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The Trojan Horse Method and its role in the electron screening effect investigation

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Owing to the presence of the Coulomb barrier at astrophysically relevant energies, it is very difficult, or sometimes impossible, to measure cross sections for charged particle induced reactions. Moreover, due to the presence of the electron screening effect in direct measurements, the relevant nuclear input for astrophysics, i.e. the bare nucleus cross section, can hardly be extracted. This is why different indirect techniques are being used along with direct measurements. The THM is an unique indirect technique that allows one to measure reactions cross sections of astrophysical interest down the thermal energies typical of the different scenarios. The basic principle and a review of the main applications of the Trojan Horse Method are given. The applications aiming at the extraction of the bare nucleus cross section and electron screening potentials U_e for several reactions of astrophysical interest are discussed.

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1. Introduction

Nuclear fusion reactions, take place in the hot interiors of remote and long-vanished stars over billions of years, are the origin of nearly all the chemical elements and their isotopes [7, 11, 21]. The detailed understanding of the origin of the chemical elements and their isotopes has combined astrophysics and nuclear physics, and forms what is called nuclear astrophysics. In turn, nuclear reactions are the heart of nuclear astrophysics: they influence sensitively the nucleosynthesis of the elements in the earliest stages of the universe and in all the objects formed thereafter, and control the associated energy generation (by processes called nuclear fusion or nuclear burning), neutrino luminosity and evolution of stars. A good knowledge of the rates of these fusion reactions is essential for understanding this broad picture [29]. Moreover only understanding the electron screening in the laboratory will shed light to astrophysical application as well as fusion reactor physics.

2. Limit of two-body reaction cross section measurements

In a stellar plasma the constituent nuclei are usually in thermal equilibrium at some local temperature T. Occasionally they collide with other nuclei, whereby two different nuclei can emerge from collision A+x$\rightarrow$ c+C. The cross section σ(E) of nuclear fusion reaction A(x,c)C, is of course, governed by the laws of quantum mechanics where, in most cases, the Coulomb and centrifugal barriers arising from nuclear charges and angular momenta in the entrance channel of the reaction strongly inhibit the penetration of one nucleus into another. This barrier penetration leads a steep energy dependence of the cross section. It is the challenge for the experimentalist to make precise σ(E) measurements over a wide range of energies, as our fragmented knowledge of nuclear physics prevents us from predicting σ(E) on purely theoretical grounds. For these reasons bare nucleus cross section measurements σ_b(E) of the (p,α) reaction at the Gamow energy (E_G) should be known with an accuracy better than 10% [21, 22] because of their crucial role in understanding the first phases of the Universe history and the subsequent stellar evolution.
Unfortunately the presence of Coulomb barrier, in the reactions with charge particles, is a limit, often insuperable, to perform measurements of the cross sections at ultralow energies. Indeed, the Coulomb barrier of height E_C in charged-particle induced reactions causes an exponential decrease of the cross section $\sigma_b(E)$ at $E < E_C$, $\sigma_b(E) \sim exp(-2\pi\eta)$, leading to a low-energy limit of direct $\sigma_b(E)$ measurements. Owing to the strong Coulomb suppression, the behavior of the cross section at E_G is usually extrapolated from the higher energies by using the definition of the smoother astrophysical factor S(E):

$$S_b(E) = E\sigma_b(E)exp(2\pi\eta) \tag{2.1}$$

where $exp(2\pi\eta)$ is the inverse of the Gamow factor, which removes the dominant energy dependence of $\sigma(E)_b$ due to the barrier penetrability.
Although the S_b(E)-factor allows for an easier extrapolation, large uncertainties to $\sigma_b(E_G)$ may be introduced due to for instance the presence of unexpected resonances, or high energy tails of sub-threshold resonances. In order to avoid the extrapolation procedure, a number of experimental solutions were proposed in direct measurements for enhancing the signal-to-noise ratio at E_G.

