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One-Loop Amplitudes for Multi-Jet Production at Hadron Colliders

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We present a numerical implementation for virtual corrections to multi-jet production at Next-to-Leading order. Using the algorithm of generalised unitarity we compute primitive amplitudes from tree-level input. These basic ingredients are then used to compute full colour and helicity summed corrections.

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1. Introduction

Next-to-leading order (NLO) corrections to observables with multi-particle final states allow for precise predictions of the complicated signal and background reactions measured at the Large Hadron Collider (LHC) at CERN. Recent years have seen rapid progress in the development of theoretical tools to handle the computation, and automation, of these challenging processes. On-shell approaches have been particularly successful at pushing the limit of multiplicity with predictions for $2 \to 5$ or higher processes being achieved [1–4]. Multi-jet corrections at NLO present an additional level of difficulty in the larger number of parton level processes in both virtual and real radiation contributions, nevertheless full NLO predictions have recently been achieved for a four jet final state [5].

New developments are continuing to improve the algorithms and open up the range and flexibility of available predictions with a large number under discussion at this conference [6–19]

In this conference note we present some new developments in the computation of one-loop multi-parton processes using the `NGluon` [20] c++ framework. The numerical library uses on-shell methods to compute the basic building blocks of the virtual contributions at NLO which we summarise in section 2. We then look at the construction of full colour and helicity summed interference with the born level amplitudes in section 3 before presenting some results of some performance tests in section 4. Finally we outline some future directions in the conclusions.

2. Primitive Amplitudes with `NGluon`

The basic building blocks, the primitive amplitudes, of our one-loop amplitudes are computed using the generalised unitarity algorithm [21–29] (for recent reviews on the subject see e.g. [30–32]) implemented into `NGluon` . The procedure extracts the coefficients of the scalar integral basis from products of one-shell tree-level amplitudes which are computed using Berends-Giele recursion relations. Some new features including multiple fermion primitive amplitudes and speed improvements through caching and re-using common objects in helicity and permutation sums have been described in a recent conference note [33]. The scalar loop integrals functions have been implemented in a number of public codes [34–36], the default choice in `NGluon` is `FF/QCDLoop`. The procedure follows in a similar vein to other successful approaches treating multi-gluon amplitudes [37–41].

An n-particle primitive amplitude is defined by and ordered set of n external particles and n internal propagators which can be represented by a parent diagram. The full set of independent primitives fall into two categories:

- Amplitudes with a mixture of quark and gluon propagators in the loop, $A_n^{[m]}$.
- Amplitudes with an internal fermion loop, $A_n^{[f]}$.

Each parent diagram can be uniquely specified by a particle in the first position and the propagator immediately before hand, a gluon for the mixed case and a fermion for the fermion loop. Each subsequent propagator will be determined by the external particle and propagator appearing before it, either a gluon, quark or blank/dummy propagator in the case that no vertex exists. Some examples are shown in figure 1.

The on-shell unitarity method employed in `NGluon` uses the Four-Dimensional Helicity (FDH) scheme. All tree level amplitudes can be computed in four dimensions using the spinor helicity formalism with rational terms extracted from mass shifted fermion and scalar amplitudes.

A strong cross check on the validity of the rational terms comes from non-trivial relations which lead to cancellations in super-symmetric Yang-Mills theories [21, 22, 42]. For the pure gluonic amplitudes, or those with a single fermion pair, the rational parts, $\mathscr{R}$, obey:

$$\mathscr{R}\left(A_n^{[m]}(1,\dots,n) - A_n^{[f]}(1,\dots,n)\right) = 0, \tag{2.1}$$

$$\begin{aligned}&\mathscr{R}\Big(A_n^{[m]}(1_{\overline{q}},2,\dots,(k-1),k_q,(k+1)\dots,n) + (-1)^n A_n^{[m]}(1_{\overline{q}},n,\dots,(k+1),k_q,(k-1),\dots,2)\\ &-A_n^{[f]}(1_{\overline{q}},2,\dots,(k-1),k_q,(k+1)\dots,n) - (-1)^n A_n^{[f]}(1_{\overline{q}},n,\dots,(k+1),k_q,(k-1),\dots,2)\Big) = 0.\end{aligned} \tag{2.2}$$

Since the rational terms are the most expensive parts of the computation of the primitive amplitudes using these relations to reduce the total number of rational terms leads to a considerable improvement in evaluation time.

