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The triple alpha reaction

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A new three-body method (R-matrix hyperspherical harmonics) has been developed to calculate the triple alpha reaction rate at low temperature. This new method provides rates that are larger by several orders of magnitude than rates included in the standard databases, at temperatures below 0.06 GK. Rates above T=0.07 GK are consistent with previously used values. The modification of the triple-alpha rate at these low energies does not affect low-mass stellar evolution.

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1. Motivation

The triple alpha reaction is responsible for the production of ^{12}C and is, thus, considered one of the most important reactions for the existence of life. This reaction occurs in all stars with mass $> 0.8\ M_{\odot}$and its rate sets the properties of the ending white dwarf, namely the core mass and the C-O ratio. For massive stars, it is the triple-alpha reaction that controls the iron core mass and the production of some nuclides, such as ^{44}Ti and ^{60}Fe. The triple-alpha rate is also important for the ignition of classical novae [1] and type I X-ray bursts [2, 3]. For all these reasons and more, it is crucial to have a reliable rate for the whole relevant temperature range.

As soon as the alpha particles reach energies above 0.1 MeV, the ^{8}Be ground state resonance can be populated and the preferred path for the triple-alpha process is a 2-step process, where 2 alpha particles combine to form the ^{8}Be ground state, and then a third alpha captures into the ^{12}C 0^{+} resonance, the so-called Hoyle state. Once the Hoyle state is formed, it has a known decay to the first 2^{+} state in ^{12}C. For the fraction of alpha particles with relative energy below the ^{8}Be resonance the most likely process is the 1-step true three-body capture, where all three alpha fuse simultaneously.

Until recently, nuclear astrophysics databases [4, 5, 6] took extrapolations of the 2-step rates to the lowest temperatures. In the last few years, work by various groups have revisited the triple-alpha reaction at low temperatures, with genuine three-body formulations [7, 8, 9, 10]. In [10] a simple extrapolation method is proposed based on a Breit Wigner extrapolation from a three-body resonance. Already with this simple prescription, it is shown that assuming the 2-step or the 1-step approach can provide 7 orders of magnitude difference in the rates at 0.02 GK. The results in [10] provide justification for investing in a detailed three-body scattering method.

The Madrid-Aarhus group [7, 8] uses a three-body formulation based on the adiabatic hyperspherical harmonics with complex scaling, however the method could not reach temperatures below 0.1 GK. The method by the Kyushu group [9], based on the continuum discretized coupled channel method, was able to reach temperatures below 0.1 GK and produces an extraordinary increase of the rate below the Hoyle state. This extreme increase of the triple-alpha rate motivated astrophysicists to explore the consequences. Stellar evolution simulations [11, 12] demonstrate that rates from [9] are seriously inconsistent with observed stellar populations.

While the method by [9] included both resonance and non-resonant capture on equal footing, the asymptotic behavior of the truncated basis it uses may not be reliable. Since the triple-alpha reaction at low energy occurs mostly at large distances, outside the range of the nuclear force, this could represent an important drawback. Here we will summarize a method developed to handle the low energy scattering of three identical charged particles, with the aim of solving the controversy on the low temperature triple-alpha rate. The work has been reported in [13, 14]. In Section II we briefly describe the method and in Section III we compare it to other rates that are readily available in standard astrophysics databases.

