PROCEEDINGS OF SCIENCE

SPIN CONTROL BY RF FIELDS AT ACCELERATORS AND STORAGE RINGS

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The first experiments to apply RF fields for resonant beam depolarization and spin flip at the VEPP-2M storage ring were carried out more than 30 years ago [1]. Later this technique was used at VEPP-2M in the experiment for comparison of electron and positron anomalous magnetic moments. Recently, interest in RF spin control has appeared at proton machines. This paper describes a general approach for consideration of RF influence on spin dynamics at electron (positron) and hadron accelerators. Some practical applications of RF fields are discussed.

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1. General description of spin motion at accelerators

The spin motion of a particle in electromagnetic fields is described by the BMT equation:[2]

$$\frac{d\mathbf{S}}{dt} = \dot{\mathbf{S}} = [\Omega \times \mathbf{S}]$$
$$-\Omega = \left(\frac{q_0}{\gamma} + q'\right)\mathbf{B}_\perp + \frac{q_0+q'}{\gamma}\mathbf{B}_\parallel + \left(\frac{q_0}{\gamma+1} + q'\right)[\mathbf{E}\times\mathbf{V}], \tag{1.1}$$

where q_0 and q' are normal and anomalous parts of the particle gyro-magnetic ratio; $\mathbf{B}_\perp$ and $\mathbf{B}_\parallel$ are magnetic field components along and transverse to the particle velocity $\mathbf{V}$. Since in circular accelerators the one-turn energy change is relatively small, we can neglect, in the first approximation, the electric field: $\mathbf{E} = 0$. Following the usual approach for orbital motion, we use as the independent variable the generalized azimuth θ and subdivide the spin precession vector in two parts: $\Omega = \mathbf{W}_0(\theta) + \mathbf{w}(\theta)$, where $\mathbf{W}_0(\theta)$ contains only fields on the Closed Orbit $\mathbf{R}_0$, while $\mathbf{w}(\theta)$ denotes *all* of the other terms (contributions from closed orbit imperfection and orbital oscillations). One can treat $\mathbf{w}$ as a small perturbation for the spin motion.[3–6] In the accelerator vector triad $\mathbf{e}_x,\ \mathbf{e}_y = \mathbf{V}/V,\ \mathbf{e}_z = [\mathbf{e}_x \times \mathbf{e}_y]$ components of the precession vector $\mathbf{W}_0$ can be presented in the linear approximation in the next form: [1]

$$\begin{aligned}
W_{0_x} &= \nu_0 K_x; & K_x &= \frac{B_x}{B_0};\\
W_{0_y} &= (1+a)K_y; & K_y &= \frac{B_y}{B_0};\\
W_{0_z} &= \nu_0 K_z; & K_z &= \frac{B_z}{B_0},
\end{aligned} \tag{1.2}$$

where we introduce the particle magnetic anomaly $a = q'/q_0$ and denote $\nu_0 = \gamma \cdot a$. On the reference orbit the equation has one solution $\mathbf{n}_0$, which is periodic around the ring, i.e. $\mathbf{n}_0(\theta + 2\pi) = \mathbf{n}_0(\theta)$. There are also two other linearly independent solutions: the orthogonal complex vectors η and η^*, which rotate around $\mathbf{n}_0$ with spin tune ν:

$$\eta(\theta+2\pi) = e^{i2\pi\nu}\eta(\theta); \quad \eta^*(\theta+2\pi) = e^{-i2\pi\nu}\eta^*(\theta).$$

We restrict consideration to planar machines and start with a proton ring, where the spin tune is $\nu = \nu_0$ and $\mathbf{n}_0 = \mathbf{e}_z$; $\eta = (\mathbf{e}_x - i\mathbf{e}_y)e^{i\nu_0\theta}$. Let's apply on a short piece of the orbit Δl a longitudinal RF-field $\tilde{K}_y\cos(\nu_d\theta)$ (RF-solenoid) with a frequency ν_d, which is an external spin perturbation. It can be presented as a number of circular harmonics $w = \sum_k w_k e^{i(k\pm\nu_d)\Theta}$ with equal amplitudes $w_k = (1+a)\tilde{K}_y \frac{\Delta l}{4\pi R}$.

