

Hadrons in a Dynamical AdS/QCD model

W. de Paula^{*†}

Dep. de Física, Instituto Tecnológico de Aeronáutica, CTA, 12228-900 São José dos Campos, Brazil

E-mail: wayne@ita.br

T. Frederico

Dep. de Física, Instituto Tecnológico de Aeronáutica, CTA, 12228-900 São José dos Campos, Brazil

E-mail: tobias@ita.br

We analyze holographic models that attempt to describe QCD-like theories focusing on 5-dimensional perspectives. We obtain the mode equations for the hadronic fields propagating in the bulk. We present the spectra for Scalars and High Spin Mesons in back-reacted geometries and we discuss the connection between confinement and the deformed geometries. Finally we discuss the application of AdS/QCD models to calculate decay constants of Mesons.

*XXXIV edition of the Brazilian Workshop on Nuclear Physics,
5-10 June 2011
Foz do Iguaçu, Parana state, Brasil*

^{*}Speaker.

[†]Supported by FAPESP

1. Introduction

Due to the non-linear structure of quantum chromodynamics (QCD), most analytical techniques for QCD rely on perturbation theory. For very high energies, asymptotic freedom ensures a small coupling constant, that allows one to use perturbative methods to obtain physical amplitudes. In opposition, for low energies, QCD is strongly coupled and therefore the perturbative expansion is meaningless. This explains why is so difficult to find analytical tools to analyze the low-energy sector of QCD. Furthermore, important properties of the strong interaction associated with the infrared (IR) physics such as confinement, mass gap and linear Regge trajectories remains unexplained by applying analytical methods to QCD.

Nowadays, different techniques have been developed to study non-perturbative aspects of QCD. Examples of such methods are QCD sum rules[1], Dyson-Schwinger equations[2] and lattice QCD[3], that requires massive numerical computations.

A remarkable contribution was done by Juan Maldacena in 1998 [4]. He proposed an exact map between a supersymmetric gauge theory, $N = 4$ SYM theory, in 4D flat space and Type IIB string field theory in 10D space-time $AdS_5 \times S_5$. The most interesting fact of this duality is that the strong-coupling regime of large- N gauge theories can be approximated by weakly coupled classical gravities, and vice-versa. Thus, one could use weak-coupling perturbative methods in one theory to investigate the strongly coupled dual theory. As QCD is a gauge theory, we can say that this duality gives some hope for a better understanding of the non-perturbative regime of strong interactions[5].

The basic difficulty to use this method to analyze strong-force physics lies on the fact that the gauge theory within the AdS/CFT duality is very different from QCD. In short, $N=4$ SYM theory has a conformal symmetry, whereas QCD breaks this symmetry at low energies and also the $N=4$ SYM theory is supersymmetric, whereas QCD does not have this symmetry.

Consequently, one should modify the AdS geometry to build a realistic gravity dual of QCD. Here, we are interested in discussing some attempts to construct gravity duals of QCD-like theories. In particular we focus on a phenomenological approach to describe QCD-like theories using 5D holographic models. The main idea is to use several QCD properties as input to build dual models. We review the Hard Wall model[6], Soft Wall model[7] and the Dynamical AdS/QCD model[8]. We also present the mass spectrum for Scalar Mesons and Higher Spins Mesons within the Dynamical AdS/QCD model.

2. Dynamical AdS/QCD model

The first application of AdS/CFT concepts to QCD was done by Polchinski and Strassler[6]. They introduce an IR cut-off in the AdS_5 space-time[9, 10]. It can be implemented by placing boundary conditions on the wave functions of the fields propagating in a slice of AdS_5 space-time. However, in contrast to the observed approximate linear Regge behavior[11], the hard-wall predictions for the squared masses of light-flavor hadrons depend quadratically on the radial excitation.

