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Dimensional Reduction (and all that)

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I summarise the salient features of regularisation by dimensional reduction, and its relationship with the original form of dimensional regularisation.

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1. Introduction

Siegel [1] introduced *regularisation by dimensional reduction* (DRED) as a variation of *dimensional regularisation*, DREG [2], to maintain the equality of Bose-Fermi degrees of freedom characteristic of supersymmetry, an equality not preserved by DREG. As a consequence, for example, the relationship between the quark-quark-gluon and quark-squark-gluino couplings in SQCD is not preserved when DREG is used. Which is not to say that DREG cannot be used in supersymmetric theories; but it lacks convenience. For a formal discussion of the equivalence of DRED and DREG (modulo DRED ambiguities to be discussed below) see [3],[4]. A pedagogical introduction to Siegel's proposal, and a first discussion of its application to the *non-supersymmetric* case appears in Ref. [5]. Essentially DRED amounts to defining

$$\begin{aligned} x^{\mu} &\equiv (x^{i},0) \\ p^{\mu} &\equiv (p^{i},0) \\ W_{\mu} &\equiv \big(W_{i}(x^{j}),W_{\sigma}(x^{j})\big) \end{aligned} \tag{1.1}$$

where μ and i,j are 4-dimensional and D-dimensional indices respectively, and $D<4$. It is useful to define hatted quantities with $\mu,\nu\cdots$ indices whose only non-vanishing components are in the D-dimensional subspace; in particular, $\hat{g}_{\mu\nu}=(g_{ij},0)$.

So we define

$$g_{\mu\nu}=\hat{g}_{\mu\nu}+\tilde{g}_{\mu\nu} \tag{1.2}$$

where

$$\begin{aligned} g_{\mu\nu}g^{\mu\nu} &= 4 \\ g_{\mu\nu}\hat{g}^{\nu}{}_{\rho} &= \hat{g}_{\nu\rho} \\ \hat{g}_{\mu\nu}\hat{g}^{\mu\nu} &= D \\ g_{\mu\nu}\tilde{g}^{\nu}{}_{\rho} &= \tilde{g}_{\nu\rho} \\ \tilde{g}_{\mu\nu}\tilde{g}^{\mu\nu} &= \varepsilon=4-D \\ \hat{g}_{\mu\nu}\tilde{g}^{\nu}{}_{\rho} &= 0 \end{aligned} \tag{1.3}$$

A (Dirac) fermion represents 4 degrees of freedom as long as we define the Dirac matrix trace to satisfy $\mathrm{Tr}1=4$. Just as in DREG, the UV divergences of the four-dimensional theory are manifested by the occurrence of poles in $D-4$, and renormalisation is effected by subtracting these poles. The DRED version of the familiar DREG subtraction method $\overline{\mathrm{MS}}$ is often termed $\overline{\mathrm{DR}}$.

The dimensionally reduced form of the gauge transformations:

$$\begin{aligned} \delta W_{i}^{a} &= \partial_{i}\Lambda^{a}+gf^{abc}W_{i}^{b}\Lambda^{c} \\ \delta W_{\sigma}^{a} &= gf^{abc}W_{\sigma}^{b}\Lambda^{c} \\ \delta\psi^{\alpha} &= ig(R^{a})^{\alpha\beta}\psi^{\beta}\Lambda^{a} \end{aligned} \tag{1.4}$$

show that W_{σ}^{a} transform as scalars, called ε-scalars. Consequently the interactions

$$g\overline{\psi}\gamma_{\sigma}R^{a}\psi W_{\sigma}^{a} \quad \text{and} \quad g^{2}f^{abc}f^{ade}W_{\sigma}^{b}W_{\sigma'}^{c}W_{\sigma}^{d}W_{\sigma'}^{e}$$

are both gauge invariant by themselves. This means that in general they will not renormalise like the corresponding gauge interactions; and in the case of the quartic coupling, different group theory structures will be generated by radiative corrections, and require subtraction.

Moreover, a *mass* for the ε-scalars is itself gauge invariant. Since it is not forbidden, such a mass will therefore be generated by loop corrections *except* in supersymmetric theories, where supersymmetry transformations connect the ε-scalar to the gauge boson, which is of course protected from developing a mass by gauge invariance.

1.1 Evanescent Couplings and Masses

We have three classes of theories which behave differently under renormalisation using DRED.

- Supersymmetric Theories:

  The ε-scalar interactions remain in step with the corresponding gauge interactions, and its mass remains zero.