3. Colour Summed Amplitudes

The full colour amplitudes can be written as a decomposition of colour factors and partial amplitudes,

$$\mathscr{A}^{(l)}(\{p\},\{h\}) = \sum_k C_k(\{a\},\{i\}) A_k^{(l)}(\{p\},\{h\}), \tag{3.1}$$

where l is the loop order.

In the above $\{i\}$ and $\{a\}$ represent fundamental and adjoint $SU(N_c)$ indices respectively while $\{p\}$ and $\{h\}$ represent the momenta and helicity. The one-loop partial amplitudes must be further decomposed into the primitive objects described in the previous section. For the pure gluonic case and the case of a single fermion pair such a decomposition is known to all-orders. As an explicit

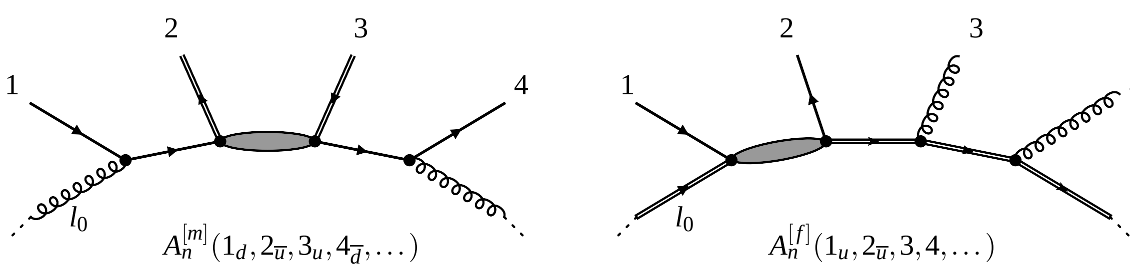

Figure 1: Examples of primitive amplitudes

example, the gluonic amplitudes are written as,

$$\mathscr{A}_n^{(0)} = \sum_{\sigma\in S_{n-1}/\mathbb{Z}} tr(1,\sigma_2,\ldots,\sigma_n)A_n^{(0)}(1,\sigma_2,\ldots,\sigma_n) \tag{3.2}$$

$$\begin{aligned}\mathscr{A}_n^{(1)} = &\sum_{\sigma\in S_{n-1}/\mathbb{Z}} N_c tr(1,\sigma_2,\ldots,\sigma_n)A_{n;1}^{(1)}(1,\sigma_2,\ldots,\sigma_n)\\ &+\sum_{c=3}^{[n/2]-1}\sum_{\sigma\in S_n/S_{n;c}} tr(1,\sigma_2,\ldots,\sigma_{c-1})tr(\sigma_c,\ldots,\sigma_n)A_{n;c}^{(1)}(1,\sigma_2,\ldots,\sigma_n),\end{aligned} \tag{3.3}$$

where

$$A_{n;1}^{(1)}(1,\ldots,n) = A_n^{[m]}(1,\ldots,n) - \frac{N_f}{N_c}A_n^{[f]}(1,\ldots,n), \tag{3.4}$$

$$A_{n;c}^{(1)}(1,\{\alpha\},\{\beta\}) = (-1)^n \sum_{\sigma\in \mathrm{COP}\{\alpha\}^T\{\beta\}} A_{n;1}^{(1)}(1,\sigma_2,\ldots,\sigma_n) \tag{3.5}$$

The sum ‘COP’ is over all possible mergings of the sets $\{\alpha\}=\{2,\ldots,c-1\}$ and $\{\beta\}=\{c,\ldots,n\}$ while keeping α^T fixed and using the cyclically ordered permutations of β. For the multi-fermion amplitudes the decompositions can be constructed by a systematic matching of terms to a Feynman diagram representation [31, 43].

The NLO corrections come from the interference between tree-level and loop-level amplitudes,

$$d\sigma^{\text{virtual}} = 2Re(\mathscr{A}^{(0),\dagger}\cdot\mathscr{A}^{(1)}) \tag{3.6}$$

Since the number of colour structures becomes extremely large with the increasing multiplicity it is important to reduce the colour basis as much as possible. We apply photon decoupling identities (and their generalisations to the multi-fermion case) in order to reduce the amplitudes to an $(n-2)!$ of tree amplitudes. As noted by Dixon, Del Duca and Maltoni [44], this has some rather striking simplification in the final representation of eq. (3.6).