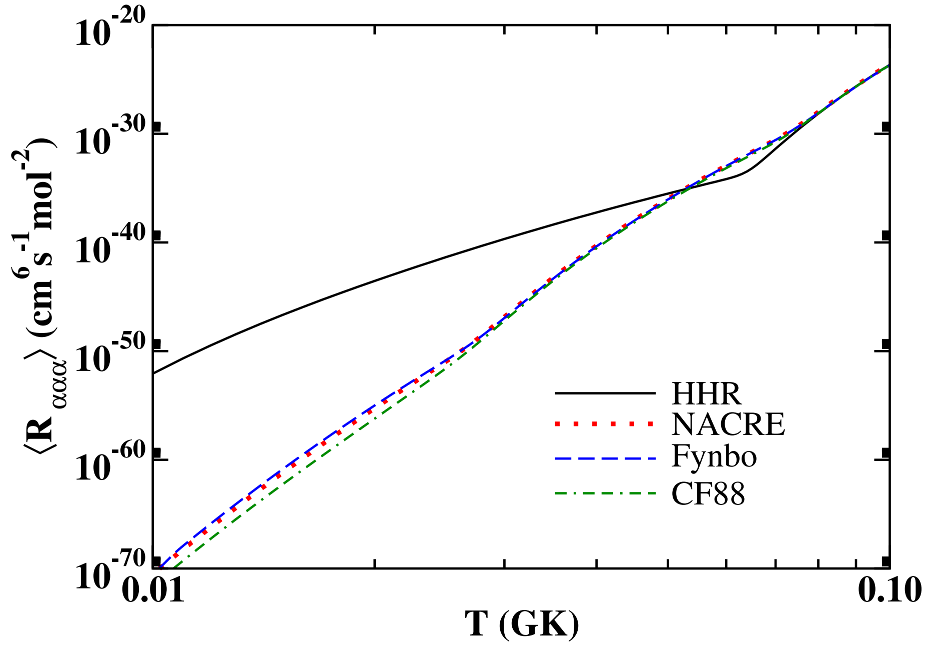

Figure 1: (Color online) The triple-alpha reaction rate: comparing the Hyperspherical Harmonic R-matrix method (solid) with NACRE (dotted), Fynbo (dashed) and CF88 (dot-dashed).

2. The method to tackle the triple-alpha reaction

The scattering problem for three-charged particles is challenging due to the long range Coulomb force. The asymptotic behavior of three particles is naturally expressed in the hyperspherical coordinates but there is no analytic form when Coulomb interactions are present. In [13] we have combined several techniques to be able to address the problem, in what we call the HHR method.

The problem is written in terms of hyperspherical coordinates [15]. We expand the three-body wavefunction in hyperspherical harmonics (the corresponding equivalent of spherical harmonic in the two-body problem) and arrive at hyperspherical coupled channel equations [14, 16]. These equations are not easy to solve through direct integration at low energies due to the strong off diagonal couplings. We use R-matrix techniques to solve the problem in a box (roughly 50 fm). Because there is no analytic solution to the full coupled channel equations in this region, we need to propagate the R-matrix to very large radii [17]. Around 10^3 fm we screen the off-diagonal couplings so enable a stable matching condition which is typically applied around 3×10^3 fm. Details of the procedure can be found in [14].

The main advantage of the HHR method over [9] is the treatment of the asymptotic behavior of the three-body scattering state. In addition, our work uses interactions that exactly reproduce the two relevant resonances (namely the ^{8}Be ground state and the ^{12}C Hoyle state). We do find a strong dependence on the choice of the alpha-alpha interaction, demonstrating it is important to constrain this interaction with elastic scattering data [18]. The normalization of that rate at temperatures higher than 0.07 GK is obviously dependent on the width of the Hoyle state and therefore this feature needs to be carefully studied [14].

The averaged rates resulting from the HHR calculations are shown in Fig.1. Our results show a

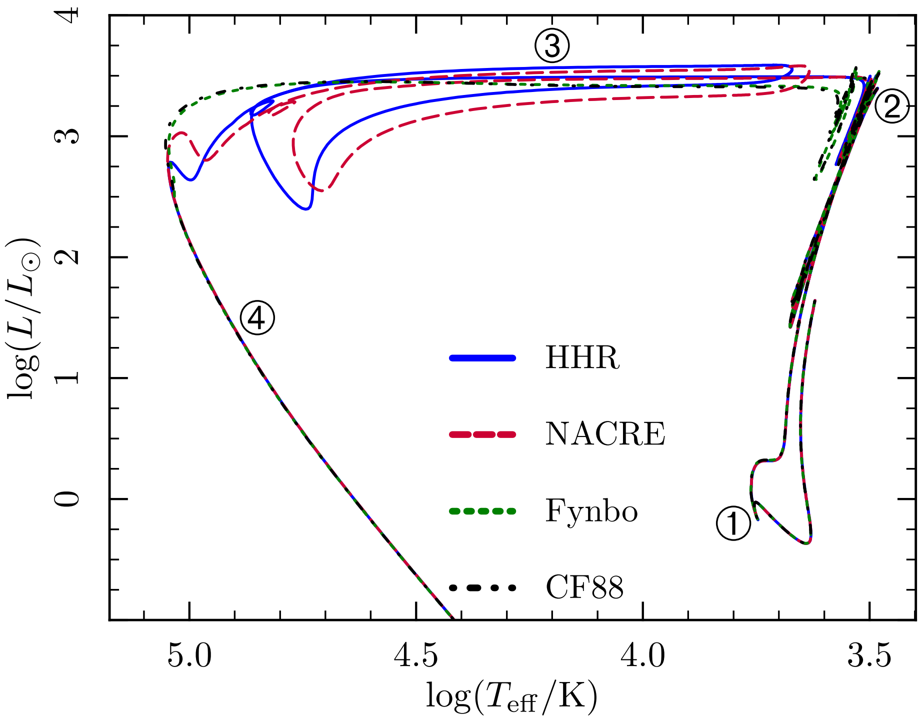

Figure 2: (Color online) HR diagram for one solar mass star: comparing the Hyperspherical Harmonic R-matrix method (solid) with NACRE (dashed), FYNBO (dotted) and CF88 (dot-dashed). See text for more detail.