A more complicated situation occurs for the application of a radial RF-field $\tilde{K}_x\cos(\nu_d\theta)$ (RF-dipole). This field disturbs not only spin motion, but it excites also forced vertical oscillations. As a result, spin gets additional kicks from off-orbit fields in dipole and quadrupole magnets around the machine. The tight frame of this paper doesn't allow a mathematical description of this process and we refer readers to papers devoted to this topic.[7–9] The linear spin response formalism has been

[1] We use dimensionless units: fields are normalized to mean guiding field $B_0 = 1/2\pi \oint B_z d\theta$; length and time are measured correspondent in units of mean radius R and revolution time.

developed for *simultaneous* treatment of the orbit and spin dynamics. Based on this formalism the code "ASPIRRIN" ([10]) calculates 5 complex response functions $F_1(\theta) - F_5(\theta)$ which satisfy the periodicity condition: $|F_i(\theta + 2\pi)| = |F_i(\theta)|$. These response functions characterize the sensitivity of the spin precession axis $\mathbf{n}$ to kicks of orbital variables $(X^T = (p_x, x, p_z, z, \delta\gamma/\gamma, 0))$ for specific machine optics and working points. In the case of an ideal flat machine, $\frac{\partial \mathbf{n}}{\partial p_z} = \nu_0 \Re(iF_3(\theta)\eta^*)$. Thereby the RF-dipole, applied at azimuth Θ, creates a set of perturbing harmonics with amplitudes $w_k = \nu_0 \tilde{K}_y F_3(\Theta) \frac{\Delta l}{4\pi R}$.

Next, it's necessary to take into account synchrotron oscillation of the particle energy, inherent to accelerators: $\gamma = \gamma_0 + \Delta\gamma \cos(\nu_s \theta)$. Usually, the synchrotron tune $\nu_s \sim 10^{-2} - 10^{-3}$ is much less than other orbital frequencies. Hence, the spin tune is modulated by the synchrotron tune. It results in a modification of the RF-field spectrum. Each line of the spin perturbation is transformed to a set of side band resonances $k \pm \nu_d \pm m\nu_s$. The side band harmonics are given by the expression:[3]

$$|w_k^m| = |w_k| J_m(\lambda), \tag{1.3}$$

where $J_m(\lambda)$ are the Bessel functions. As a rule, the modulation index $\lambda = \frac{\nu_0}{\nu_s}(\frac{\Delta\gamma}{\gamma}) \ll 1$, at proton and electron accelerators. For RF spin resonances not all sidebands overlap ($w_k^m \ll \nu_s$). It's important to emphasize that the central line of the spectrum corresponds to the mean energy γ_0 of the beam. Moreover, the width of this spectrum line, averaged over the synchrotron oscillations, shrinks to a size of $\sigma_\nu \sim (\delta\gamma/\gamma)^2$.[11] Assuming a Gaussian particle distribution in the longitudinal phase space, mean value of the central resonance strength $(w_k^0 \equiv w)$ and its rms deviation σ_w are given by the expressions:

$$w^2 = w_k^2 I_0(\Lambda) e^{-\Lambda}; \quad \sigma_w^2 = w_k^2 \frac{\Lambda}{2} I_1(\Lambda) e^{-\Lambda}, \tag{1.4}$$

where I_0, I_1 are the modified Bessel functions from the argument $\Lambda = \frac{\nu_0}{\nu_s}\sigma_\gamma$; $(\sigma_\gamma^2 = \overline{\frac{\Delta\gamma^2}{\gamma}})$.

Evidently, spin control operations have to be done at the central resonance. The choice of frequency ν_d is determined by concrete conditions of an experiment, but always one must satisfy the resonance condition: $|\nu_d \pm k - \nu_0| \ll 1$. It's more visual to consider this situation in a frame rotating at the resonant frequency, where the spin motion is simply precession in a "field" $\mathbf{h} = \varepsilon \mathbf{e}_z + w \mathbf{e}_y$, where $\varepsilon = \nu_0 - \nu_d \pm k$ is the resonance tune, see Fig. 1. The precession frequency $h = \sqrt{\varepsilon^2 + w^2}$ in the resonance case $(\varepsilon = 0)$ decays to w, which can be taken as a strength of the resonance.

