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Generalized Parton Distributions of the Photon in Position Space

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We report on a calculation of the generalized parton distributions of the photon when there is non-zero momentum transfer both in the transverse and longitudinal directions. By taking Fourier transforms of the GPDs with respect to the transverse and longitudinal momentum transfer, we obtain the parton distributions of the photon in position space.

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1. Introduction

In recent years, experimental facilities have allowed access not only to inclusive processes like deep inelastic scattering but also exclusive processes like the deeply virtual Compton scattering (DVCS) $\gamma^* p \to \gamma p$ or hard exclusive meson (M) production (HEMP)$\gamma^* p \to Mp$ .These processes are theoretically described by the generalization of the parton distributions (pdfs) to generalized parton distributions (GPDs) [1][2][3][4]. The generalized parton distributions are expressed as non-forward matrix elements of bilocal operators on the light-cone. These new distributions have been shown to be interesting theoretical tools for the study of hadrons and they connect through sum rules, to the hadronic form factors. Also their forward limit connects to the usual pdfs. Much of the interest in these quantities has been triggered by their potential to help unravel the spin structure of the nucleon, as they contain information not only on the helicity carried by partons, but also on their orbital angular momentum. The off-forward nature of the GPDs requires an additional variable for the description of the initial and final proton states, called the skewness variable, ζ, which reflects the longitudinal momentum transfer. Hence the generalized parton distributions, at a given scale, are functions of three variables namely they depend on x, the longitudinal momentum fraction of the struck parton, ζ and the square of the momentum transfer in the process $(-t)$.

The photon is one of the fundamental gauge bosons of the Standard Model without self couplings and without intrinsic structure. But at high energies when the virtuality of the probe is large the interactions are dominated by quantum fluctuations of photon into fermion-antifermion pairs. This is called photon structure. The photon structure function is understood fairly accurately and agrees well with experimental results [5]. For the GPDs of the photon, deeply virtual Compton scattering on a photon target $\gamma^*\gamma \to \gamma\gamma$ was considered in [6] in the kinematic region of large Q^2 but small squared momentum transfer ($-t << Q^2$). The momentum transfer was taken purely to be in longitudinal direction ($\Delta_\perp = 0$) in [6] and the photon GPDs were related with the Fourier transform of the matrix elements of light-front bilocal currents between initial and final photon states. The case when the momentum transfer is purely in the transverse direction ($\Delta_\perp \neq 0, \zeta = 0$) was studied in [7]. We consider here the case when there is non-zero momentum transfer both in the transverse and longitudinal directions ($\Delta_\perp \neq 0, \zeta \neq 0$). As the GPDs involve a momentum transfer (off-forwardness), they do not have probabilistic interpretation, unlike ordinary parton distributions (pdfs). However, it has been shown that when the momentum transfer is purely in the transverse direction ($\zeta = 0$), if one performs a Fourier Transform (FT) with respect to the transverse momentum transfer $\Delta_\perp$, one gets the so-called impact parameter dependent parton distributions $q(x, b^\perp)$ (ipdpdfs) [8][9][10]. Ipdpdfs of the photon tell us how the quarks of a given longitudinal momentum fraction x are distributed in the transverse position or impact parameter space at a distance $b^\perp$ from the center of the photon. These obey certain positivity conditions and unlike the GPDs, have probabilistic interpretation. We consider here the ipdpdfs of the photon when $\zeta \neq 0$. Unlike the case when $\zeta = 0$, photon GPDs do not have a probabilistic interpretation when $\zeta \neq 0$, however they now describe the parton distributions when the initial photon is displaced from the final photon in the transverse impact parameter space. The same description holds for proton GPDs as shown in [11]. We also consider the parton distribution of the photon in the longitudinal position space by introducing a longitudinal impact parameter σ conjugate to the skewness ζ. For an electron dressed with a photon in QED, it was found that the DVCS amplitude show a diffraction-like pat-

