

## Statistical properties of hot nuclei

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One of the basic ingredients of calculations of the statistical decay of hot nuclei is the density of states. Bonche, Levit and Vautherin [1] studied the statistical properties of hot nuclei using a static Hartree-Fock calculation at finite temperature that describes a hot nucleus in equilibrium with an external nucleon vapor and then extracting the contribution of the vapor. An alternative manner of calculating such properties is through the contribution of the bound particle states alone [2]. Here we perform calculations of the properties of hot nuclei using both formalisms in the relativistic Hartree approximation with the NL3 and DDME1 interactions. We compare the results obtained in the two formalisms.

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## 1. Introduction

The statistical decay of the compound nucleus is determined by the squared matrix element of the transition and the density of final states. In the usual Hauser-Feshbach and Weisskopf-Ewing evaporation models, the transition matrix element is expressed in terms of the transmission coefficients or the cross section of the inverse formation process. In the Fermi breakup and statistical multifragmentation models, the transition matrix element is taken to be equal for all decay channels. In all cases the density of quasi-bound final fragment states plays an important role.

The density of quasi-bound states of a nucleus is the density of states in which all neutrons are in bound single-particle states and all protons are in single particle states that are either bound or in long-lived single particle states well below the Coulomb barrier. We associate these states with the long-lived states of Bohr's conception of the compound nucleus.[1] The excitation energy dependence of the density of states was first estimated by Bethe [2] and has been the center of a great deal of theoretical and experimental effort [3-5]. All of these calculations of the density of quasi-bound states begin with a static set of single-particle states and analyze their occupation as a function of the temperature. The Helmholtz free energy determined in this manner,  $F^*(T)$  is related to the density of states  $\omega(E^*)$  by a Laplace transform,

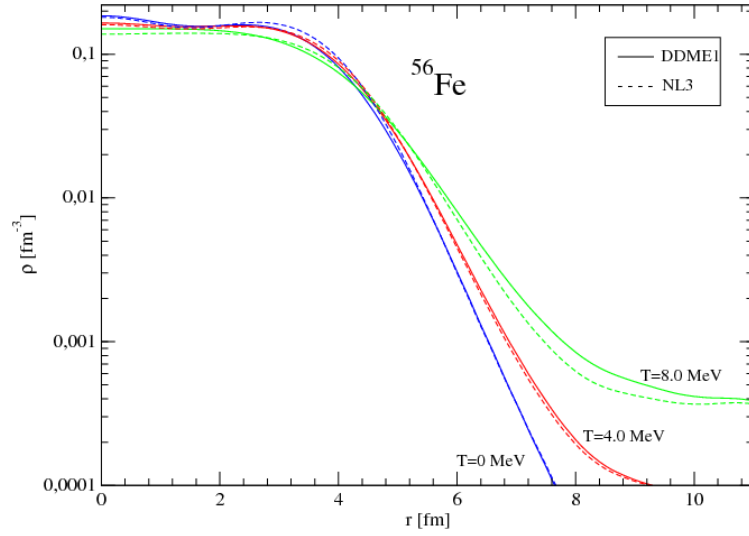
$$e^{-F^*(T)/T} = \int_0^{\infty} e^{-E^*/T} \omega(E^*) dE^*.$$

The Laplace transformation is then inverted to determine the density of states.

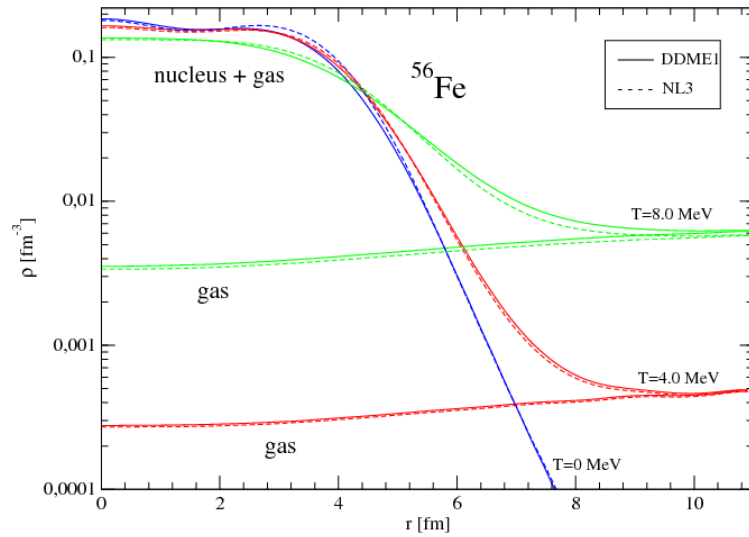
However, as a nucleus is heated, it expands and becomes less bound. At a sufficiently high temperature, it will evaporate completely. These effects are not taken into account when a static set of single-particle levels are used to calculate the density, but can be estimated from self-consistent calculations of hot nuclei. Here we take two approaches to such calculations. In the simplest, we perform temperature-dependent self-consistent mean field calculations restricted to the states that are quasi-bound at the given temperature. Similar calculations were performed years ago by Brack and Quentin. [6] Typical results of this type of calculation are shown in the Fig. 1, for the case of  $^{56}\text{Fe}$ . As the temperature increases, the nucleus expands and decreases in density. Although it is not visible in the figure, the curve associated with temperature  $T = 8.0$  MeV in Figure 1 goes to zero at  $r = 14$  fm.

