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Relations between Supergravity and Gauge Theory and Implications for UV Properties of $\mathcal{N} = 4$ Supergravity

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Z. Bern*
Department of Physics and Astronomy
University of California and Los Angeles
Los Angeles, CA 90095-1547, USA

S. Davies
Department of Physics and Astronomy
University of California and Los Angeles
Los Angeles, CA 90095-1547, USA

T. L. Dennen
Niels Bohr International Academy and Discovery Center
Blegdamsvej 17
DK-2100 Copenhagen, Denmark

Y. T. Huang
Michigan Center for Theoretical Physics
Randall Laboratory of Physics
University of Michigan, Ann Arbor, MI 48109, USA

In this talk we explain how the duality between color and kinematics helps us determine the ultraviolet properties of supergravity theories. In particular, we use the associated double-copy relation to demonstrate that half-maximal $\mathcal{N} = 4$ supergravity in four space-time dimensions is ultraviolet finite at three loops, contrary to previous expectations from standard symmetry considerations

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1. Introduction

Recent years have seen a resurgence of interest in the study of the ultraviolet properties of supergravity theories. The consensus opinion from the late 70's and early 80's was that all such theories would likely diverge by three loops. (See ref. [1] for a review of the situation in that era). However, recent years have made it clear that the supergravity amplitudes are much better behaved, though it is not yet clear if a perturbatively finite supergravity theory is possible or not.

In the past few years significant progress has been made on this question, especially for the cases of $\mathcal{N}=8$ [2] and $\mathcal{N}=4$ supergravity [3]. In particular, there have been major advances in developing powerful tools for carrying out the explicit computations needed for unraveling the ultraviolet behavior. These include the unitarity method [4, 5] and its refinement known as the method of maximal cuts [6]. These tools have played a central role in uncovering a duality between color and kinematics [7, 8], which was then used in a variety of new supergravity calculations [9, 10, 11, 12]. There has also been enormous progress in carrying out the loop integration needed to determine the explicit values of the divergences. (See e.g. ref. [13].) There has also been enormous progress in understanding the role of supersymmetry and duality symmetry in constraining the counterterms (see e.g. ref. [14].)

In this talk we focus on the divergence properties of the three-loop four-point amplitudes of half-maximal $\mathcal{N}=4$ supergravity [11]. This case is especially interesting because the potential R^4 counterterm [15] does appear to be fully consistent with all known symmetry constraints even if not expressible as a full superspace integral [16].

The question of whether supergravity theories diverge in perturbation theory is still an open one. (For a recent optimistic opinion in favor of ultraviolet finiteness of $\mathcal{N}=8$ supergravity see ref. [17]. For a recent pessimistic opinion see ref. [18].) While conventional counterterm symmetry considerations suggest that all supergravity amplitudes will diverge at some loop order, studies of multiloop scattering amplitudes suggest that additional ultraviolet cancellations exist beyond these [19]. Surprisingly, even pure Einstein gravity at one loop exhibits remarkable cancellations as the number of external legs increases [20]. The double-copy structure also implies that gravity amplitudes are much more highly constrained than implied by the standard symmetries.

To settle the debate on the question of perturbative finiteness, we need to carry out further explicit calculations. Such calculations of expected divergences would either prove divergences or demonstrate their absence. This type of information is obviously very useful in guiding further studies. While there has been some recent progress on understanding the general structure of ultraviolet cancellations in gravity amplitudes by linking them to cancellations in gauge-theory amplitudes [12], more work is needed before the general structure can be unraveled.