In recent years the availability of hight-current low-energy accelerators, such as that at the underground Laboratories, together with improved target and detection techniques have allowed us to perform $\sigma_b(E)$ measurements in some cases down to E_G or at least close to E_G [6]. Then in principle no $\sigma_b(E)$ extrapolation would be needed anymore for these reactions. However, the measurements in laboratory at ultralow energies suffer from the complication due to the effects of electron screening [1, 29]. This leads to an exponential increase of the laboratory measured cross section $\sigma_s(E)$ [or equivalently of the astrophysical factor $S_s(E)$] with decreasing energy relative to the case of bare nuclei. This can be described by an enhancement factor defined by the relation

$$f_{lab}(E) = \sigma_s(E)/\sigma_b(E) \approx exp(\pi\eta U_e/E) \tag{2.2}$$

In this equation U_e is the electron screening potential in the laboratory which is different from the U_{pl} present in the stellar environment. Clearly, a good understanding of U_e is needed in order to calculate σ_b from the experimental data σ_s using equation (2). The effective cross section $\sigma_{pl}(E)$ in the stellar plasma, is connected to the bare nucleus cross section $\sigma_b(E)$ and to the the stellar electron screening enhancement factor f_{pl} by the relation

$$\sigma_{pl}(E) = \sigma_b(E) f_{pl}(E) \approx \sigma_b(E) \cdot exp(\pi\eta U_{pl}/E) \tag{2.3}$$

with U_{pl} is the plasma potential energy, η the Sommerfeld parameter.
If $\sigma_b(E)$ is measured at the ultralow energies E_G and U_{pl} is estimated within the framework of the Debye-Hückel theory, it is possible toestimate the effective cross section $\sigma_{pl}(E)$ in the stellar plasma from equation (3). In turn, the understanding of U_e may help to better understand U_{pl}, needed to calculate σ_{pl}.
Then, although it is possible to measure cross sections in the Gamow energy range, the bare nucleus cross section σ_b is extracted by extrapolating the direct data behavior at higher energies where negligible electron screening contribution is expected. In order to decrease uncertainties in the case of charged particle induced reactions a rather striking conclusion could be achieved: to avoid extrapolations, experimental techniques were improved. After improving measurements (at very low energies), electron screening effects were discovered. Finally to extract from direct (shielded) measurements the bare astrophysical S_b(E)-factor, extrapolation were performed from higher energy. In any case the extrapolation procedure is necessary and in consequence we find again the uncertainties problem in direct measurements.

3. The Trojan Horse Method

Alternative methods for determining bare nucleus cross sections of astrophysical interest are needed. In this context a number of indirect methods, e.g. the Coulomb dissociation (CD) [4, 5], the Asymptotic Normalization Coefficient method (ANC)[30, 2, 17, 15, 16] and the Trojan-horse method (THM), were developed.
In particular, the THM is a powerful tool that selects the quasi-free (QF) contribution of an appropriate three-body reaction performed at energies well above the Coulomb barrier to extract a charged particle two-body cross section. In the framework of the extrapolation problems linked to the presence of electron screening effects, a number of experimental measurements were carried

out in order to measure the bare nucleus cross section in reactions of astrophysical interest with the purpose of determining the electron screening potential by comparison with the shielded direct data.

The idea of the THM [3] is to extract the cross section of an astrophysically relevant two-body reaction

$$A + x \rightarrow c + C \tag{3.1}$$

at low energies from a suitable chosen three-body QF reaction

$$A + a \rightarrow c + C + S \tag{3.2}$$

This is done with the help of direct reaction theory assuming that the nucleus a has a strong $x \oplus S$ cluster structure. In many applications [27, 28, 14], this assumption is trivially fulfilled e.g. a = deuteron, x = proton, S= neutron. This three-body reaction can be described by a Pseudo-Feynmam diagram, where only the first term of the Feynman series is retained. The upper pole describes the virtual break-up of the target nucleus a into the clusters x and S; S is then considered to be spectator to the $A + x \rightarrow c + C$ reaction which takes place in the lower pole (a sketch of the process is reported in figure 1).