Linear Regge trajectories for high Spin Mesons were implemented in Ref. [7]. In this model an inert dilaton field is introduced in the AdS_5 metric background. However the vacuum expectation value (vev) of the Wilson loop in this model does not exhibit the area-law behavior that a linearly confining static quark-antiquark potential would generate. This is because the vev of the Wilson

loop in AdS/CFT is determined by the area of the dual string world sheet in the five-dimensional spacetime[12], i.e. it depends exclusively on the background geometry. Since the latter remains exact AdS₅ (and thus conformal) in this Soft-Wall model, the Wilson loop vev shows a non-confining perimeter law. A second, common shortcoming of a Soft-Wall background is that it is not solution of the 5D Einstein equations, i.e. it has no dynamics.

The main goal of the Dynamical AdS/QCD model that we have proposed in Ref.[8] is to overcome these two drawbacks of the AdS/QCD models. This model remove both difficulties by deriving a rather minimal AdS/QCD background which implements both the area law, i.e. linear confinement, dynamically and asymptotically linear Regge trajectories, consistent with the available data[13]. The model is composed by a scalar field (dilaton) propagating in a deformed AdS₅ metric. We write the metric as

$$ds^2 = e^{-2A(z)} \left(\eta_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}} - dz^2 \right), \quad (2.1)$$

where $\eta_{\hat{\mu}\hat{\nu}} = (1, -1, -1, -1)$, $x^{\hat{\mu}} = \{x^1, x^2, x^3, x^4\}$ is the four-dimensional coordinates and z is the holographic coordinate. We look for solutions of Einstein-equations coupled to a dilaton field that reproduces some features of QCD. In the following we present our results.

3. Higher Spins Mesons

The spin S string modes of the massive tensor fields $\phi_{M_1 \dots M_S}$ (in axial gauge) in the dilaton-gravity background can be rewritten in terms of reduced amplitudes $\psi_{n,S}$ which satisfy a Sturm-Liouville equation:

$$\left[-\partial_z^2 + \mathcal{V}_S(z) \right] \psi_{n,S} = m_{n,S}^2 \psi_{n,S}, \quad (3.1)$$

where the spin-dependent string-mode potential is $\mathcal{V}_S(z) = \frac{B'^2(z)}{4} - \frac{B''(z)}{2}$, $B = (2S-1)A + \Phi$.

The gauge/gravity dictionary identifies the eigenvalues $m_{n,S}^2$ with the squared meson mass of the boundary gauge theory. For Higher Spins Mesons, we found the following solution of Einstein equations

$$A(z) = \log(z\Lambda_{QCD}) + \frac{1 + \sqrt{3}}{2S + \sqrt{3} - 1} \frac{(z\Lambda_{QCD})^2}{1 + e^{(1-z\Lambda_{QCD})}}, \quad (3.2)$$

that reproduces the linear Regge trajectories. The associated dilaton field and potential are obtained by solving Einstein equations numerically. We thus obtain a complete solution of the Einstein-dilaton equations. For the particular case of the metric (3.2), we can obtain numerically the behavior of the dilaton potential $V(\phi)$ presented in figure 1.

A good analytical approximation to the spectrum for $\Lambda_{QCD} = 0.3$ GeV is (in units of GeV)

$$m_{n,S}^2 \simeq \frac{1}{10} (11n + 9S + 2), \quad (n \geq 1) \quad (3.3)$$

which implements the approximate universality of the linear trajectory slopes for light flavors explicit.

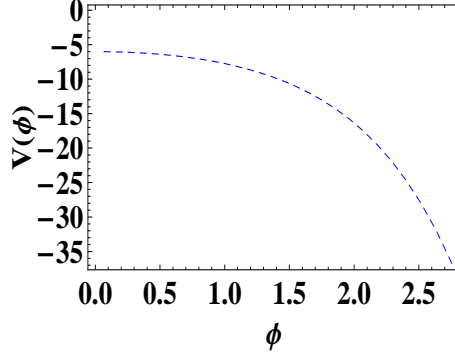


Figure 1: Dilaton potential for $\Lambda_{QCD} = 0.3$ GeV.