- Softly-broken supersymmetric theories:

  Radiative corrections generate a mass for the ε-scalar.

- Un-supersymmetric theories:

  Again a mass for the ε-scalar, and both its Yukawa coupling and the quartic interaction renormalise differently from the gauge coupling. New quartic group theory structures are generated.

1.2 ε-scalar quartic couplings

Let us consider QCD with gauge group $SU(N)$ for example: a basis for tensors K^{abcd} in $SU(N)$ is given by

$$\begin{aligned} K_1 &= \delta^{ab}\delta^{cd} \quad K_4 = d^{abe}d^{cde} \quad K_7 = d^{abe}f^{cde} \\ K_2 &= \delta^{ac}\delta^{bd} \quad K_5 = d^{ace}d^{bde} \quad K_8 = d^{ace}f^{bde} \\ K_3 &= \delta^{ad}\delta^{bc} \quad K_6 = d^{ade}d^{bde} \quad K_9 = d^{ade}f^{bce}. \end{aligned}$$

So for ε-scalars a natural basis is

$$\begin{aligned} H_1 &= \tfrac{1}{2}K_1, \quad H_2 = \tfrac{1}{2}(K_2+K_3) \\ H_3 &= \tfrac{1}{2}K_4, \quad H_4 = \tfrac{1}{2}(K_5+K_6), \end{aligned}$$

reducible in $SU(3)$ since then $H_3+H_4 = \frac{1}{3}(H_1+H_2)$.

This makes DRED ponderous in non-susy theories. Nevertheless, DRED and DREG are equivalent. For some loop calculations where the above interactions arise, see Refs. [6], [7].

1.3 DRED ambiguities

Siegel himself[8] drew attention to the following issue. Given $D < 4$, it would seem one can define $\hat{\varepsilon}^{\mu\nu\rho\sigma}$ as

$$\hat{\varepsilon}^{\mu\nu\rho\sigma} = \hat{g}^{\mu\alpha}\hat{g}^{\nu\beta}\hat{g}^{\rho\gamma}\hat{g}^{\sigma\delta}\varepsilon_{\alpha\beta\gamma\delta} \tag{1.5}$$

where $\varepsilon_{\alpha\beta\gamma\delta}$ is the usual 4-dimensional tensor. Then, using Eq. (1.3), one can use the basic relation

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\delta} = g^{\mu\alpha}g^{\nu\beta}g^{\rho\gamma}g^{\sigma\delta} - g^{\mu\beta}g^{\nu\alpha}g^{\rho\gamma}g^{\sigma\delta} + \cdots \tag{1.6}$$

to show that

$$\hat{\varepsilon}^{\mu\nu\rho\sigma}\hat{\varepsilon}^{\alpha\beta\gamma\delta} = \hat{g}^{\mu\alpha}\hat{g}^{\nu\beta}\hat{g}^{\rho\gamma}\hat{g}^{\sigma\delta} - \hat{g}^{\mu\beta}\hat{g}^{\nu\alpha}\hat{g}^{\rho\gamma}\hat{g}^{\sigma\delta} + \cdots \tag{1.7}$$

and hence by consideration of

$$A^{\mu\nu\rho\sigma} = \hat{\varepsilon}^{\mu\nu\rho\sigma}\hat{\varepsilon}^{\alpha\beta\gamma\delta}\hat{\varepsilon}_{\alpha\beta\gamma\delta} \tag{1.8}$$

that

$$(D+1)(D-4)(D^2-3D+6)\hat{\varepsilon}^{\mu\nu\rho\sigma} = 0 \tag{1.9}$$

This indicates that for $D < 4$ the definition of $\hat{\varepsilon}^{\mu\nu\rho\sigma}$ is problematic. Of course Eq. (1.5) is not true if $D > 4$ so one cannot define $\hat{\varepsilon}^{\mu\nu\rho\sigma}$ in this manner, and the problem does not arise.