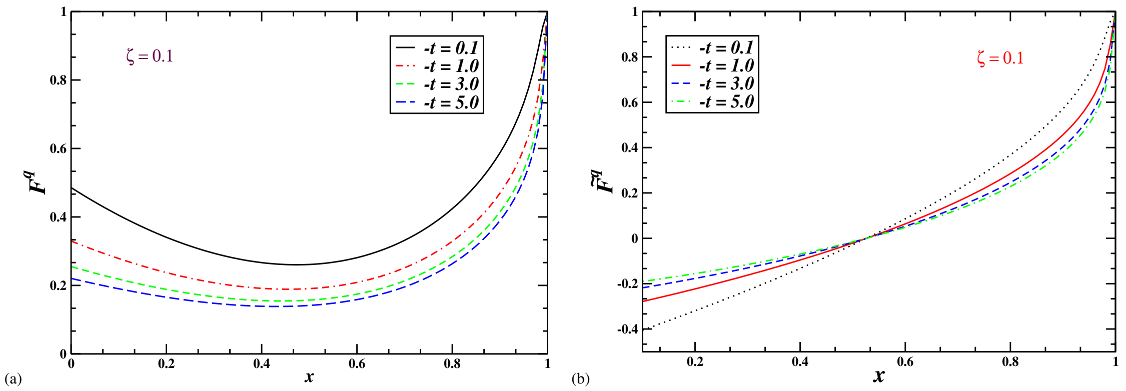

Figure 1: (Color online) (a) Plot of unpolarized GPD F^q vs x for fixed values of $-t$ in GeV^2 and at $\zeta = 0.1$ and (b) polarized GPD $\tilde{F}^q$ vs x for fixed values of $-t$ in GeV^2 and at $\zeta = 0.1$, $\Lambda = 12$GeV.

tern in longitudinal position space [12][13]. Similar diffraction patterns were observed for proton GPDs in the longitudinal position space in [14][15] which were found to be model dependent. We consider here the photon GPDs in both the transverse as well as longitudinal position space when the skewness is non-zero.

2. GPDs of the photon for non-zero skewness

For a real photon target state, we can define the photon GPDs as the following non-forward matrix elements [6]:

$$F^q = \int \frac{dy^-}{8\pi} e^{\frac{-iP^+y^-}{2}} \langle \gamma(P') \mid \bar{\psi}(0)\gamma^+\psi(y^-) \mid \gamma(P)\rangle;$$
$$\tilde{F}^q = \int \frac{dy^-}{8\pi} e^{\frac{-iP^+y^-}{2}} \langle \gamma(P') \mid \bar{\psi}(0)\gamma^+\gamma^5\psi(y^-) \mid \gamma(P)\rangle. \tag{2.1}$$

F^q and $\tilde{F}^q$ give the GPDs for unpolarized and polarized photon respectively. We choose the light-front gauge $A^+ = 0$. We consider the region $1 > x > \zeta$ and $-1 < x < \zeta - 1$ which corresponds to the diagonal contributions coming from the particle number conserving overlap of the light-front wave functions [16].

We work in the standard light-cone co-ordinates $x^\pm = x^0 \pm x^3$. We choose our frame such that:

$$P = \Big(P^+ \,,\, 0^\perp \,,\, 0 \Big) \,, \tag{2.2}$$

$$P' = \left((1-\zeta)P^+ \,,\, -\Delta^\perp \,,\, \frac{\Delta^{\perp 2}}{(1-\zeta)P^+} \right) \,, \tag{2.3}$$

The four-momentum transfer from the target is

$$\Delta = P - P' = \left(\zeta P^+ \,,\, \Delta^\perp \,,\, \frac{t + {\Delta^\perp}^2}{\zeta P^+} \right) \,, \tag{2.4}$$