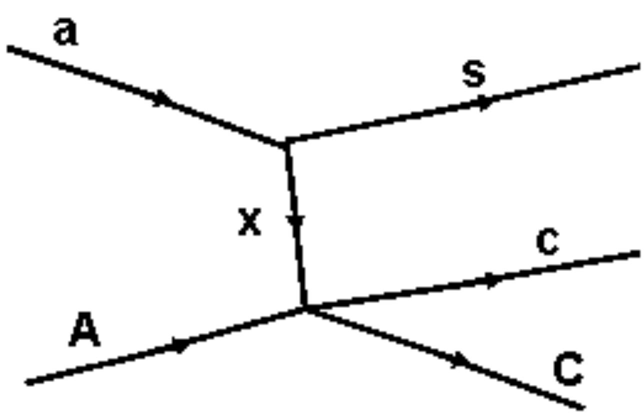

Figure 1: Sketch of a quasi-free process for the reaction $A + a \rightarrow c + C + S$. See the text for details.

We will refer to other papers for the THM theory [27, 28, 14]. In Plane Wave Impulse Approximation (PWIA) the cross section of the three body reaction can be factorized into two terms corresponding to the two poles and it is given by [18]:

$$\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C} \propto KF \left(\frac{d\sigma}{d\Omega_{cm}}\right)^{off} \cdot |\Phi(\vec{p}_s)|^2 \tag{3.3}$$

where:

- $[(d\sigma/d\Omega)_{cm}]^{off}$ is the half-off-energy-shell differential cross section for the two body A(x,c)C reaction at the center of mass energy E_{cm}, given in post collision prescription, by:

$$E_{cm} = E_{c-C} - Q_{2b} \tag{3.4}$$

where Q_{2b} is the two body Q-value of the $A+x \to c+C$ reaction and E_{c-C} is the relative energy between the outgoing particles c and C;

- KF is a kinematical factor containing the final state phase-space factor and it is a function of the masses, momenta and angles of the outgoing particles:

$$KF = \frac{\mu_{Aa} m_c}{(2\pi)^5 \hbar^7} \frac{p_C p_c^3}{p_{Aa}} \left[\left(\frac{\vec{p}_{Bx}}{\mu_{Bx}} - \frac{\vec{p}_{Cc}}{m_c} \right) \cdot \frac{\vec{p}_c}{p_c} \right]^{-1} \tag{3.5}$$

- $\Phi(\vec{p}_s)$ is the Fourier transform of the radial wave function $\chi(\vec{r})$ for the x-S inter-cluster motion, usually described in terms of Hänkel, Eckart or Hulthen functions depending on the x-S system properties.

We stress that with THM, one cannot obtained the absolute value of the two-body cross section. However, the absolute value can be extracted through normalization to the direct data available at energies above and/or below the Coulomb barrier. Thanks to this, since we select the region of low momentum p_s for the spectator ($p_s \leq 40$ MeV/c) the PWIA approach can be used for further analysis of the experimental results. If $|\Phi(\vec{p}_s)|^2$ is known and KF is calculated, it is possible to derive $[(d\sigma/d\Omega)_{cm}]^{exp}$ from a measurement of $d^3\sigma/dE_c d\Omega_c d\Omega_C$ by using Eq. 3.3.

$$\left(\frac{d\sigma}{d\Omega}\right) \propto \left[\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C}\right] \cdot [KF|\Phi(\vec{p}_s)|^2]^{-1} \tag{3.6}$$

