

## Indirect methods for nuclear astrophysics: reactions with RIBs. The ANC method

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**Abstract.** Indirect methods that use nuclear reactions with rare isotope beams (RIB) at laboratory energies (1-10-100 MeV/nucleon) to extract nuclear structure information, which is then employed in nuclear astrophysics, are presented. We use data from reactions at such large energies to evaluate reaction cross sections at very low energies: 10s-100s keV. In many cases the indirect methods are our only choice. I select only a few types of reactions, including transfer and nuclear breakup reactions at intermediate energies. I will insist on the need for good theories and codes, as well as for better data with stable beams/targets, to relate the experimental data we measure with RIBs to astrophysical S-factors.

**Keywords:** nuclear astrophysics; H-burning; rare isotope beams; indirect experimental methods.

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## 1. Introduction

The use of indirect methods in nuclear astrophysics is prompted by the known difficulties that one encounters in attempting to make direct nuclear astrophysics measurements. Direct experiments mean trying to measure exactly the reactions that happen in stars, in those exact conditions (targets and projectiles, energies, charge states, etc...). The main difficulties arise because:

- In stars many reaction partners are unstable nuclei, and some are so shortly leaved that even with the recent advances in the rare isotope production they are not available, or not easily available, for the exact projectile-target combination at the energies they have in stars.
- Stars are cold! Compared with the energies typical in the nuclear laboratories, the energies of the partners in stars are very small (10s-100s keV) and the corresponding cross sections, in particular when charged particles are involved, are very small, therefore difficult to measure.

We have to resort to indirect methods. Several such methods are known in literature, some dedicated and labeled as such, some not. All these experiments are done at laboratory energies (1-10-100 MeV/nucleon) to extract nuclear structure information. This nuclear structure information is then used for nuclear astrophysics, that is, to evaluate reaction cross sections at low energies (10s-100s keV) and the resulting reaction rates at stellar temperatures. There are two steps here where theoretical calculations occur, and these calculations need to be seriously tested, well parameterized and tested using a large variety of data. For this, the use of good quality data with stable beams is still crucial. Another important practice is also to check the results of indirect methods with those from direct methods, whenever possible!

In this lecture I will present two of these indirect methods:

- A. One-nucleon transfer reactions (the ANC method)
- B. Breakup reactions at intermediate energies

In another lecture I will present a third one: decay spectroscopy. In all three cases they are being used to evaluate reaction rates for radiative proton capture, with the difference that the first two are applied to find the continuum (non-resonant) component of the reaction cross sections, while the latter is used for resonant capture.

This being a school, I will not attempt below to be exhaustive in the description of the two methods, but rather to be illustrative. I will also prefer to use relevant cases as illustrations, not necessarily 'newest' data. Moreover, most if not all of the examples will be from work done in the group I am working at Texas A&M University, even though many groups in the world have by now accepted these methods and are using them.

## 2. The ANC method

### 2.1 One-nucleon transfer reactions

A direct transfer reaction is characterized by the rearrangement of only a few nucleons during a fast process. In the early days of nuclear physics, nucleon transfer reactions were the tool to study the single-particle degrees of freedom of nuclei and were crucial in establishing our current understanding of the structure of nuclei. Typically spectra of final states and angular distributions were measured. Due to the direct character of the interaction, the tool of choice for the description of transfer reactions was the Born Approximation, either in the Plane Wave (PWBA), or the Distorted Wave (DWBA) form. By comparing the shape of the measured angular distributions with DWBA, the quantum numbers  $nlj$  of the single-particle orbitals involved could be determined, and by comparing the absolute values of experimental cross sections with those calculated, the spectroscopic factors  $S_{nlj}$  were found for the states populated. The spectroscopic factor is proportional to the "probability" that a many-body system is found in a given configuration. In the case we are talking about, single particle orbitals  $nlj$ , the classical definition (from Macfarlane and French, 1960 to Bohr and Mottelson, 1969 etc...) relates the spectroscopic factors to the occupation number for the  $nlj$  orbital in question. One nuclear state may present several spectroscopic factors: e.g. the ground state (g.s.) of  $^8\text{B}$  has  $S(p_{3/2})$ ,  $S(p_{1/2})$ ... related to the probability that the last proton is bound around the g.s. of the  $^7\text{Be}$  core in a  $1p_{3/2}$ , or a  $1p_{1/2}$  orbital. The determination of spectroscopic factors from one-nucleon transfer reactions was and is crucial in building our current understanding of the fermionic degrees of freedom in nuclei and their coupling to other types of excitations. The Asymptotic Normalization Coefficient (ANC) method is an indirect nuclear astrophysics (NA) method introduced by our group more than a decade ago to determine astrophysical S-factors for the non-resonant component of radiative proton capture at low energies (tens or hundreds of keV) from one-proton transfer reactions involving complex nuclei at laboratory energies (about 10 MeV/u) [1]. It uses essentially the same determination of the proton spectroscopic factors, but avoids one uncertainty of the latter by removing the dependence of the result on the geometry of the proton binding potential assumed (but not well known!) in the DWBA cross section calculations. This is possible when using peripheral reactions. The method was explained in detail in many publications, I summarize the main ideas in the slide shown in Figure 1.