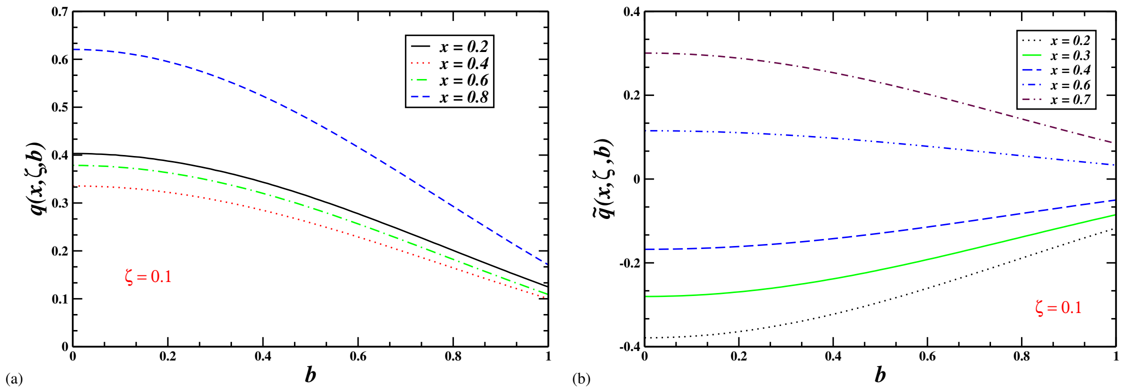

Figure 2: (Color online) (a) Plot of $q(x,\zeta,b)$ vs b and (b) $\tilde{q}(x,\zeta,b)$ vs b for fixed values of x and at $\zeta = 0.1$,where we have taken $\Lambda = 12$ GeV and Δ_{max}= 3 GeV where Δ_{max} is the upper limit in the Δ integration. The distributions are given in GeV^2 and b is in GeV^{-1}.

where $t = \Delta^2$. In addition, overall energy-momentum conservation requires $\Delta^- = P^- - P'^-$, which connects ${\Delta^{\perp}}^2$, ζ, and t according to

$$(1-\zeta)t = -{\Delta^{\perp}}^2. \tag{2.5}$$

The quark contribution comes from $1 > x > \zeta$ region and the anti-quark contribution comes from $-1 < x < \zeta - 1$ region. The photon GPDs are calculated in terms of the light-front wave functions of the target photon, which can be evaluated in perturbation theory [16].

3. Photon GPDs in impact parameter space

Parton distribution of photon in the transverse impact parameter space for non-zero skewness is defined as :

$$\begin{aligned} q(x,\zeta,b) &= \frac{1}{4\pi^2}\int d^2\Delta^{\perp} e^{-i\Delta^{\perp}\cdot b^{\perp}} F^q(x,\zeta,t) \\ \tilde{q}(x,\zeta,b) &= \frac{1}{4\pi^2}\int d^2\Delta^{\perp} e^{-i\Delta^{\perp}\cdot b^{\perp}} \tilde{F}^q(x,\zeta,t). \end{aligned} \tag{3.1}$$

As described in the introduction the parton distributions in impact parameter space describe the probability of finding a parton of definite momentum fraction x at a distance $b^{\perp}$ from the center of the photon. Here $b = \mid b^{\perp} \mid$ is the transverse impact parameter which is a measure of the transverse distance between the struck quark and the center of momentum of the photon. Unlike the case when skewness is zero, for non-zero skewness the transverse location of the photon is not the same before and after the scattering. This shift is independent of x but depends on the skewness ζ and $b^{\perp}$ as a result the information related to this transverse shift does not vanish when the GPDs are integrated over x. The impact parameter dependent pdfs would have been a delta function for a single quark. Thus the smearing in the $b_{\perp}$ space reveals the partonic content of the photon.

We introduce a coordinate b conjugate to the momentum transfer Δ such that $b.\Delta = \frac{1}{2}b^+\Delta^- + \frac{1}{2}b^-\Delta^+ - b_{\perp}\Delta_{\perp}$. Also $\frac{1}{2}b^-\Delta^+ = \frac{1}{2}b^-P^+\zeta = \sigma\zeta$ where we have defined the boost invariant variable