If the bombarding energy E_A is chosen high enough to overcome the Coulomb barrier in the entrance channel of the three-body reaction, both Coulomb barrier and electron screening effects are negligible. In this way, it is possible to extract the two-body cross section from Eq.(9) after inserting the appropriate penetration function G_l in order to account for the penetrability effects affecting direct data below the Coulomb barrier [9, 25]. The complete formula is given by:

$$\left(\frac{d\sigma}{d\Omega}\right) \propto \left[\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C}\right] \cdot [KF|\Phi(\vec{p}_s)|^2]^{-1} \cdot G_l \tag{3.7}$$

As shown above, since in the experimental works the IA validity conditions are fulfilled, the PWIA was applied for the extractions of the two-body cross-section. In this approximation the differential two-body cross-section of Eq.9 is expressed by:

$$\left(\frac{d\sigma}{d\Omega}\right) = CG_l \left(\frac{d\sigma}{d\Omega}\right) \tag{3.8}$$

being C the normalization constant to the direct data. As already mentioned, the THM data are not affected by electron screening effects. Therefore, once the behavior of the absolute bare $S_b(E)$ factor from the two-body cross-section is extracted, a model-independent estimate of the screening potential U_e can be obtained from comparison with the direct screened S(E)-factor.

Table 1: Two-body reaction results in recent experiments as regards the astrophysical S(E)-factor and the electron screening potential

	Reaction	$U_e^{Adiab.}$ (eV)	U_e^{Dir} (eV)	U_e^{THM} (eV)	$S(0)^{THM}$ (MeVb)
[1]	^{7}Li + p $\rightarrow \alpha + \alpha$	186	300±160	330± 40	0.055
[2]	^{6}Li + d $\rightarrow \alpha + \alpha$	186	330±120	340±50	16.9
[3]	^{6}Li + p $\rightarrow ^3He + \alpha$	186	440±150	450±100	3.0
[4]	^{9}Be + p $\rightarrow ^6Li + \alpha$	240	830±130	676±86	21.0
[6]	d + $^3He \rightarrow p + \alpha$	115	219±7	180±40	6.08±1.42

4. Discussion and conclusions

An experimental program has already been undertaken to study p-capture reactions on $^{6,7}Li$, main responsible for their destruction [20, 19]. The extracted S(E) factors as well as electron screening potentials, extracted as stated above are shown in table I. Recently, $^3He(d,p)^4He$ [12] and $^9Be(p,\alpha)^6Li$ [23] were also investigated. Their importance is indeed strongly related to the cosmology as well as to stellar structure and evolution. The bare nucleus S(E)-factor for the $^3He(d,p)^4He$ and $^9Be(p,\alpha)^6Li$ are reported in table 1 as well.

In picture 1,2 and 3 we present the results for the $^6Li(d,\alpha)^4He$, $^9Be(p,\alpha)^6Li$ and $^7Li(p,\alpha)^4He$. The bare nucleus S(E)-factor is reported as black dots and more details are given in each caption.

In the three pictures it is clear how the bare nucleus behaviour is given by the THM measurement. The difference between the THM and the direct data, at lower energies (around and below100 keV), is assumed to be due to the electron screening enhancement. Therefore the bare nucleus parameterization based on THM data is corrected for the electron screening effect by using expression 2.2, with U_e the only free parameter. The obtained result is then compared with results from other experiments (direct ones) as well as with the adiabatic limit (see table 1).
From the obtained results we can see how the THM measurements are in agreement with other experimental data (at least within the experimental errors) and how both direct and indirect measurements systematically exceed the adiabatic limit, i.e. the maximum value of energy of the electron screening potential predicted by theory. Nevertheless, as in the direct experiments, the isotopic invariance of the electron screening potential is confirmed as it can be seen in table 1.

The present paper shows the basic features of the THM and a review of recent applications to several reactions of importance in astrophysics. In particular, these results show the possibility of extracting the bare nucleus two-body cross section via THM. However, a lot remains to do in the future to achieve reliable information for many key reactions and processes. New theoretical developments are strongly needed especially for the study of electron screening effects in fusion reactions in order to meet progress in the application field (fusion reactors). This information and the evaluation of fusion reaction cross sections will help to determine the basic properties of future reactors as well as plasma confinement devices.