1.4 γ_5

From

$$\{\gamma_\mu, \gamma^5\} = 0 \tag{1.10}$$

we have, *if* $D < 4$, then

$$\{\hat{\gamma}_\mu, \gamma^5\} = 0 \tag{1.11}$$

and hence that

$$(D-4)\mathrm{Tr}\left[\gamma^5\hat{\gamma}^\mu\hat{\gamma}^\nu\hat{\gamma}^\rho\hat{\gamma}^\sigma\right] = 0. \tag{1.12}$$

Once again, for $D > 4$, there is no problem: Eq. (1.11) does not hold and so Eq. (1.12) no longer follows. In that case you can impose

$$[\gamma_\sigma, \gamma^5] = 0, \quad \text{for} \quad 4 < \sigma < D \tag{1.13}$$

giving an unambiguous DREG derivation of the anomaly. In Ref. [9] it was proposed that the relation Eq. (1.13) simply be used also in DRED (effectively by analytical continuation from $D > 4$ to $D < 4$). This proposal was successful in that the authors then found that the Adler-Bardeen theorem held, in other words that the two-loop corrections to the axial anomaly summed to zero at two loops in both DREG and DRED.

To avoid all ambiguities we must avoid assuming relations like Eq. (1.6) and Eq. (1.10) even though they are true in “normal" four-dimensional space. For example, in two dimensional σ models[10] the relation

$$\hat{\varepsilon}^\mu{}_\nu\hat{\varepsilon}^{\nu\rho} = (1+c\varepsilon)\hat{g}^{\mu\rho} \tag{1.14}$$

can be used instead without ambiguity; the dependence on the c-parameter can then be absorbed into redefinitions of the renormalised metric and torsion. Stöckinger has formalised this[11] by distinguishing “normal” $D = 4$ space, from a *quasi* $D = 4$ space, $Q4S$, in which Eqs. (1.6), (1.14) are not true. By starting with our theory defined in such a space one avoids the problem.

2. The NSVZ β-function

There is a third scheme [12],[13] which differs from both DRED and DRED, in that there exists a particular form for β_g in $N=1$ supersymmetric gauge theories:

$$\beta_g^{NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{Q-2r^{-1}\mathrm{Tr}\left[\gamma^{NSVZ}C(R)\right]}{1-2C(G)g^2(16\pi^2)^{-1}}\right]. \tag{2.1}$$

Here $Q=T(R)-3C(G)$, $T(R)\delta^{ab}=\mathrm{Tr}(R^aR^b)$, $C(G)\delta^{ab}=f^{acd}f^{bcd}$, $r=\delta^{aa}$, $C(R)^i{}_j=(R^aR^a)^i{}_j$ and where γ^{NSVZ} is anomalous dimension in the NSVZ scheme of the the chiral supermultiplet, which transforms according to the representation R^a of the gauge group, which has structure constants f^{abc}.

The NSVZ scheme is like the DRED scheme in that it preserves (under renormalisation) coupling constant relations that are a consequence of supersymmetry; but β_g and γ calculated using DRED begin to deviate from the NSVZ results at three loops. However, there is an analytic redefinition of g, $g\to g'(g,Y)$ which connects them. It is *non-trivial* that the redefinition exists; in the abelian case for example, the redefinition consists of a single term, but it affects four distinct terms (with different tensor structure) in the β-functions. Exploiting the fact that $N=2$ theories are finite beyond one loop it was possible to determine β_g^{DRED} for $N=1$ at three and four loops for a general $N=1$ theory by (comparatively) simple calculations. Subsequently, special cases of these results were confirmed by the Karlsruhe group [14], [15].

2.1 The NSVZ↔DRED connection

In the abelian case (for simplicity)

$$\beta_g^{(3)DRED} = r^{-1}g\left\{3X_1+6X_3+X_4-6g^6Q\mathrm{Tr}[C(R)^2]\right\} \tag{2.2}$$

and

$$\beta_g^{(3)NSVZ} = r^{-1}g\left\{2X_1+4X_3-4g^6Q\mathrm{Tr}[C(R)^2]\right\}. \tag{2.3}$$

Here

$$\begin{aligned}
X_1 &= g^2Y^{klm}P^n{}_lC(R)^p{}_mY_{knp}\\
X_2 &= g^4Y^{klm}C(R)^n{}_lC(R)^p{}_mY_{knp}\\
X_3 &= g^4\mathrm{Tr}[PC(R)^2]\\
X_4 &= g^2\mathrm{Tr}[P^2C(R)]
\end{aligned} \tag{2.4}$$

where $P^i{}_j=\frac{1}{2}Y^{ikl}Y_{jkl}-2g^2C(R)^i{}_j$ and the cubic part of the superpotential is $W=\frac{1}{6}Y^{ijk}\phi_i\phi_j\phi_k$.