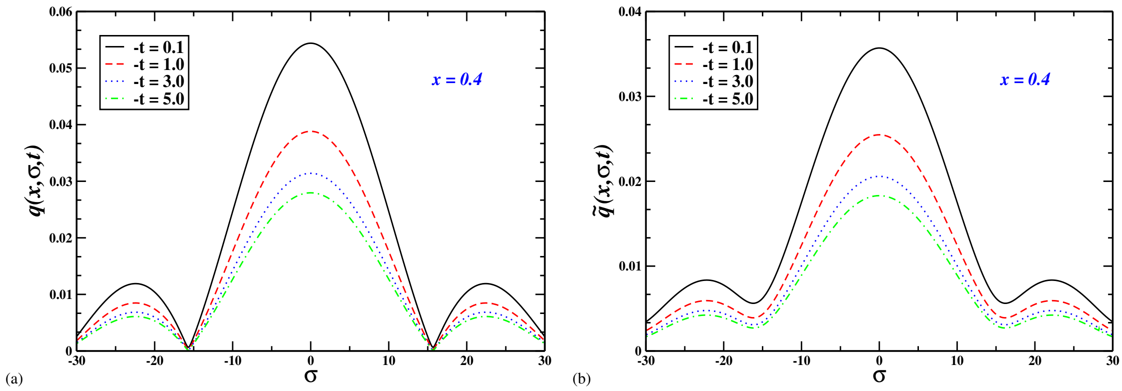

Figure 3: (Color online) (a) Plot of $q(x,\sigma,t)$ vs σ for a fixed value of $x = 0.4$ and different values of $-t$ in GeV^2 and (b) $\tilde{q}(x,\sigma,t)$ vs σ for a fixed value of $x = 0.4$ and different values of $-t$ in GeV^2 we have taken Λ = 12 GeV.

σ which is an 'impact parameter' in the longitudinal position space. The Fourier transform of the GPDs with respect to ζ allows one to determine the longitudinal structure of the target hadron in terms of the variable σ.

The photon GPDs in longitudinal position space is given by:

$$q(x,\sigma,t) = \frac{1}{2\pi}\int_0^{\zeta_{max}} d\zeta e^{i\zeta\sigma} F^q(x,\zeta,t)$$
$$\tilde{q}(x,\sigma,t) = \frac{1}{2\pi}\int_0^{\zeta_{max}} d\zeta e^{i\zeta\sigma} \tilde{F}^q(x,\zeta,t) \tag{3.2}$$

The upper limit of ζ integration ζ_{max} comes out as x in the region $\zeta < x < 1$. It was shown that DVCS amplitude shows a diffraction pattern in longitudinal impact parameter space in [12][13]. Similar diffraction patterns were observed for the proton GPDs in the longitudinal position space in [11]. Photon GPDs also show a diffraction like pattern in the longitudinal position space with distinct features as compared to the proton GPDs.

4. Results

The nature of the photon GPDs for non-zero skewness with respect to x is same as that observed for the case when skewness is zero [7]. This is true for both the polarized and unpolarized photon GPDs. Both F^q and $\tilde{F}^q$ become independent of t as $x \to 1$ as seen in Figs. 1(a) and (b) because in this limit all the momentum is carried by the quark in the photon. We took the upper limit in the momentum integration $\Lambda = 12$GeV and the quark mass to be $m = 0.0033$GeV. The Fourier transform of F^q and $\tilde{F}^q$ with respect to $\Delta^\perp$ for non-zero ζ as a function of b and a fixed value of x and for different values of $-t$ is shown in Figs. 2 (a) and (b). The GPDs in transverse impact parameter space for non-zero ζ probe partons inside the target photon when the initial photon is displaced from the final photon in the transverse impact parameter space. The upper limit in the $\Delta_\perp$ integration is taken to be $\Delta_{max} = 3$GeV. The Fourier transform of F^q and $\tilde{F}^q$ with respect to ζ as a function of the boost invariant variable σ at fixed value of x and for different values of $-t$ is shown in Figs. 3(a) and (b). The unpolarized photon GPD shows diffraction pattern in the longitudinal position space with a central maxima and several secondary maxima separated by well-defined minima as in Fig. 3(a). However no such prominent diffraction pattern is observed for the polarized GPDs as seen in Fig. 3(b). An analogy with the diffraction pattern in optics was given in [12][13]. The finite range of ζ integration acts as a slit of finite width necessary to produce the diffraction pattern. The height of the maxima of both $q(x,\sigma,t)$ and $\tilde{q}(x,\sigma,t)$ decrease with increasing $-t$.

5. Conclusions

We calculated the photon GPDs, both polarized and unpolarized in the general case when the skewness parameter was non-zero and when the momentum transferred was both in the longitudinal as well as transverse directions. We considered the distribution of the partons in the impact parameter space both in the longitudinal and transverse direction. We calculated the photon GPDs for the case when the helicity of the initial and the final photon states are the same or in other words the non-helicity-flip photon GPDs. The photon GPDs in the transverse impact parameter space for non-zero skewness shows the same features as observed for the photon GPDs when skewness is zero [7]. The photon GPDs in the longitudinal impact parameter space show interesting diffraction pattern analogous to those observed in optics. The diffraction pattern appears because of the finiteness of the ζ integration and it depends on x and $-t$.

6. Acknowledgments

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