2. Duality between color and kinematics

The duality between color and kinematics uncovered by Carrasco, Johansson and one

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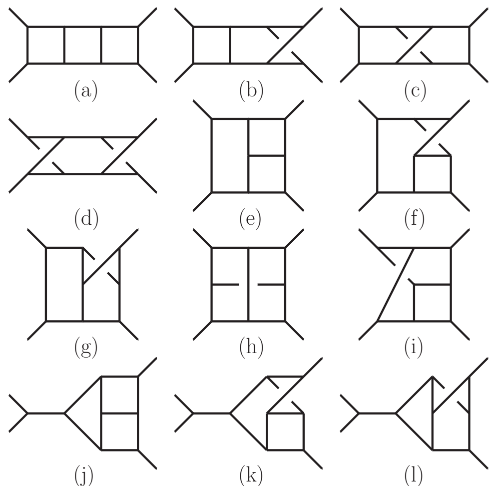

Figure 1: The twelve graphs of the $\mathcal{N}=4$ sYM three-loop four-point amplitude in a representation satisfying the duality between color and kinematics [8].

of the authors (BCJ) [7, 8] allows us to convert gauge-theory amplitudes into gravity ones via a double-copy formula [7, 8]. Whenever a representation of the gauge-theory amplitude can be found that satisfies the duality [21, 10, 11], it enormously simplifies the construction of gravity amplitudes, especially in the context of unitarity based computations [4, 6],

Any m-point L-loop gauge-theory amplitude with all particles in the adjoint representation can be written in the form,

$$\frac{(-i)^L}{g^{m-2+2L}}\mathcal{A}_m^{L-\mathrm{loop}} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{\prod_{\alpha_j} p_{\alpha_j}^2}\,, \tag{2.1}$$

where the sum labeled by j runs over the set of distinct m-point L-loop graphs with only cubic vertices, including distinct relabelings of external legs. The factor S_j is the symmetry factor of graph j which removes overcounts from internal symmetry. In this representation contact terms are absorbed into graphs with cubic vertices by multiplying and dividing by appropriate inverse propagators. The integrals are over L independent D-dimensional loop momenta. The product in the denominator runs over all Feynman propagators of graph j. The c_j are the color factors obtained by dressing every three-vertex with a group-theory structure constant, $\widetilde{f}^{abc} = i\sqrt{2}f^{abc}$, and the n_j are kinematic numerators of graph j depending on momenta, polarizations and spinors. We note that there is enormous freedom in the choice of finding valid numerators due to generalized gauge invariance [7, 8, 22, 23].

According to the duality conjecture of refs. [7, 8], a representation of L-loop m-point amplitudes should exist where kinematic numerators satisfy the same algebraic properties as color factors. For Yang-Mills theory this amounts to imposing the same Jacobi identities on the kinematic numerators as satisfied by the color factors,

$$c_i = c_j - c_k \;\Rightarrow\; n_i = n_j - n_k\,, \tag{2.2}$$

where the indices i, j, k denote the graph to which the color factors and numerators belong. Moreover, the numerator factors are required to have the same antisymmetry properties as color factors under interchange of any two legs attached to a cubic vertex. As explained in some detail in refs. [21, 10], the numerator relations are functional equations. For four-point tree amplitudes such relations were noticed long ago using Feynman diagrams [24]. Beyond the four-point tree level, the relations are highly nontrivial and hold only after appropriate rearrangements of the amplitudes.

At tree level, explicit forms of amplitudes satisfying the duality have been found for an arbitrary number of legs [25]. An interesting consequence of the duality is that color-ordered partial tree amplitudes satisfy nontrivial relations [7], proven in gauge theory and in string theory [26, 23]. Although we do not yet have a satisfactory Lagrangian understanding, some progress in this direction can be found in refs. [22, 27]. The duality (2.2) has also been expressed in terms of an alternative trace-based representation [28]. Progress has also been made in understanding the underlying infinite-dimensional Lie algebra [27, 29] responsible for the duality. The duality between color and kinematics also appears to hold in three-dimensional theories based on three algebras [30], as well as in some cases with higher-dimension operators [31].

At loop level, the duality has been confirmed to hold up to four loops for the four-point amplitudes of $\mathcal{N}=4$ super-Yang-Mills (sYM) theory [8, 10], and for the five-point one- and two-loop amplitudes of this theory [32]. Moreover, the infrared singularities do appear to be consistent with BCJ duality to all loop orders [33]. The duality is also known to hold for the identical-helicity one- and two-loop four-point amplitudes of pure Yang-Mills theory [8]. There has also been progress in understanding more general one-loop amplitudes [34].

