at least up to 4 qp states. Moreover, the QRPA can say nothing regarding the DSR given by (2.7), nor it can be used to describe the $\beta\beta$-decays to excited final states. How this can be implement is explained in Ref. [17].

4. Relativistic charge-exchange RQRPA

The RQRPA is based on the relativistic HB (RHB) approximation for the RMFT. It was formulated in Ref. [34] for charge-conserving excitations, and extended to charge-exchange excitations in Ref. [35]. In the present work we approximate the RHB equations and put them in a form that resembles the non-relativistic BCS equations. To do this we start from the variational functional

$$\begin{aligned} W &= \int d^3x d^3y \sum_t (U_t^\dagger(\mathbf{x}), V_t^\dagger(\mathbf{x})\gamma_0) \\ &\times \begin{pmatrix} (\omega_t+\mu_t)\delta(\mathbf{x}-\mathbf{y}) - h_t(\mathbf{x},\mathbf{y}) & \overline{\Delta}_t^\dagger(\mathbf{x},\mathbf{y}) \\ \overline{\Delta}_t(\mathbf{x},\mathbf{y}) & (\omega_t-\mu_t)\delta(\mathbf{x}-\mathbf{y}) - h_t(\mathbf{x},\mathbf{y}) \end{pmatrix} \begin{pmatrix} U_t(\mathbf{x}) \\ \gamma_0 V_t(\mathbf{x}) \end{pmatrix}, \end{aligned} \tag{4.1}$$

based on the Dirac-Gorkov equation [33, Eq.(39)] with notation $t = p$ or n. Here $U_t(\mathbf{x})$ and $V_t(\mathbf{x})$ are the normal and time-reversed Dirac spinors corresponding to solutions of this equation with positive and negative-frequency ω_t. The Lagrange multipliers μ_t are determined by requiring that the expectation values of the baryon number operators yield the desired values of Z and N. Dirac Hamiltonian operators $h_t(\mathbf{x},\mathbf{y})$ and pairing fields $\overline{\Delta}_t(\mathbf{x},\mathbf{y})$ are given by [33, Eqs. (40) and (49)]. Next we use the anzatz

$$\begin{pmatrix} U_t(\mathbf{x}) \\ \gamma_0 V_t(\mathbf{x}) \end{pmatrix} \longrightarrow \begin{pmatrix} u_t \mathscr{U}_t(\mathbf{x}) \\ v_t \mathscr{U}_t(\mathbf{x}) \end{pmatrix}, \tag{4.2}$$

where u_t and v_t are numbers, and $\mathscr{U}_t(\mathbf{x})$ are the Hartree mean-field wave functions, satisfying the equation $\int d^3y h_t(\mathbf{x},\mathbf{y})\mathscr{U}_t(\mathbf{y}) = \varepsilon_t \mathscr{U}_t(\mathbf{x})$, with ε_t being the single-particle energies. After performing this replacement one maximizes (4.1) with respect to the coefficients u_t, obtaining in this way the relativistic BCS (RBCS) equations for u_t and v_t, similar to the non-relativistic ones.

Since the pion does not participate in the RBCS, the Lagrangian density is determined once the masses of the nucleon and mesons σ, ω and ρ, the coupling constants of mesons with the

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nucleon, and the self-interaction constants of the meson σ, g2 and g3 are given. Several sets for these parameters are known in the literature. Here we use the $NL1$ set[36].

We solve the RBCS and the Klein-Gordon equations numerically by expanding the mesons fields and the fermions wave functions in complete sets of eigenfunctions of harmonic oscillator (HO) potentials. In actual calculations, the expansion is truncated at a finite number of major shells, with the quantum number of the last included shell denoted by N_F (N_B) for fermions (bosons). The maximum values are selected so as to assure the physical significance of the results. The oscillator frequency for fermions is given by $\hbar\omega_0 = 41A^{-1/3}$ MeV and the maximum number of oscillator shells for fermions and bosons is given by $N_F = N_B = 20$. The Coulomb field is calculated directly in configuration space. The same procedure was used by Ghambir *et al* [37] in their approach to the relativistic mean field.