The figure shows that we can choose peripheral proton transfer reactions to extract the ANCs, which can be used to evaluate  $(p,\gamma)$  cross sections important in different types of H-burning processes. The idea behind it is that in peripheral processes it is sufficient to know the overlap integral at large distances, and this is given by a known Whittaker function times a normalization coefficient  $C_{nlj}$  (the formula in the bottom right corner), to be determined by experiment. Figure 1 also stresses the importance of having good and reliable optical model potentials (OMP) to make the DWBA calculations, a problem I want to discuss in the next subsection.

## B. Transfer reactions: the ANC method

**Transfer reaction**  $B+d \rightarrow A+a$  peripheral (absorption)

- Transfer matrix element:

$$M = \langle \chi_f^{(-)} I_{Bp}^A | \Delta V | I_{ap}^d \chi_i^{(+)} \rangle$$

$$\frac{d\sigma}{d\Omega} = \sum_{mj} \left[ S_i S_f \left( \frac{d\sigma}{d\Omega} \right)_{DWBA} \right]_{n_1 l_1 j_1 \rightarrow n_2 l_2 j_2}$$

$$\frac{d\sigma}{d\Omega} = \sum (C_{Bp, A, j, A}^A)^2 (C_{apl, a, j, d}^d)^2 \frac{\sigma_{I_{A, j, A, j, d}^{DW}}}{b_{Bp, A, j, A}^2 b_{apl, a, j, d}^2}$$

ANC - independent on binding potential geometry!

**OMP knowledge crucial for reliable absolute values!**

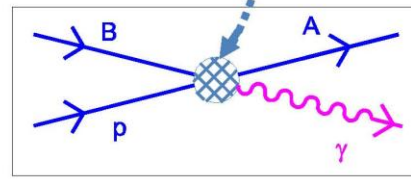
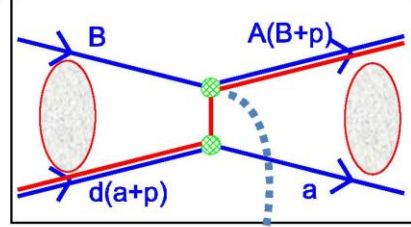
Semi-micr proc. JLM interaction (LT ea, PRC, 2000)

**NA: proton-nucleus also peripheral**

$$\sigma_{(p, \gamma)} \propto (C_{Bp}^A)^2$$

(Christy and Duck, 1963

Parker and Tombrello, 1964)



$$I_{Bp}^A \approx C_{Bp}^A \frac{W_{-n_u, l+1/2}(2\kappa_{Bp} r_{Bp})}{r_{Bp}}$$

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Figure 1. Schematic presentation of the ANC method, which relates one-proton transfer reactions to radiative proton capture in NA.