clear transition around $T = 0.06$ GK. Below this energy, the non-resonant direct capture mechanism is dominant. Above this energy, it is the 2-step process, going through the Hoyle state, that dictates the rate. Note, however, that these two processes are coherent: they are not obtained through separate calculations and added afterwards. As such, interference effects can be observed when both processes are competing. This is exactly what happens around $T = 0.06 - 0.07$ GK, where a dip in the rate appears.

3. Comparing our rate with rates in databases

The HHR method aims at solving the problem for low temperature. For the higher temperatures, energies and widths of resonances need to be accurately reproduced. Although we fit the three-body force in our problem to reproduce exactly the location of the Hoyle state, our estimated width differs somewhat from the experimental value. We have thus chosen to normalize our rates at the higher temperatures (above 0.08 GK) to NACRE [4]. This choice is rather arbitrary and in this section we explore its consequences.

In Fig.1 the HHR rate is compared to NACRE and to other rates typically used in astrophysics, namely Fynbo [5] and CF88 [6]. We focus on temperatures below 0.1 GK and show that even though there are significant differences in the rates typically used in astrophysical modeling, our HHR rate represents a huge increase relative to all the above references.

Next we use MESA (version 3635) [19] to produce stellar evolution tracks of $1M_\odot$stars. Fig.2 contains the evolutionary tracks for a one solar mass star and Fig.3 illustrates the corresponding surface luminosity relative to solar as a function of age. In both Fig.2, we compare the results obtained taking our triple-alpha rate with those when rates are taken from [4], [5] and [6]. Since we have chosen to normalize our rates at higher temperature to NACRE, differences between HHR and NACRE reflect the influence of the low temperature region only. As discussed in [13], stars with the HHR triple-alpha rate evolve just like the NACRE reference: they have the same behavior in the H burning phase (indicated with (1) in Fig.2), and the AGS phase all the way up to the He flashes (indicated with (2) in Fig.2). We consider insignificant the minor differences during the He flashes and the ejection of the envelope (indicated with (3) in Fig.2). These stars end up in the same white dwarf (indicated with (4) in Fig.4), with unchanged C-O ratio (1 : 2.08). Similar results were found for $1.2M_\odot$stars.

The reason why the dramatic increase of the triple-alpha rate does not significantly impact stars around one solar mass is that in stars He burning occurs mostly above 0.07 GK, and thus the triple-alpha process is dominated by resonances. However there are other sites, where conditions are colder and where He burning can occur below 0.07 GK, such as novae with slow accretion rates and super bursts. Studies along these lines are in progress and will be reported elsewhere.

Fynbo and CF88 follow identical tracks, but there are noticeable differences between these and the NACRE track. Given that the Fynbo, CF88 and NACRE rates below 0.1 GK are relatively consistent, these differences must arise due to the treatment of high energy resonances [20]. The $1M_\odot$star results in a final white dwarf with C-O ratio of 1 : 2.37 when using Fynbo reaction rate. This ratio is about 12% smaller than that from NACRE rate. This issue, unrelated to the behaviour at low temperature, is important and must be resolved.

4. Summary

In summary, we have developed a three-body method based on the hyperspherical harmonics R-matrix method (HHR) to calculate the triple-alpha rate at low temperatures. It pays special attention to the inclusion of the long-range Coulomb force and the impact on the asymptotic behavior of the three-alpha scattering wavefunction. We find that our rates are largely enhanced at temperatures below 0.06 GK compared to the standard rates used previously and available in astrophysics databases. There is no impact of our new rates in stellar evolution for stars around one solar mass. Studies of the impact of our HHR results on other astrophysical environments, where He burning occurs at lower temperature, is underway.

In comparing the rates available in databases, we found that the differences between the rates from NACRE and from Fynbo, caused by a different treatment of the higher energy resonances, impact the evolution of low-mass stars. This aspect can have impact in other astrophysics simulations and should be quickly resolved.

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