The spin resonance crossing was described by Froissart-Stora [12], who found that in result of passing from $\varepsilon = \infty$ up to $\varepsilon = -\infty$ with a tune rate $\dot{\varepsilon} = const$ the residual projection averaged over spin phases $S_z = (\mathbf{S} \cdot \mathbf{n}_0)$ is described by a formula:

$$S_z = S_z(0)\left(2e^{-\frac{\pi w^2}{2\dot{\varepsilon}}} - 1\right). \tag{1.5}$$

The final result depends on the spin phase advance near the resonance center: $\Psi = \int h\,dt \sim w^2/\dot{\varepsilon}$. The polarization is preserved by an adiabatic change of parameters $(\Psi \gg 1)$ and flips down together with the $\mathbf{n} = \mathbf{h}/h$-axis. In the opposite case of a fast crossing $(\Psi \ll 1)$, the spin only tilts slightly from its initial direction with a depolarization $\Delta S_n \simeq w^2/\dot{\varepsilon}$. Polarization losses may attain $\sim 100\%$ in intermediate situations $\Delta\phi \leq 1$.

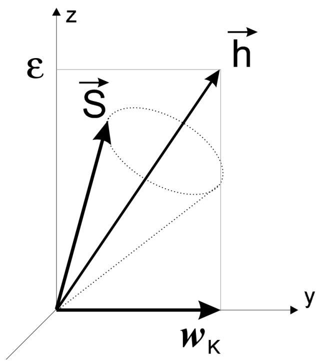

Figure 1: Resonant frame.

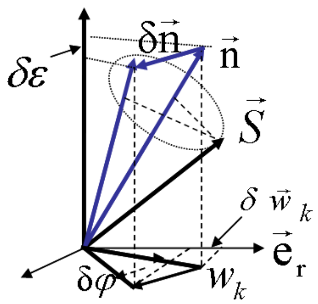

Figure 2: Spin diffusion.

2. Spin flip at electron accelerators

At electron accelerators there are other limitations for RF-device usage. Quantum fluctuations of the synchrotron radiation lead to so called, "spin diffusion", which is enhanced at spin resonances.[4] In a general case, a quantum emission in the resonant frame brings at the same instant jumps of the resonance tune ($\delta\varepsilon$), the resonant harmonic (δw) and its phase ($\delta\phi$) (see Fig. 2). At that, the precession axis $\mathbf{n} = \mathbf{h}/h$ gets a kick $\delta\mathbf{n}$:

$$(\delta\mathbf{n})^2 = \left(\delta\arctan\left(\frac{|w|}{\varepsilon}\right)\right)^2 + \frac{w^2}{h^2}(\delta\phi)^2, \tag{2.1}$$

but the spin vector does not change (we neglect here "spin-flip" quantum emissions). However, the projection S_n undergoes a change $\delta S_n = -1/2\,(\delta\mathbf{n})^2 S_n$. A consequence of random kicks results in a polarization loss with a decay time $\tau_d = 1/2\left\langle\frac{d}{dt}(\delta\mathbf{n})^2\right\rangle$, where the angle brackets $\langle\rangle$ denote an average over time and an ensemble.

In the case of the RF-resonance ($\delta\phi = 0$), we assume the time of the resonance crossing $\Delta t \sim w/\dot{\varepsilon}$ is much longer than the radiative damping time τ_0 and the characteristic times of the orbital motion. Therefore, we can consider, in average, numerous jumps of $\delta\varepsilon$ and δw only as a diffusion around mean values $\overline{\varepsilon}$ and $\overline{w}$ with corresponding rms deviations σ_ν, and σ_w. So, from (2.1) we find the depolarization time, caused by the quantum fluctuations:

$$\tau_d^{-1} = \frac{1}{2}\left\langle\frac{d}{dt}\left(\frac{\varepsilon\,\delta w - w\,\delta\varepsilon}{w^2+\varepsilon^2}\right)^2\right\rangle \simeq \frac{\overline{\varepsilon}^{\,2}\sigma_w^2 + \overline{w}^{\,2}\sigma_\nu^2}{(\overline{w}^{\,2}+\overline{\varepsilon}^{\,2})^2}\cdot\tau_0^{-1}. \tag{2.2}$$

To obtain the above expression, we ignored the interference term by averaging over time $(\overline{\delta\varepsilon\cdot\delta w} = 0)$ and applied usual formulas for an equilibrium state: $\sigma_\varepsilon^2 = 1/2\left\langle\frac{d}{dt}(\delta\varepsilon)^2\right\rangle\cdot\tau_0$; $\sigma_w^2 = 1/2\left\langle\frac{d}{dt}(\delta w)^2\right\rangle\cdot\tau_0$ with σ_w from (1.4), where $\sigma_\gamma^2 = 1/2\left\langle\frac{d}{dt}(\delta\gamma)^2\right\rangle\cdot\tau_0$.