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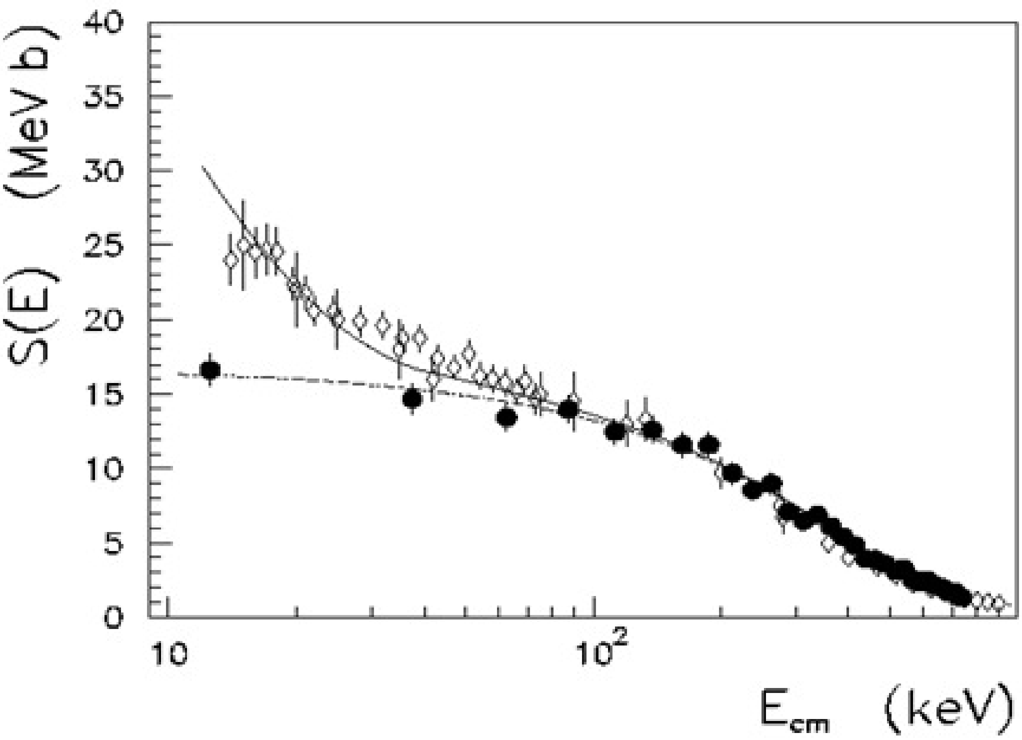

Figure 2: Bare nucleus astrophysical S(E) factor for ^{6}Li + d $\rightarrow \alpha + \alpha$ compared with direct data. The black dots represent THM data, the diamonds direct ones while the solid line is a fit to the shielded data and the dashed line a fit to THM data.