4. Scalar Mesons

We assume the same universal form of the metric, as given by

$$A(z) = \log(z\Lambda_{QCD}) + \frac{(\xi z\Lambda_{QCD})^2}{1 + e^{(1-\xi z\Lambda_{QCD})}}, \quad (4.1)$$

to describe the f_0 and pion families with a single parameter ξ .

- *Scalar Sector*

For the f_0 family we found $\xi = 0.58$ from the fit (see figure 2).

Comparing to the vector sector, the slope of the Regge trajectory for the scalar excitations is reduced (see figure 2) ($\xi < 1$). This means that in our model the size of the $f_0(600)$ should be larger than the size of other light mesons. In particular, scalar mesons were also analyzed in ([14] and [15]).

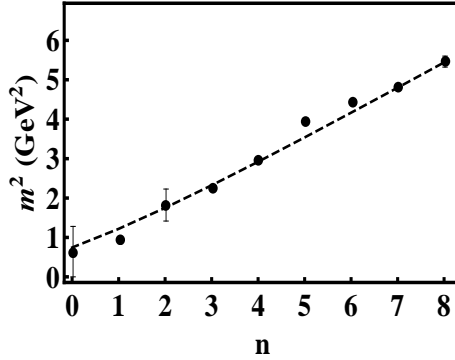


Figure 2: Regge trajectory for f_0 from the Dynamical AdS/QCD model[16] with $\Lambda_{QCD} = 0.3$ GeV (dashed line). Experimental data from [13].

- *Pseudoscalar Sector*

The first striking point is the slope of about 1 GeV^2 for the pion Regge trajectory, with a value twice that found for the scalars. This suggests that the scaling factor of the holographic coordinate

for the pseudoscalars should be changed with respect to that of the f_0 family. A scaling factor of $\xi = 0.76$ makes the IR effective potential of the pion the same as the one found for higher spin mesons[8]. By allowing a fine-tuning variation of about 15% to fit the actual data, we found $\xi = 0.88$. The almost vanishing pion mass is implemented by rescaling the fifth dimensional mass according to $M_5^2 \rightarrow M_5^2 - \lambda z^2$ (see [17]), where λ is uniquely determined as $\lambda = 2.19\text{GeV}^2$ [18].

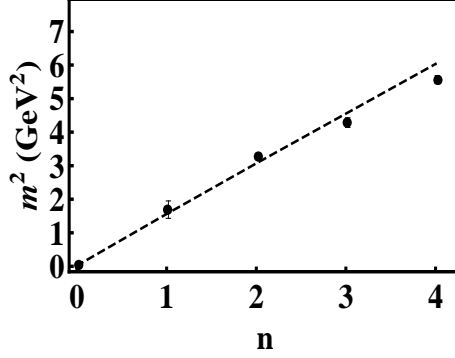


Figure 3: Regge trajectory for pion from the Dynamical AdS/QCD model[16] with $\Lambda_{QCD} = 0.3$ GeV (dashed line). Experimental data from [13].

We suggest that the f_0 's partial decay width into $\pi\pi$ can be calculated from the overlap integral of the normalized string amplitudes (Sturm-Liouville form) in the holographic coordinate dual to the scalars (ψ_n) and pion (ψ_π) states,

$$h_n = \lambda \Lambda_{QCD}^{-\frac{3}{2}} \int_0^\infty dz \psi_\pi^2(z) \psi_n(z). \quad (4.2)$$

We have introduced the parameter λ in the transition amplitude considering that it gives the natural scale for the coupling between the pion and a scalar, as has been obtained through the pion mass shift. Through dimensional analysis, one has to consider that the coupling has the dimension of $\sqrt{\text{mass}}$ and therefore a factor of Λ_{QCD} must be introduced to provide the correct dimension. We find that $\lambda \Lambda_{QCD}^{-\frac{3}{2}} = 13 [\text{GeV}]^{\frac{1}{2}}$, for $\Lambda_{QCD} = 0.3$ GeV, giving the results shown in table 1.