The coupling constant redefinition linking the two schemes is uniquely determined up to an overall constant:

$$\delta g = -(16\pi^2)^{-2}\tfrac{1}{2}r^{-1}g^3\mathrm{Tr}\left[PC(R)\right] \tag{2.5}$$

and generates just the right shift in β_g:

$$(16\pi^2)^3\delta\beta_g = r^{-1}g\left(-X_1-2X_3-X_4+2g^6Q\mathrm{Tr}[C(R)^2]\right). \tag{2.6}$$

The lesson of this result is that the form of β-functions beyond one loop is *not* as arbitrary as generally believed; the set of renormalisation schemes which is spanned by parameter redefinitions does not permit arbitrary assignments of coefficients of all terms.

3. Physical quantities and the schemes

A QCD example. In DRED:

$$m_t^{\text{pole}} = m_t^{DRED}(\mu)\left[1+\frac{\alpha_3^{DRED}(\mu)}{3\pi}\left(5-3\ln\frac{m_t^2}{\mu^2}\right)\right] \tag{3.1}$$

whereas in DREG:

$$m_t^{\text{pole}} = m_t^{DREG}(\mu)\left[1+\frac{\alpha_3^{DREG}(\mu)}{3\pi}\left(4-3\ln\frac{m_t^2}{\mu^2}\right)\right] \tag{3.2}$$

from which we can deduce that

$$m_t^{DREG}(\mu) = m_t^{DRED}(\mu)\left[1+\frac{\alpha_3}{3\pi}\right]. \tag{3.3}$$

Thus to establish the relation between the schemes we must calculate a *physical* quantity in both schemes.

4. The DRED SQCD β-function

This has been calculated through four loops[16]:

$$\begin{aligned}
16\pi^2\beta_g^{(1)} &= (N_f-3N_c)\,g^3,\\
(16\pi^2)^2\beta_g^{(2)} &= \left(\left[4N_c-\tfrac{2}{N_c}\right]N_f-6N_c^2\right)g^5,\\
(16\pi^2)^3\beta_g^{(3)} &= \left(\left[\tfrac{3}{N_c}-4N_c\right]N_f^2\right.\\
&\quad+\left.\left[21N_c^2-\tfrac{2}{N_c^2}-9\right]N_f-21N_c^3\right)g^7,\\
(16\pi^2)^4\beta_g^{(4)} &= \left(-\tfrac{2}{3N_c}N_f^3+\left[132N_c^3-66N_c-\tfrac{8}{N_c}-\tfrac{4}{N_c^3}\right]N_f\right.\\
&\quad+\left[44+\tfrac{36\zeta(3)-20}{3N_c^2}-(42+12\zeta(3))N_c^2\right]N_f^2\\
&\quad-\left.102N_c^4\right)g^9.
\end{aligned}$$

Note that the higher order group theory invariants of the form $(\mathrm{Tr}F^aF^bF^cF^d+\cdots)^2$ and $(\mathrm{Tr}R^aR^bR^cR^d+\cdots)^2$ found in the corresponding 4-loop QCD calculation (using DREG or DRED) do not appear here; and indeed they cancel in those calculations when the fermion representation R^a is replaced by the adjoint, F^a, thus rendering the theory supersymmetric.

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It is possible that β_g^{DRED} for SQCD is free of such structures to all orders (manifestly so for β_g^{NSVZ} in the absence of chiral superfields, of course, see Eq. (2.1)).

These new terms in QCD cannot be removed by analytic coupling constant redefinitions; it follows that the DRED↔DREG↔ NSVZ linkage described above does *not* extend to the QCD β-function ansatz of Ryttov and Sannino[17]. I will return to this issue in the discussion of soft β-functions.

5. DRED and soft breaking

In DRED the ε-scalar mass mixes with the $\phi\phi^*$ mass terms of genuine particles under renormalisation [18]:

$$\begin{aligned}\beta_{\tilde{m}^2} &= A(g,Y)\tilde{m}^2 + \sum_i B_i(g,Y)m_i^2 + \cdots ,\\ \beta_{m_i^2} &= C_i(g,Y)m_i^2 + D_i\tilde{m}^2 + \cdots ,\end{aligned} \tag{5.1}$$

where the $+\cdots$ denotes terms involving gaugino masses and A-parameters.

By an analytic redefinition of the form

$$m_i^2|_{\mathrm{DRED}'} = m_i^2|_{\mathrm{DRED}} - C_i(g)\tilde{m}^2 + \cdots \tag{5.2}$$

we can make $\beta_{m_i^2}$ is independent of $\tilde{m}^2$. The DRED$'$ scheme can be extended to all orders [19].