Associated with the conjectured duality between color and kinematics is a double-copy formula for gravity amplitudes given by [8]

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}_n^{\rm loop} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{\prod_{\alpha_j} p_{\alpha_j}^2}\,, \tag{2.3}$$

where n_j and $\tilde{n}_j$ are kinematic numerator factors from gauge-theory amplitudes and κ is the gravitational coupling. The particular gravity theory obtained by the double-copy formula (2.3) is dictated by the choice of numerators. In eq. (2.3) only one of the two copies needs to satisfy duality (2.2) [8, 22]. The other gauge-theory amplitude can be any convenient representation arranged into graphs with only cubic vertices. At tree level, eq. (2.3) encodes the Kawai-Lewellen-Tye [35] relations between gravity and gauge theory [7].

3. Constructing the integrand

We obtain pure $\mathcal{N}=4$ supergravity amplitudes with no additional matter by taking one component gauge theory to be $\mathcal{N}=4$ sYM theory and the second component to be nonsupersymmetric pure Yang-Mills theory [9, 3]. The $\mathcal{N}=4$ sYM copy is obtained from ref. [8], which contains a representation with BCJ duality manifest. This representation of the $\mathcal{N}=4$ sYM amplitude is described by the twelve graphs displayed in fig. 1. As

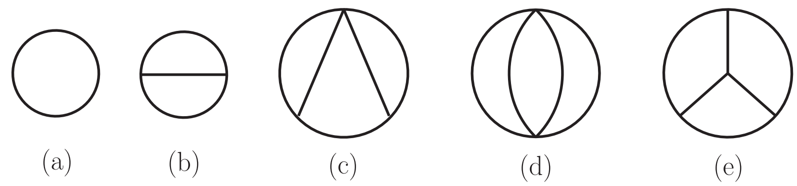

Figure 2: A basis of vacuum integrals used in the three-loop calculation of ref. [11].

discussed in ref. [11], for the pure Yang-Mills copy, a convenient representation is to use ordinary Feynman gauge Feynman diagrams. Any contact contributions are assigned to diagrams with only cubic vertices according to their color factor. In this construction, most Feynman graphs are irrelevant because in the double-copy formula, they get multiplied by vanishing $\mathcal{N} = 4$ sYM graph numerators. In this way, the complete three-loop four-point integrand of $\mathcal{N} = 4$ supergravity was determined in ref. [11] for all external states of the theory.

4. Extracting ultraviolet divergences

As discussed in ref. [11], to extract the ultraviolet divergences from the constructed integrand, we expand in large loop momenta or equivalently in small external momenta. This gives a set of vacuum graphs containing both infrared and ultraviolet divergences. We use the four-dimensional-helicity regularization scheme [36] variant of dimensional regularization because it preserves supersymmetry and has been used in analogous multiloop pure gluon and supersymmetric amplitudes [37]. In this scheme, the number of states circulating in the loops remain at their four-dimensional values. To separate out the infrared divergences at the level of the vacuum integrals, we introduce a uniform mass m, following the procedure in ref. [38]. Although ultimately there are no one- and two-loop ultraviolet divergences in $\mathcal{N} = 4$ supergravity, individual integrals generally will contain subdivergences. Extractions of ultraviolet divergences in higher-dimensional $\mathcal{N} = 8$ supergravity were discussed recently in refs. [39, 10].

The introduced mass regulator induces unphysical regulator dependence in individual integrals arising entirely from subdivergences. However, after systematically subtracting the subdivergences we obtain results independent of the details of the regulator choice [38]. Further details can be found in ref. [11]

The next task is to calculate the ultraviolet divergences of the vacuum integrals. To evaluate these integrals, we first use Lorentz invariance to replace tensors composed of loop momenta in the numerators by linear combinations of products of metric tensors $\eta^{\mu\nu}$ and dot products of momenta amongst themselves. (See ref. [10] for a more detailed discussion of simplifying tensor vacuum integrals.) To carry out the reduction of the scalar integrals to a basis, we use integration by parts as implemented in FIRE [40]. The resulting basis is given by the scalar vacuum integrals shown in fig. 2 (along with products of lower-loop integrals), with a single massive propagator corresponding to each line. As cross checks we