At the next step we employ the FS-formula for electron machines, taking into account the spin diffusion under the condition of adiabatic crossing ($\psi \gg \frac{w^2}{\dot{\varepsilon}}$). We calculate the beam polarization $\zeta(t) = \langle S_n\rangle = \zeta(0)\cdot\int_0^t e^{-t/\tau_d}\,dt$, while resonance crossing with different w and $\dot{\varepsilon}$. For example, we use the parameters of the VEPP-2M storage ring (E=700 MeV).

The initial polarization is always $\zeta(0) = 1$. Fig. 3 shows three curves $\zeta(t)$ versus $\varepsilon(t)$ for different RF resonance amplitudes and spin tune spreads but for the same crossing rate $\dot{\varepsilon}\cdot f_0 =$

$400\,Hz/sec$: (red line) $w = 1\cdot 10^{-5}$; $\sigma_\nu = 3\cdot 10^{-7}$; (blue line) $w = 4\cdot 10^{-5}$; $\sigma_\nu = 1\cdot 10^{-6}$; and (magenta line) $w = 1\cdot 10^{-4}$; $\sigma_\nu = 3\cdot 10^{-6}$.

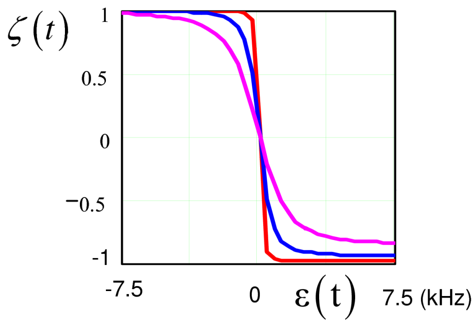

Figure 3: "Simulations" of spin flip.

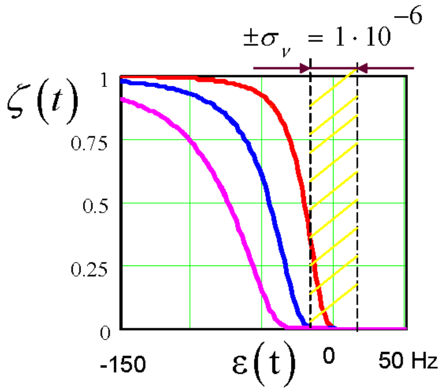

Figure 4: Resonant depolarization.

These calculations demonstrate clearly the influence of the spin diffusion and spin tune spread. Even increasing the RF amplitude by ten times does not help to avoid some depolarization, when the spin tune spread grows up to three times (compare curves in Fig. 3). So, using such "simulations", it's possible to choose RF device parameters for successful spin flip. Moreover, the measurement of a residual polarization in the case of a reasonable polarization loss appears as a way to measure the spin tune spread and minimize it, if that is necessary.[11]

It's interesting also to study the opposite case of a small RF amplitude. Fig. 4 presents three other curves, where we fixed the spin tune spread $\sigma_\nu = 1\cdot 10^{-6}$ and the crossing rate $\dot{\varepsilon}\cdot f_0 = 2\,Hz/sec$, but changed the amplitude w: (red line) $w = 2\cdot 10^{-7}$; (blue line) $w = 5\cdot 10^{-7}$ and (magenta line) $w = 1\cdot 10^{-6}$. One can see from Fig. 4 the resonant depolarization by the RF-field. Decreasing the RF power provides a measurement of the spin tune with accuracy up to its spread σ_ν. In turn, the spin tune determination is simultaneously the absolute mean energy measurement, since the magnetic anomalies are well known.[13] For instance: $a_e = 1.159652193\times 10^{-3}$.