5.1 The soft β-functions

Using DRED$'$, we can prove [20], [21] that the β-functions for the soft parameters corresponding to the gaugino mass M, the ϕ^3 interaction h, and the ϕ^2 mass terms b are given by:

$$\begin{aligned}\beta_M &= 2\mathcal{O}\left(\frac{\beta_g}{g}\right)\\ \beta_h^{ijk} &= \gamma_l^{(i}h^{jk)l} - 2\Gamma_l^{(i}Y^{jk)l}\\ \beta_b^{ij} &= \gamma_l^{(i}b^{j)l} - 2\Gamma_l^{(i}\mu^{j)l}\end{aligned} \tag{5.3}$$

where γ_j^i is the chiral supermultiplet anomalous dimension in DRED. The same results hold in the NSVZscheme, with of course γ then being the anomalous dimension in that scheme.

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h\frac{\partial}{\partial Y}\right), \quad (\Gamma)^i{}_j = \mathcal{O}\gamma^i{}_j$$

Thus the underlying supersymmetric theory determines these soft β-functions to all orders.

5.2 The soft scalar mass β-function

The β-function for the $\phi\phi^*$ mass term m^2 is a bit more tricky [22]:

$$\beta_{m^2} = \left[2\mathcal{O}\mathcal{O}^* + 2|M|^2g^2\frac{\partial}{\partial g^2} + \left(\tilde{Y}\frac{\partial}{\partial Y} + cc\right) + X\frac{\partial}{\partial g}\right]\gamma$$

where $Y_{lmn} = (Y^{lmn})^*$, and $\tilde{Y}^{ijk} = Y^{l(jk}(m^2)^{i)}{}_l$

Here

$$X_{\mathrm{NSVZ}} = -2\frac{g^3}{16\pi^2}\frac{r^{-1}\mathrm{Tr}[m^2C(R)] - MM^*C(G)}{1 - 2C(G)g^2(16\pi^2)^{-1}}.$$

$X_{\mathrm{DRED}'}$ is known through three loops, but there is no all orders form for it.

6. The RS anzatz for QCD

Ref. [17] presents the following ansatz for the QCD β-function:

$$\beta_g^{RS} = \frac{g^3}{16\pi^2}\left[\frac{Q - \frac{2}{3}\gamma_m N_f T(R)}{1 - 2C(G)g^2(16\pi^2)^{-1}\left(1 + 2\frac{N_f T(R) - C(G)}{Q}\right)}\right] \tag{6.1}$$

where γ_m is the fermion mass anomalous dimension. In the special case of a single fermion adjoint multiplet (corresponding to $N = 1$ susy) they equate this to

$$\beta_g^{NSVZ} = -3\frac{g^3 C(G)}{16\pi^2}/(1 - 2C(G)g^2(16\pi^2)^{-1}) \tag{6.2}$$

to deduce that then

$$\gamma_m = -6\frac{g^2 C(G)}{16\pi^2}/(1 - 2C(G)g^2(16\pi^2)^{-1}) \tag{6.3}$$

But in this case the γ_m is precisely the β-function for the gaugino mass; a soft breaking term. Consequently it is given in the NSVZ scheme by the formula

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right), \tag{6.4}$$

i.e.

$$\gamma_m^{NSVZ} = -6\frac{g^2 C(G)}{16\pi^2}\frac{1}{(1 - 2C(G)g^2(16\pi^2)^{-1})^2} \tag{6.5}$$

which is *not* equal to the RS deduction Eq. (6.3) beyond one loop.

So: I can calculate in the DRED and DREG schemes, and relate the results to each other and to the NSVZ scheme; but I don't know how to calculate in the RS scheme. Moreover, it is unclear to me why there should exist a scheme in which Eq. (6.1) and Eq. (6.2) are simultaneously valid.

7. Summary

The introduction of DREG was a remarkable achievement, and of crucial *practical* value in making loop calculations in non-abelian gauge theories much simpler to organise than they are with other regulators such as explicit UV cut-offs. An important part of this is the fact that DREG preserves gauge invariance; DRED attempts to extend this advantage to encompass supersymmetry as well. In spite of the difficulties with defining the method in a rigorous manner that we have described above, the practical advantages mean that DRED is universally adopted for calculations beyond one loop in supersymmetry; with DRED$'$ employed in the softly-broken case. For a recent example relevant to LHC physics, with a careful discussion of the relationship between DRED and DREG and also the matching to the $\overline{\text{MS}}$ scheme, see Ref. [23].

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