The technique was used in many other experiments of this type; I will mention the latest study on the  $^{12}\text{N}(p, \gamma)^{13}\text{O}$  proton capture reaction at stellar energies using the proton transfer reaction  $^{14}\text{N}(^{12}\text{N}, ^{13}\text{O})^{13}\text{C}$  with a  $^{12}\text{N}$  beam at 12 MeV/u [2]. Figure 2 below, also the image of a slide shown during the lecture, summarizes the whole process. Going from bottom left, clockwise: we have measured the elastic scattering and the one-proton transfer using a  $^{12}\text{N}$  beam produced and separated with the MARS spectrometer [3] at Texas A&M University. The elastic scattering data were used to determine the OMP needed in the DWBA calculations for transfer. The ANC for the system  $^{13}\text{O} \rightarrow ^{12}\text{N}+p$  was extracted from the transfer data after which was used to evaluate the non-resonant component of the astrophysical S-factor for the radiative proton capture  $^{12}\text{N}(p, \gamma)^{13}\text{O}$  and the corresponding reaction rate as a function of stellar temperature. Finally, the astrophysical consequences are shown in a plot (bottom right) which shows the region of density-temperature where the capture process competes with its competitor ( $\beta$ -decay), in first stars. For comparison, the curves from literature before our data were measured are shown.

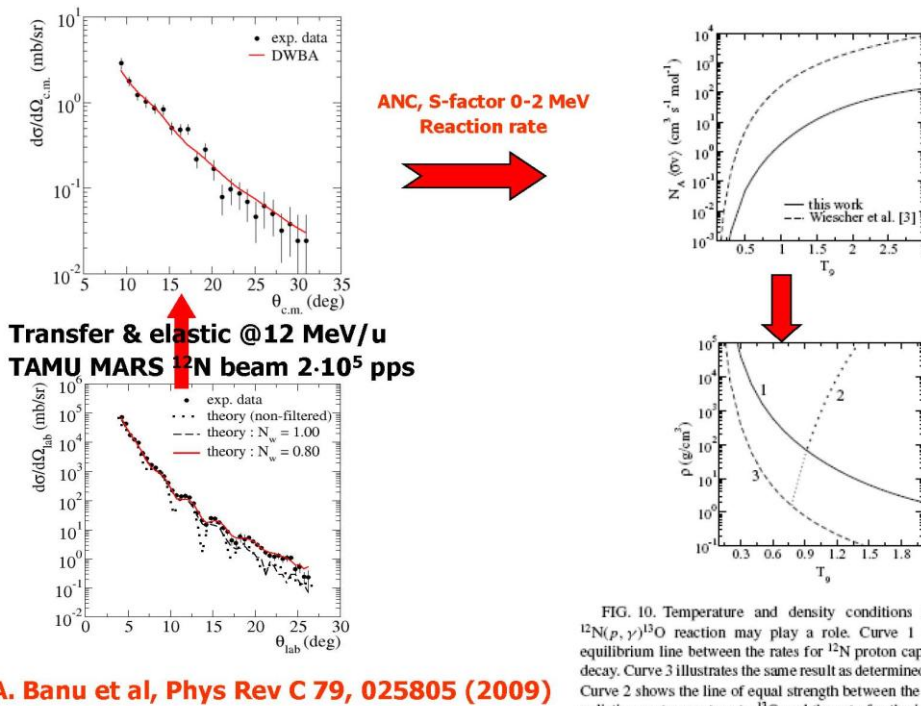


FIG. 10. Temperature and density conditions at which the  $^{12}\text{N}(p,\gamma)^{13}\text{O}$  reaction may play a role. Curve 1 represents the equilibrium line between the rates for  $^{12}\text{N}$  proton capture and  $^{12}\text{N}$   $\beta$  decay. Curve 3 illustrates the same result as determined from Ref. [3]. Curve 2 shows the line of equal strength between the rate of the  $^{12}\text{N}$  radiative proton capture to  $^{13}\text{O}$  and the rate for the inverse process,  $^{13}\text{O}$  photodisintegration. See text for details.

Figure 2. Summary of how elastic and one-proton transfer data measured with secondary RIB (left side) are transformed in nuclear astrophysics information (right side).