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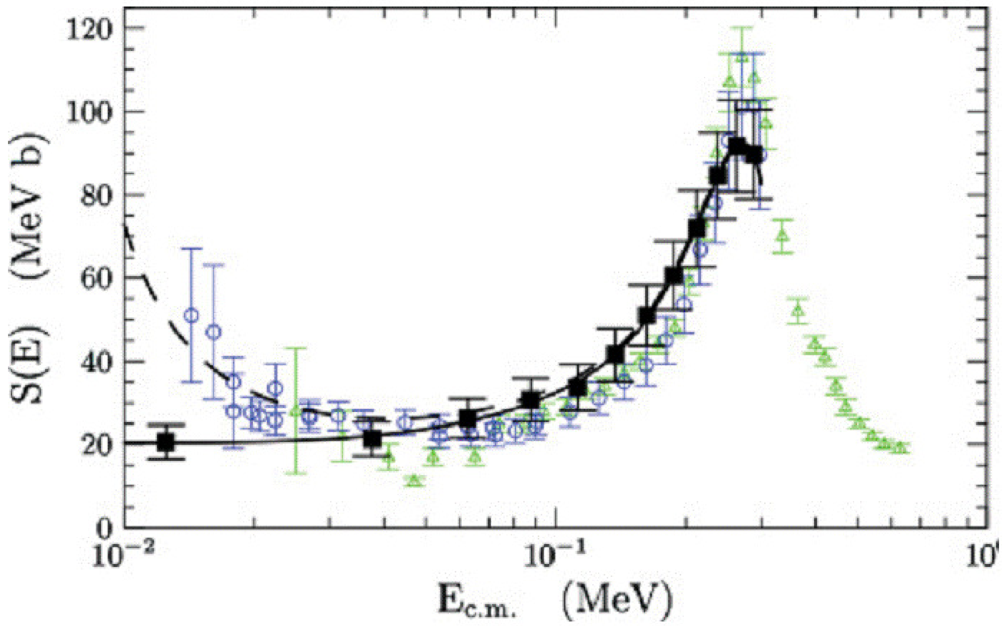

Figure 3: Bare nucleus astrophysical S(E) factor for $^9Be + p \rightarrow ^6 Li + \alpha$ compared with direct data. The black dots represent THM data, the empty circles direct ones while the solid line is a fit to the THM data and the dashed line a fit to direct ones.

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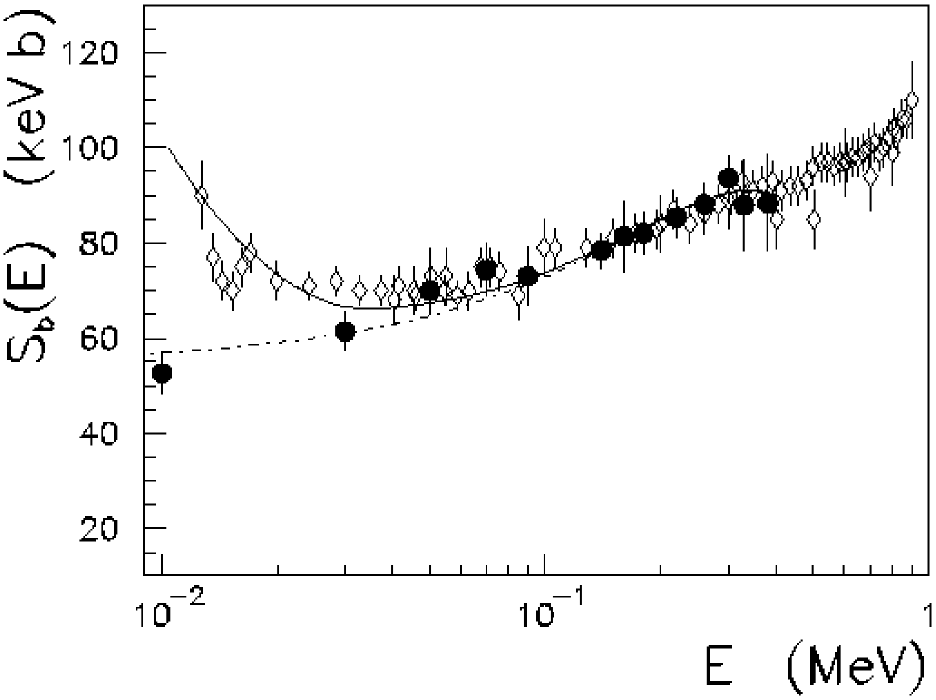

Figure 4: Bare nucleus astrophysical S(E) factor for $^{7}Li + p \rightarrow \alpha + \alpha$ compared with direct data. The black dots represent THM data, the diamonds direct ones while the solid line is a fit to the shielded data and the dashed line a fit to THM data.

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