The RQRPA equations (3.1) are solved by employing for the residual interaction the same parameters used in the RMFT to obtain the discrete basis of qp states within the RBCS approximation. Yet, in dealing with isovector excitations it is essential to include, together with the ρ meson, the π meson as well [5, 6]. Here, the experimental values of the pseudoscalar pion-nucleon coupling and the pion mass were used, *i.e.,* $f_\pi = 1.00$, and $m_\pi = 138.0$ MeV [5, 6]. Since Fock terms are ignored in the RMFT, for the sake of self-consistency we must omit the exchange matrix element of the residual interaction $V = V_\pi + V_\rho$ in the sub-matrices A and B.

5. Results

Table 1: Matrix elements $\mathscr{M}_{2\nu}$ in units of (MeV)$^{-1}$. The experimental values are obtained from the half-lives $T_{2\nu}$ compiled by Barabash [38] and the $\mathscr{G}_{2\nu}$ values from Ref. [39] for the bare axial-vector coupling constant $g_A^0 = 1.25$.

$\beta\beta$-Decay	EXP	SM [40]	SMren [40]	QRPA [29]	RQRPA
^{48}Ca $\rightarrow^{48}$Sc $\rightarrow^{48}$Ti	0.049 ± 0.003	0.047	0.055	0.058	0.066
^{76}Ge $\rightarrow^{76}$As $\rightarrow^{76}$Se	0.140 ± 0.005	0.116	0.206	0.064	0.055
^{82}Se $\rightarrow^{82}$Br $\rightarrow^{82}$Kr	0.098 ± 0.004	0.126	0.224	0.077	0.088
^{100}Mo$\rightarrow^{100}$Tc$\rightarrow^{100}$Ru	0.239 ± 0.007	–	–	0.065	0.062
^{128}Te$\rightarrow^{128}$I$\rightarrow^{128}$Xe	0.049 ± 0.006	0.059	0.116	0.076	0.076
^{130}Te$\rightarrow^{130}$I$\rightarrow^{130}$Xe	0.034 ± 0.003	0.043	0.085	0.061	0.070

In Table 1 the experimental values of the matrix elements $\mathscr{M}_{2\nu}$ are compared with values obtained from several calculations. The later critically depend on the adopted value for the effective axial-vector coupling constant g_A. The QRPA results [29], as well as the present ones correspond to $g_A = 1$, *i.e.,* to a quenching factor of $q = g_A/g_A^0 = 0.8$ when the bare value is $g_A^0 = 1.25$ [39]. Bearing in mind that only a very tiny fraction ($< 0.1\%$) of the sum rule $S_{GT}^{\beta\beta}$ goes into the final state $J_f^\pi = 1_1^+$ [17], we can say that both agree reasonably well with experiment. One should also remembered that the QRPA calculations have been monitored by the restoration of the $SU(4)$ symmetry while the RQRPA were not. In the shell model (SM) study [40] different q values were used in different nuclei, namely $q = 0.74$ in ^{48}Ca, $q = 0.60$ in ^{76}Ge, and ^{82}Se, and $q = 0.57$ in ^{128}Te, and ^{130}Te. These results are listed in the third column of Table 1. For the sake of comparison, the same results renormalized to $q = 0.8$ are shown in the fourth column (labelled as "SMren").

6. Conclusion

A variational functional based on the Dirac-Gorkov equation is used to obtain the RHB equations in the form of the non-relativistic BCS equations. The RQRPA equations (3.1) are solved for the residual $\pi + \rho$ interaction by employing for the latter meson the same parameters used in the RMFT. The RQRPA results for the $2\nu\beta\beta$ matrix elements are of the same order magnitude as those obtained within the QRPA and the SM. Bearing in mind the small fraction of double GT strength going to the 0^+ final state compared with the GT DSR, as well as the uncertainty involved in the quenching factor q, it is difficult to discern which of the three calculations is better and which is worse. Despite this we are planning to apply the RQRPA model to study the neutrinoless $\beta\beta$-decays using the formalism developed in Ref. [21] as well.

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