A variation of the ANC method uses one-neutron transfer reactions to obtain information about the mirror nuclei, for example studying the  $^{13}\text{C}(^7\text{Li}, ^8\text{Li})^{12}\text{C}$  reaction to determine the ANC for  $^8\text{Li}$  which we then translate into the corresponding structure information (the proton ANC) for its mirror  $^8\text{B}$  and from there  $S_{17}(0)$  for the reaction important in the neutrino production in Sun  $^7\text{Be}(p,\gamma)^8\text{B}$  [4]. We did this using the mirror symmetry of these nuclei: the similarity of their wave functions, expressed best by the identity of the neutron and proton spectroscopic factors for the same  $nlj$  orbital in the two nuclei  $S_p(nlj)=S_n(nlj)$  (of course, the radial wave functions are not identical!). The experiment using these concepts and the results were published in Ref. [5]. Later similar ideas were expanded in a theoretical paper [6].

I mentioned before that in order to extract data, either the spectroscopic factors, or the ANCs, the experiments have to be compared with calculations, and in the above conditions, the knowledge of the optical potentials is crucial. I will insist on this in the next subsection.

## 2.2 Elastic scattering and OMP for RIBs

Elastic scattering can be a good, sensitive probe of the surfaces of nuclei, and (sometimes!) even of their interior. For this to happen we need to measure angular

distributions on a large range of angles. The measurements at small angles probe the surface only (and are known to lead to ambiguities in optical potentials), while measuring at larger angles probe more toward the interior of the nuclei. Unfortunately, this is not easy with RIBs; the cross sections can vary with several orders of magnitude (6-7) from small to large angles. Also, the (important) diffraction patterns can be washed out if the angular resolutions of the beam and/or of the detection systems are not good enough. This is the case in many reactions with RIB, which are yet of low quality. The elastic scattering data are described with optical model potentials, which are used to reduce a complex many-body dynamical interaction to a simpler one-body problem. The OMPs are complex, one-body potentials, which account for the refractive part of the scattering through the real part, and take into account the absorption into other channels through the imaginary part. If well understood and used, the elastic scattering data can give important information about the *structure* of the two partners and about the *reaction mechanism*.

OM potentials used to describe elastic scattering data are either phenomenological: potentials of different types (volume, surface, spin-orbit...) and typically Woods-Saxon shapes are used to fit the available data, or semi-microscopic. While much was learned when using the former for stable beams data, no clear rules or parameter dependences could be extracted for nucleus-nucleus collisions on large mass and energy ranges in more than 30-40 years of work on this problem. Moreover, it is clear that it is not appropriate to expect to extrapolate those to the cases of RIBs, because the surfaces of unstable nuclei can be different from those of nuclei on stability valley. This is where the semi-microscopic, double folding models, are expected to do better. There was much work in this field also in the last three decades or so, with no clear cut answers either. I will mention here one such attempt that we made, at Texas A&M, in collaboration with Bucharest, and which gave us surprisingly good results in the last decade or more, in particular in describing, and even predicting, elastic scattering of RIBs from the *p*- and *sd*-shell regions of the nuclides chart [7,8].

Figure 3 (also a slide from the lecture) shows some of the data and the results of this work, including both stable and radioactive beams. The work was generated by the need to find OMP for the DWBA description of transfer reactions which we use as indirect method in nuclear astrophysics, as described in the previous subsection. We found that using a careful calculation of the density distributions of the projectile and target nuclei (using well adjusted Hartree-Fock-Bogoliubov theories and codes [9]) and appropriate nucleon-nucleon effective interactions (we used the interaction of Jeukenne, Lejeune and Mahaux [10], for short JLM) we can describe a large amount of data with only a simple renormalization of the real and imaginary potentials. The double-folded potentials from the procedure found in Refs. [7,8] proved to be very good starting points for all RIB cases we have studied so far.