Table 1: Two-pion decay width and masses for the f_0 family. Experimental values from PDG[13]. (\dagger Mixing angle of 20° .)

Meson	$M_{exp}(\text{GeV})$	$M_{th}(\text{GeV})$	$\Gamma_{\pi\pi}^{exp}(\text{MeV})$	$\Gamma_{\pi\pi}^{th}(\text{MeV})$
$f_0(600)$	0.4 - 1.2	0.86	600 - 1000	602
$f_0(980)$	0.98 ± 0.01	1.10	$\sim 15 - 80$	47^\dagger
$f_0(1370)$	1.2 - 1.5	1.32	$\sim 41 - 141$	159
$f_0(1500)$	1.505 ± 0.006	1.52	38 ± 3	42
$f_0(1710)$	1.720 ± 0.006	1.70	$\sim 0 - 6$	6
$f_0(2020)$	1.992 ± 0.016	1.88	—	0.0
$f_0(2100)$	2.103 ± 0.008	2.04	—	1.4
$f_0(2200)$	2.189 ± 0.013	2.19	—	2.8
$f_0(2330)$	2.29 - 2.35	2.33	—	3.2

The overlap integral is the dual representation of the transition amplitude $S \rightarrow PP$ and therefore the decay width is given by $\Gamma_{\pi\pi}^n = \frac{1}{8\pi} |h_n|^2 \frac{p_\pi}{m_n^2}$, where p_π is the pion momentum in the meson rest frame.

5. Conclusions and Perspectives

To summarize, we discuss a Dynamical AdS/QCD model, solution of the five-dimensional Einstein-dilaton equations, which provides linear Regge trajectories for Scalar and Higher Spin Mesons. The method used in its derivation applies to essentially all asymptotically AdS₅ (and hence UV conformal) spacetimes with a Poincaré-invariant boundary.

The vacuum properties of the boundary gauge theory, including quark confinement, are dynamically encoded in this solution without the need for additional background fields. In particular, our background generates a confining area law for the Wilson loop (in contrast to the soft-wall model). We also calculated the f_0 's partial decay width into $\pi\pi$ with good agreement to experimental data. The next challenge is to describe Baryons[17] within this Dynamical model.

References

- [1] M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. **B 147** (1979) 385.
- [2] C. Roberts and A. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477.
- [3] J. Kogut, Rev. Mod. Phys. **55** (1983) 775.
- [4] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231.
- [5] M. Bianchi and W. de Paula, JHEP **04** (2010) 113.
- [6] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. **88**, 031601 (2002).
- [7] A. Karch, E. Katz, D.T. Son and M.A. Stephanov, Phys. Rev. **D 74**, 015005 (2006).
- [8] W. de Paula, T. Frederico, H. Forkel and M. Beyer, Phys. Rev. **D 79** (2009) 075019.
- [9] H. Boschi, N. Braga and H. Carrion, Eur. Phys. J. **C 32** (2004) 529.
- [10] G. F. de Téramond and S. J. Brodsky, Phys. Rev. Lett. **94** (2005) 0201601.
- [11] A.V. Anisovich and V.V. Anisovich and A.V. Sarantsev, Phys. Rev. **D 62** (2000) 051502.
- [12] J. Maldacena, Phys. Rev. Lett. **80** (1998) 4859.
- [13] C. Amsler *et al.*, Phys. Lett. **B** (2008) 667.
- [14] A. Vega and I. Schmidt, Phys. Rev. **D 78** (2008) 477.
- [15] P. Colangelo, F. De Fazio, F. Jugeau, S. Nicotri, Phys. Lett. **B 652** (2007) 73.
- [16] W. de Paula and T. Frederico, Phys. Lett. **B 693** (2010) 287.
- [17] H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
- [18] W. de Paula and T. Frederico, Int. J. Mod. Phys. **D 19** (2010) 1351.