	$A_n^{[0]}$	$A_n^{[m]}$	$A_n^{[f]}$
$\mathscr{A}_4(g,g,g,g)$	2	3	3
$\mathscr{A}_4(\overline{d},d,g,g)$	2	6	0
$\mathscr{A}_4(\overline{d},d,\overline{u},u)$	1	4	1

	$A_n^{[0]}$	$A_n^{[m]}$	$A_n^{[f]}$
$\mathscr{A}_5(g,g,g,g,g)$	6	12	12
$\mathscr{A}_5(\overline{d},d,g,g,g)$	6	24	9
$\mathscr{A}_5(\overline{d},d,\overline{u},u,g)$	3	16	3

	$A_n^{[0]}$	$A_n^{[m]}$	$A_n^{[f]}$
$\mathscr{A}_6(g,g,g,g,g,g)$	24	60	60
$\mathscr{A}_6(\overline{d},d,g,g,g,g)$	24	120	59
$\mathscr{A}_6(\overline{d},d,\overline{u},u,g,g)$	12	80	14
$\mathscr{A}_6(\overline{d},d,\overline{u},u,\overline{s},s)$	4	32	4

Table 1: The number of independent primitive amplitudes appearing in the colour sums at tree-level ($A_n^{[0]}$) and at one-loop for the mixed ($A_n^{[m]}$) and fermion loop ($A_n^{[f]}$) cases. Like-flavour amplitudes for multiple fermions are obtained by (anti-)symmetrization and therefore contain larger bases of primitives.

4. Performance Tests

4.1 Infra-Red Poles

The first important check on the implementation is the verification of the universal Infra-Red and Ultra-Violet poles in the dimensional regularisation parameter ε [45, 46]. For the interfered amplitude in massless QCD these take a rather simple form (in this case unrenormalised in the FDH scheme):

$$\begin{aligned} Re(\mathscr{A}^{(0),\dagger}.\mathscr{A}^{(1),FDH,U}) &= -\frac{1}{\varepsilon^2}\left(N_c n_g + \frac{N_c^2-1}{2N_c}n_q\right)|\mathscr{A}^{(0)}|^2 \\ &+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{1}{\varepsilon}\log\left(\frac{\mu_R^2}{|s_{ij}|}\right)|\mathscr{A}_{ij}^{(0)}|^2 - \frac{1}{\varepsilon}\left(\beta_0 + n_q\left(\frac{\beta_0}{2}-\frac{3}{2}\frac{N_c^2-1}{2N_c}\right)\right)|\mathscr{A}^{(0)}|^2 \\ &+ \text{ finite terms}, \end{aligned} \tag{4.1}$$

where n_q is the number of external quarks and n_g is the number of external gluons. N_c is the number of colours and n_f is the number of light flavours and $\beta_0 = \frac{11N_c-2N_f}{3}$. The quantity $|\mathscr{A}_{ij}^{(0)}|^2$ is the colour correlated Born amplitude defined by

$$|\mathscr{A}_{ij}^{(0)}|^2 = \mathscr{A}^{(0),\dagger}\cdot T_i\cdot T_j\cdot \mathscr{A}^{(0)}. \tag{4.2}$$

where $T_g = if^{abc}$ and $T_q = T^a_{ij}, (T_{\overline{q}} = -T^a_{ji})$. Scheme conversion to Conventional Dimensional Regularisation (CDR) and renormalisation are all corrections proportional to the Born amplitude and are defined as follows:

$$\frac{Re(\mathscr{A}^{(0),\dagger}.(\mathscr{A}^{(1),FDH,R}-\mathscr{A}^{(1),FDH,U}))}{|\mathscr{A}^{(0)}|^2} = -\frac{(n_g+n_q-2)}{2}\left(\frac{\beta_0}{\varepsilon}-\frac{N_c}{3}\right) \tag{4.3}$$

$$\frac{Re(\mathscr{A}^{(0),\dagger}.(\mathscr{A}^{(1),CDR,U}-\mathscr{A}^{(1),FDH,U}))}{|\mathscr{A}^{(0)}|^2} = -\frac{N_c}{3}-\frac{n_q}{4N_c}\left(\frac{N_c^2}{3}-1\right) \tag{4.4}$$

$$\frac{Re(\mathscr{A}^{(0),\dagger}.(\mathscr{A}^{(1),CDR,R}-\mathscr{A}^{(1),FDH,U}))}{|\mathscr{A}^{(0)}|^2} = -\frac{(n_g+n_q-2)\beta_0}{2\varepsilon}-\frac{N_c}{3}-\frac{n_q}{4N_c}\left(\frac{N_c^2}{3}-1\right) \tag{4.5}$$

4.2 Accuracy

We examine the accuracy of the amplitudes against the known pole structure and by using a re-scaling of the external momenta to test the finite terms. 10^5 phase-space points were generated using the RAMBO algorithm using rather weak kinematics cuts requiring only that $p_i\cdot p_j > 10^{-4}s$ for a centre of mass energy of $s = 7$ TeV.