Beam energy calibration has been routinely used at electron-positron colliders in precise experiments for secondary particle mass measurements.[14] The coherent spin rotation by 90 degrees and full spin flip were crucially important at VEPP-2M in the experiment for electron and positron anomalous magnetic moments comparison.[15]

3. Spin flip at hadron accelerators

RF spin flip for proton and deuteron seems an easier procedure, due to the absence of quantum radiative effects. However, there is always noise in the system, which can influence the polarization, especially while flipping. The noise, modulating the tune $\varepsilon = \nu - \nu_d$, can come from a guiding field ripple and from RF frequency driver. Fluctuations in an amplifier create the noise for the resonance amplitude w. These effects can be easily estimated, if we know the Power Spectrum Density $P(\omega)$ of each noise source.

$$P(\omega) = \int \chi(\tau)\, e^{i\omega\tau} d\tau; \qquad \chi(\tau) = \frac{1}{T}\int x(t)\,x(t-\tau)dt,$$

where $\chi(\tau)$ is a noise correlation function. Then, taking into account infinitive quality of the spin (now damping), a mean square value and a diffusion rate follow from expressions:

$$\langle x^2 \rangle = \int_0^\infty P(\omega) d\omega; \qquad \frac{d\bar{x}^2}{dt} \cong P(\omega_c).$$

Using this approach, one can get, similar to (2.2), a formula for the depolarization time τ_d, caused by the noises and "simulate" protons polarization losses while flipping, as it was done for electrons ($\zeta(t) = \langle S_n \rangle = \zeta(0) \cdot \int_0^t e^{-t/\tau_d} dt$). By that, $\omega_c = h(t) = \sqrt{\varepsilon(t)^2 + w(t)^2}$ is the spin precession frequency in the resonance frame at the moment "t".

Till now, RF spin flip has been studied for protons and deuterons at the IUCF storage ring (see, for instance [16]) and at the synchrotron COSY.[17] At first, the both machines have demonstrated spin flip by RF-fields was going, as a rule, with very low polarization losses ($\ll 10^{-2}$ per pass). It's interesting to make an analysis for understanding a nature of this depolarization. Does it come from the noise or from violation of the condition of adiabatic crossing?

The experiments at IUCF and COSY machines were carried out with RF-solenoids and RF-dipoles. Authors have found that flipper strength depends on machine type and particle species. Suddenly, the results of these measurements have been claimed "unexpected discrepancies" between the measured values and "the theoretical expressions for the spin flip resonance widths".

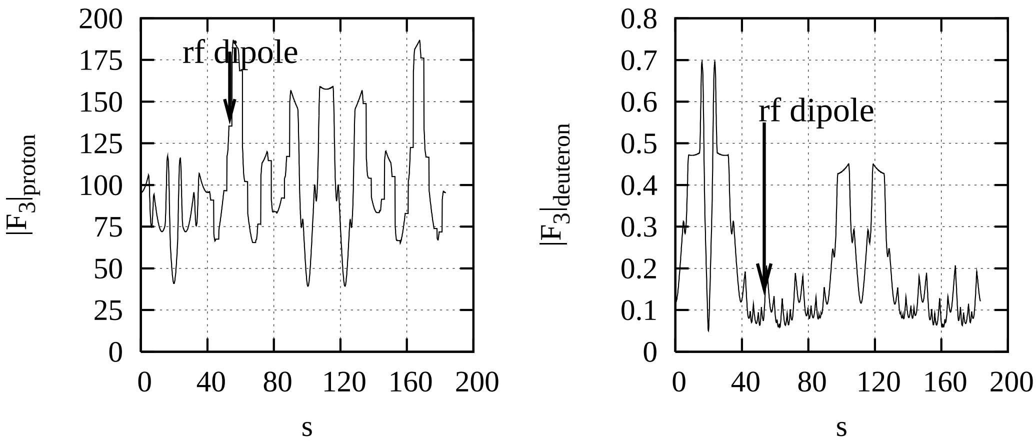

Figure 5: $|F_3|$ along the COSY orbit for protons and deuterons.

However, recent analysis of the COSY data, based on the formalism of the spin response functions, has found an amazing accordance of the experiment and the theory as for both kinds of the flippers, for both types of particles.[8] The main reason for the"unexpected discrepancies" is explained by Fig. 5, which presents results of ASPIRRIN calculations of $|F_3|$ values along the COSY orbit for protons and deuterons, that are distinguished in more than 1000 times in the point of the RF flipper.

A separate interest exists for the spin flip at machines with Siberian snakes.[18] In case of a perfect snake, the spin tune $\nu \equiv \frac{1}{2}$ and the spin flip is impossible by a one directional field, because besides the resonant harmonic $k + \nu_d$ its "mirror" partner $k + 1 - \nu_d$ contributes also essential to the spin motion. In a near vicinity of the resonance both harmonics give a field, which oscillates in the resonant frame with frequency 2ε. In this case the adiabatic condition will be surely violated.[19]

But a relatively small snake imperfection $\nu - 1/2 = \varepsilon \gg |w|$ is enough to provide successful spin flip. This method has been successfully at a few machines.