Another approach, developed by Bonche, Levit and Vautherin, [7] takes into account all single-particle levels of a heated nucleus in the self-consistency calculation. To extract the contribution of the quasi-bound nucleons alone, the contribution of gas of unbound nucleons must be subtracted out. This is done by performing two calculations - one calculation of the nucleus plus gas and another of the gas alone (both with identical Fermi energies) and subtracting extensive quantities (entropy, excitation energy, baryon density) obtained for the latter from those for the former. Typical results from this type of calculation are shown in the Fig. 2. Again, as the temperature increases, the nucleus expands and decreases in density. The gas vanishes at zero density and increases with temperature until the nucleus disappears completely, typically at a temperature of about 12 MeV.

We have performed all calculations using the self-consistent relativistic Hartree approximation and the nonlinear NL3 [8] and density-dependent DDME1 [9] parameter sets. To our knowledge, these sets provide the best agreement with ground state nuclear masses obtained using the relativistic Hartree approximation.



**Figure 1:** Density with a function of radius for  $^{56}\text{Fe}$  with two different parametrizations NL3 and DDME1 for three different temperatures when we consider quasi-bound states only



**Figure 2:** Density with a function of radius for  $^{56}\text{Fe}$  with two different parametrizations NL3 and DDME1 for three different temperatures taking into account all single-particle levels

## 2. Results

As stated above, the extensive quantities that can be obtained from the self-consistent calculations are the entropy, the excitation energy, the baryon and charge density and density related quantities, such as deformations and rms radii. Deformation and pairing are found to vanish at extremely low temperatures. Normal isovector pairing usually disappears at temperatures below 1 MeV and almost all calculations yield nuclei that are spherical at 2 MeV [10].

The entropy and excitation energy are related to the Helmholtz free energy through

$$S^* = -\frac{\partial F^*}{\partial T},$$

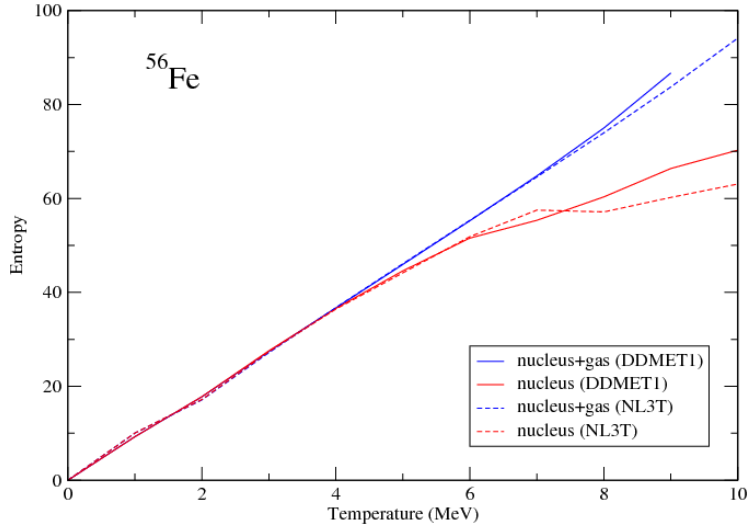
and

$$E^* = F^* - T \frac{\partial F^*}{\partial T}$$

The entropy  $S^*(T)$  can be determined directly from the calculations, while the excitation energy is obtained as the difference between the binding energy  $E_{bnd}(T)$  at a temperature  $T$  and that at temperature zero,