Graph	(divergence)/$(\langle 12\rangle^2[34]^2 st A^{\rm tree}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3}+\frac{205}{27648}\frac{1}{\epsilon^2}+\left(-\frac{5551}{768}\zeta_3+\frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3}-\frac{1}{4}\frac{1}{\epsilon^2}+\left(\frac{593}{288}\zeta_3-\frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3}+\frac{2057}{6912}\frac{1}{\epsilon^2}+\left(\frac{10769}{2304}\zeta_3-\frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3}-\frac{41}{1536}\frac{1}{\epsilon^2}+\left(\frac{3227}{2304}\zeta_3-\frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3}-\frac{29}{1024}\frac{1}{\epsilon^2}+\left(-\frac{2087}{2304}\zeta_3-\frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3}+\frac{9}{64}\frac{1}{\epsilon^2}+\left(\frac{101}{12}\zeta_3-\frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3}+\frac{89}{1152}\frac{1}{\epsilon^2}+\left(-\frac{377}{144}\zeta_3+\frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3}-\frac{251}{1152}\frac{1}{\epsilon^2}+\left(-\frac{835}{144}\zeta_3+\frac{7385}{3456}\right)\frac{1}{\epsilon}$

Table 1: The divergences for each graph in the four-graviton amplitude with helicities $(1^-2^-3^+4^+)$ as computed in ref. [11]. Each expression includes a permutation sum over external legs, with the symmetry factor appropriate to the graph. To simplify the expression, spinor helicity with the choice of reference momenta $q_1=q_2=k_3$ and $q_3=q_4=k_1$ is used.

also used MB [41] and FIESTA [42]. The resulting scalar integrals shown in fig. 2 have been evaluated in refs. [43].

In table 1, we collect the derived divergences of the three-loop four-graviton amplitude for each graph in fig. 1 for external gravitons with the indicated choices of spinor-helicity reference momenta (defined in, e.g., ref. [44]). The individual graphs are not gauge invariant; in particular, the vanishing the contribution (a)-(d) in table 1 is due to the choice of spinor-helicity reference momenta. The results shown in the table for each graph are summed over independent permutations and include symmetry factors. We have divided out a prefactor depending on the four-point color-ordered sYM tree amplitude, spinor inner products and the usual Mandelstam invariants s and t. If we sum the contributions of all graphs they vanish, so the three-loop four-graviton amplitude is ultraviolet finite. The actual calculation in ref. [11] does not rely on spinor helicity and makes use of formal polarization vectors and is valid for all states in the theory, proving that there are no three-loop four-point divergences in $\mathcal{N}=4$ supergravity.

5. Conclusions and outlook

The calculation of the coefficient of the potential three-loop four-point divergence of $\mathcal{N}=4$ supergravity [11] gives us a concrete example of an ultraviolet cancellation in supergravity that has not yet been understood from symmetry considerations [16]. The key future tasks are to find further examples of unexpected ultraviolet cancellations in supergravity theories and to see whether they can plausibly be explained by the standard symmetries of supergravity. For example, it may turn out that the inability to write the counterterm as a full superspace integral [16] might ultimately lead to an explanation for its finiteness. On the other hand, the two-loop counterterm of half-maximal supergravity in

$D=5$ does appear to be expressible as a full superspace integral [45], yet the corresponding divergence vanishes [46, 12]. (The counterterm is apparently also duality invariant after integration [45].) A novel explanation for these supergravity cancellations was proposed in ref. [12] by tying them to cancellations of divergences in forbidden color factors of gauge theories. An alternative proposal based on a hidden superconformal symmetry of $\mathcal{N}=4$ supergravity has also been given in ref. [47]. These results emphasize the importance of carrying out further explicit computations to determine if potential divergences are present. Obvious next steps are to determine the ultraviolet properties of $\mathcal{N}=8$ supergravity at five loops and $\mathcal{N}=4$ supergravity at four loops.

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