An alternative way to create a rotating RF field is very difficult to realize. In principle, it's possible to use two RF devices (AC dipoles or solenoids) with one directional fields, that are located along the orbit thus their mirror harmonics compensate each other, whereas the main harmonics reinforce.[20] Of course, a design of a system with two AC magnets requires one to know the spin response functions. They are modified by the snake magnets, but do not cancel out. It's especially important for high energy machine like RHIC, where a combination of spin rotators strongly changes the phases and moduli of F_1 and F_3. For instance, one can compare Fig. 6 and Fig. 7, where the graph of $|F_3|$ is given in the drift space L232 of RHIC at 100 GeV and 205 GeV for three scenarios: (a) solid (purple) no rotators, (b) dash (green) longitudinal polarization at both STAR and PHENIX, (c) dotdash (red) no rotators at STAR, radial polarization at PHENIX. The horizontal axis shows the arc length in meters, measured from the 6 o'clock point.[21] From comparison it's clear, that is impossible practically to construct an universal flipper, which works in arbitrary machine lattice.

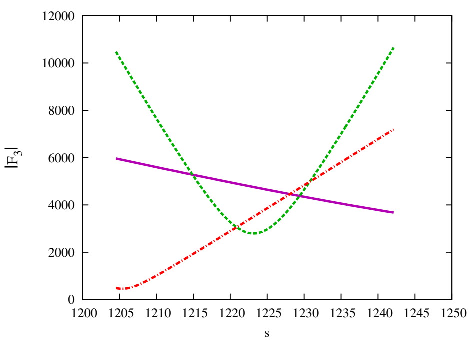

Figure 6: Graph of |F3| at 100 GeV.

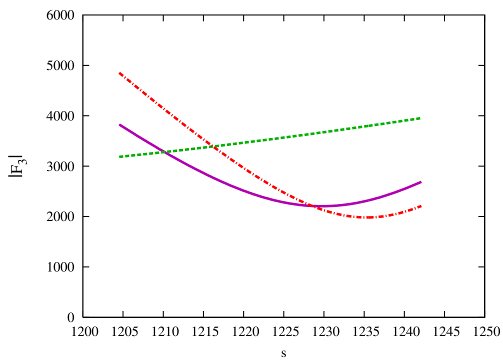

Figure 7: Graph of |F3| at 205 GeV.

Till now we discussed the spin flipping for adiabatic resonance crossing. But, there is another method. That is, so called, the resonant spin flip, when the RF field is switched on exactly on the resonant frequency during a time Δt, which is enough for spin rotation by the angle 180 degrees: $(w \cdot \Delta t = \pi)$. Of course, this approach requires high precision of the spin tune knowledge $\Delta\nu \ll w$. But, on the other hand, it's promising method for machines with many bunches. For beginning, let's try to reverse spins in one separate bunch. To do that, we have to supply a flipper by shot pulses, synchronized with the bunch. At next step, it's easy to apply this method for any bunches, if the pulse meander will be modified in an arbitrary form. If we extend meander up to the revolution time, spins of all bunches will be reversed. The simplest case for this method application takes place at the machine with perfect Siberian snake (or two snakes). One has to change only the pulse polarity from turn to turn (with some gaps with zero amplitude, if needed).

The simplicity of this approach does not free us from the necessity to know the response functions for right flipper design. But it's not so crucial as in the mirror harmonic compensation. In any case, the perfect spin flip will be finally indicated by polarimeters. Based on this measurement, it's not difficult to adjust the amplitude of the resonant harmonic w or the reversal time Δt.

Conclussion

At the present moment spin motion is a well known topic in the accelerator theory. Application of radio frequency devices for the spin control is not "terra incognita". The spin response formalism is a powerful, elegant, general technique with applications far beyond only spin flippers. The formalism treats rings of arbitrary structure (including Siberian Snakes), and both radiative and nonradiative polarization. For example, the formalism has been used with success to explain "unexpected discrepancies" between the measured values and "the theoretical expressions for the spin flip resonance widths" at IUCF and COSY.

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