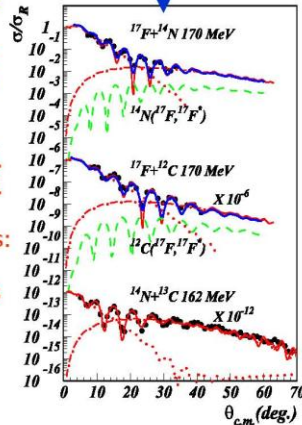
### Works for RNBs

**Optical Model Potentials for Nucleus-Nucleus collisions for RNBs**

- Essential to make credible DWBA calc needed in transfer studies  
 Have established semi-microscopic double folding using JLM effective interaction:
- Established from expts with stable loosely bound p-shell nuclei:  ${}^6,7\text{Li}$ ,  ${}^{10}\text{B}$ ,  ${}^{13}\text{C}$ ,  ${}^{14}\text{N}$  ... @ 10 MeV/u
  - Independent real and imaginary parts, energy and density depend.
  - Parameters: renormalization coeff. ( $N_v \sim 0.4-0.5$ ,  $N_w = 1.0$ )
  - Predicts well elastic scatt for RNBs:  ${}^7\text{Be}$ ,  ${}^8\text{B}$ ,  ${}^{11}\text{C}$ ,  ${}^{12}\text{N}$ ,  ${}^{13}\text{N}$ ,  ${}^{17}\text{F}$ ,  ${}^{14}\text{C}$ , ...
  - Good results for transfer reactions (tested where possible)
- L. Trache ea, PRC 61 (2000), F Carstoiu ea PRC 70 (2004)

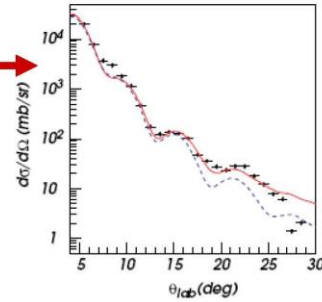
**TAMU exps @ 12 MeV/u**  
 G. Tabacaru ea, PRC 73, 025808 (2006)

**ORNL exps @ 10 MeV/u**

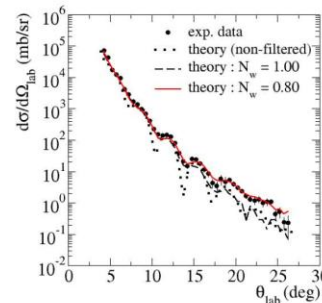


J. Blackmon ea, PRC 73, 034606 (2005)

**${}^7\text{Be}$  on melamine**



**${}^{12}\text{N}$  on melamine**



A. Banu ea, PRC 79, 2009. 3

Figure 3. Elastic scattering data with stable and radioactive beams and their fit with double folding potentials from JLM NN-interaction. Data and parameters are from the references cited in the slide.

### 3. Breakup at intermediate energies

Work done in the last decade in several laboratories has demonstrated that one-nucleon removal reactions (or breakup reactions) can be a good and reliable spectroscopic tool. In a typical experiment a loosely bound projectile at energies above the Fermi energy impinges on a target and loses one nucleon. The momentum distributions (parallel and/or transversal) of the remaining core measured after reaction give information about the momentum distribution of the removed nucleon in the wave function of the ground state of the projectile. The shape of the momentum distributions allows determining the quantum numbers  $nlj$  of the s.p. wave function (unambiguously only  $l$  is determined; shell model systematics are needed for the others). It was shown in Ref. 11 that on a large range of projectile energies breakup reactions are peripheral and, therefore, the breakup cross sections can be used to extract asymptotic normalization coefficients. For this to be true, we need, again, careful and reliable reaction model calculations. They need to reproduce all available data from such measurements if they are to be believed. We have investigated this aspect in detail in Ref. 12. This is a very important point, which I stressed in the lecture. The method to use breakup reaction for nuclear astrophysics was first applied in [11,12] to the breakup of  ${}^8\text{B}$  to determine again  $S_{17}(0)$ . All available breakup data, on targets from C to Pd and at energies from 27

MeV/u to 1400 MeV/u were used to determine the ANC for  ${}^8\text{B} \rightarrow {}^7\text{Be} + p$ . Different reaction models – appropriate for the energies in question - and different nucleon-nucleon effective interactions were used, without any further parameter adjustment. Consistent ANC values were obtained, with an overall uncertainty estimated at about 10%. This is a very good agreement, a fact that validates both the  $S_{17}(0)$  adopted in the neutrino production calculations [3] and the validity of indirect methods in NA.

Another example is the breakup of  ${}^{23}\text{Al}$  at intermediate energies. It is a good example as it takes a case where several configurations contribute to make the ground state of the projectile of which only one is important for nuclear astrophysics. The participating configurations were disentangled using the detection of gamma-rays from the de-excitation of the remaining core after a proton is removed from the projectile moving at 50-60 MeV/nucleon. It is treated in the paper by A. Banu *et al.* and I refer the reader to it [13].

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