$$E^*(T) = E_{bnd}(T) - E_{bnd}(0)$$

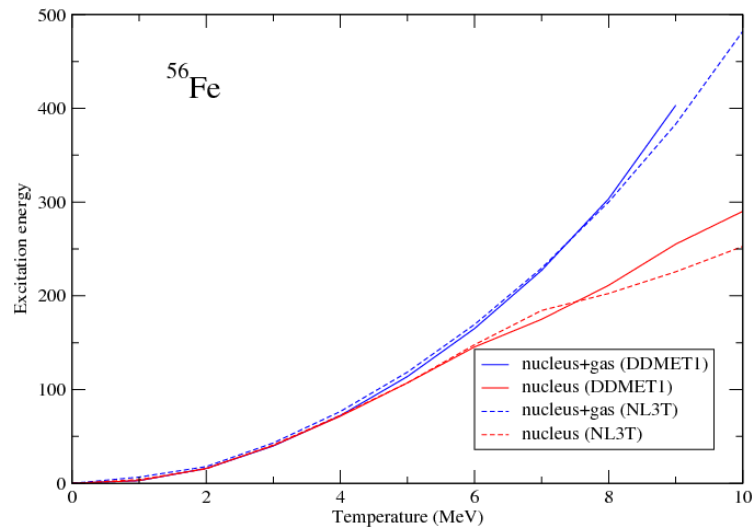
In the Fig. 3, we display the entropy obtained for the four different calculations in the case of  $^{56}\text{Fe}$ . All calculations furnishes similar values at temperatures below about 4 MeV. Above that value, the calculations using only the bound single-particle states display a deviation from the linearly increasing entropy that might be expected in this case.



**Figure 3:** Entropy with a function of temperature for  $^{56}\text{Fe}$  with two different parametrizations NL3 and DDME1

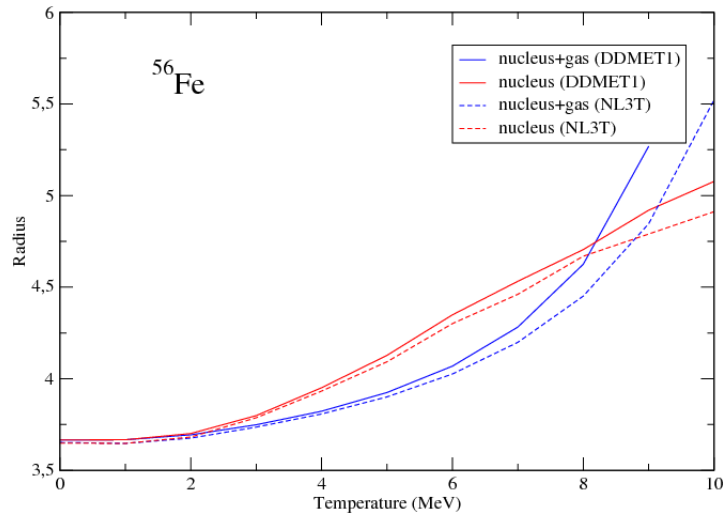
In the Fig. 4, we show the excitation energy obtained for the four different calculations in the case of  $^{56}\text{Fe}$ . Again, all calculations furnishes similar values at temperatures from zero up to 4 MeV and the expected deviation from a quadratically-increasing excitation energy above this value for the calculations using only the bound single-particle states.

Despite the very similar behavior of the entropy and excitation energy of the two sets of calculations below about 4 MeV, we see in the Fig. 5 that their rms radii are very different. The rms radius increases almost linearly in the calculations including only bound single-particle states while it increases quadratically and, except at very high temperatures, remains smaller in the case in which the nucleus is in equilibrium with a surrounding gas. We are examining the temperature



**Figure 4:** Excitation energy with a function of temperature for  $^{56}\text{Fe}$  with two different parametrizations NL3 and DDME1

dependence of the energy and rms radii of the single-particle levels to better understand the very different variations of the radius with the temperature.



**Figure 5:** Radius with a function of temperature for  $^{56}\text{Fe}$  with two different parametrizations NL3 and DDME1

## References

- [1] N. Bohr, Nature **137** (1936) 344.
- [2] H. A. Bethe, Rev. Mod. Phys. **9** (1937) 69.
- [3] C. Bloch, Phys. Rev. **93** (1954) 1094.
- [4] C. Bloch, *Statistical Nuclear Theory*, Les Houches Lectures (1968) 305.
- [5] A. Bohr and B.R. Mottelson, Nuclear Structure, vol. 1 (Benjamin, Reading, Mass., 1969) p.281
- [6] M. Brack and Ph. Quentin, Physica Scripta **A10** (1974) 163.
- [7] P. Bonche, S. Levit and D. Vautherin, Nuclear Physics **A427** (1984) 278.
- [8] G. A. Lalazissis, J. König and P. Ring, Phys.Rev. C **55**, 540 (1997).
- [9] T. Niksic, D. Vretenar, P. Finelli, and P.Ring, Phys. Rev. C **66**, 024306 (2002).
- [10] R. Lisboa, M. Malheiro, and B. V. Carlson, IJMP **16** (2007) 3032.