The thinner lined histograms show the number of points re-evaluated in higher precision (quadruple precision using the `qd` package [47]). In every case, except for the six gluon amplitude, all re-evaluated points achieved the required relative accuracy of 10^{-4}. The number of points for re-evaluation reaches a maximum of 2.2% for six gluons and decreases rapidly with the number of fermion pairs. In this most complicated channel 4 from 100,000 events required octuple precision to reach the desired threshold, nevertheless they would likely be excluded by common LHC cuts. Rough evaluation times for the most complicated $2\to 4$ processes are of order $15-20s$ for full helicity and colour and including re-scaling to compute an accuracy estimate (see table 2).

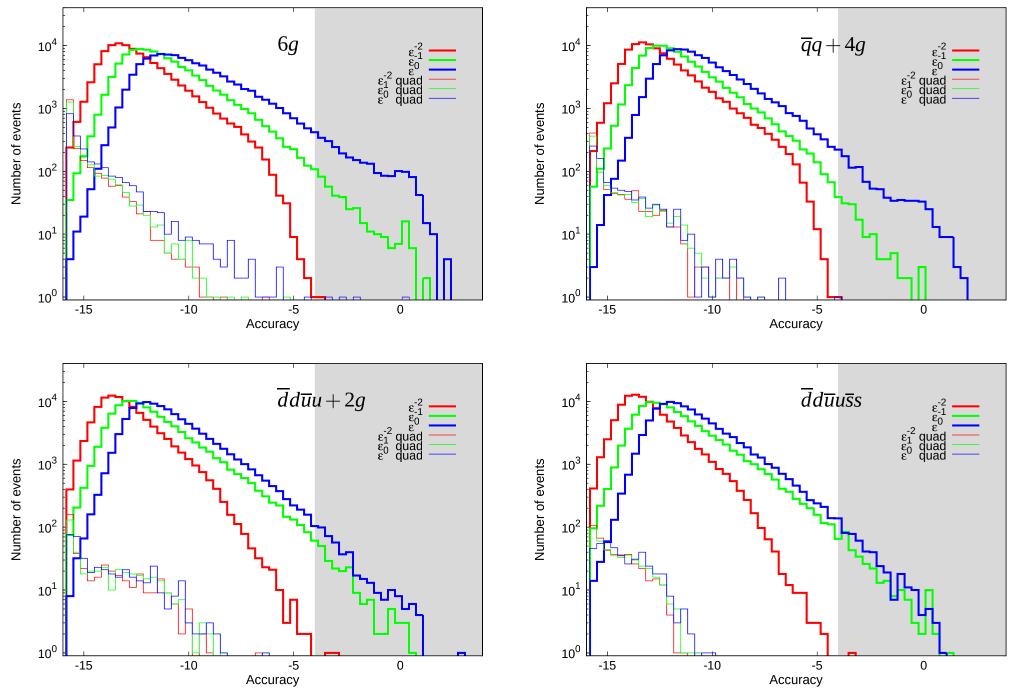

Figure 2: Accuracy plots for the $2 \to 4$ processes contributing to the 4-jet rate. Relative accuracy of 10^5 points over a flat phase-space. Points appearing in the shaded region are re-evaluated in quadruple precision.

n	4	5	6
n-gluons	0.03s	0.5s	15.0s
$q\bar{q} + (n-2)$-gluons	0.03s	0.75s	17.0s

Table 2: Estimated evaluation times (s) for n-gluon and $q\bar{q} + n$-gluon amplitudes using Intel Core i7 2.7GHz.

5. Conclusions

We have described the implementation of full colour QCD virtual contributions to NLO multi-jet production at hadron colliders. The primitive amplitude decomposition and use of on-shell generalised unitarity shows to be a fast and accurate method for the evaluation which scales well with the number of final state particles. We are looking forward to some phenomenological applications in the near future.

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