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The Sudakov form factor to three loops in $\mathcal{N}=4$ super Yang-Mills

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We review the results for the Sudakov form factor in $\mathcal{N}=4$ super Yang-Mills theory up to the three-loop level. At each loop order, the form factor is expressed as a linear combination of only a handful scalar integrals, with small integer coefficients. Working in dimensional regularisation, the expansion coefficients of each integral exhibit homogeneous transcendentality in the Riemann ζ-function. We find that the logarithm of the form factor reproduces the correct values of the cusp and collinear anomalous dimensions. Moreover, the form factor in $\mathcal{N}=4$ super Yang-Mills can be related to the leading transcendentality pieces of the QCD quark and gluon form factor. Finally, we comment briefly on the ultraviolet properties of the $\mathcal{N}=4$ form factor in $D>4$ dimensions.

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1. Introduction and definition

In recent years the investigation of scattering amplitudes in gauge theories – in particular $\mathcal{N}=4$ super Yang-Mills (SYM) theory – has experienced tremendous progress, and revealed a lot of insight into their structure, see [1] for a review.

Quantities closely related to scattering amplitudes are form factors. For example, planar amplitudes can be factorised into an infrared divergent part, given by a product of form factors, and an infrared finite remainder [2]. The relation to form factors makes it possible to give an operator definition of the latter. In addition, one observes that both scattering amplitudes and form factors have uniform degree of transcendentality in their loop and/or ε-expansion.

For both, the planar four-particle amplitude and the form factor, the general form of the result is known in principle. For the former, this is due to dual conformal symmetry, for the latter it is due to the exponentiation of infrared divergences. However, it is a non-trivial task to obtain these a priori known results from an explicit linear combination of loop integrals. The final result, however, is simple and suggests that there should be more structure hidden in the loop integral expressions. Hence by studying them further one might gain insights into better ways of evaluating them.

Despite the apparent simpler structure of form factors compared to scattering amplitudes (the former have a trivial scale dependence), less is known about the loop expansion of form factors in $\mathcal{N}=4$ SYM than about scattering amplitudes. For example, the calculation of the planar four-point amplitude has been carried out to the four-loop order, see e.g. [3]. On the other hand, the Sudakov (or scalar) form factor in $\mathcal{N}=4$ SYM has long been known only to two loops owing to a calculation by van Neerven [4], and has only recently been extended to one higher loop [5].

Although generalisations of the Sudakov form factor to the case of more external on-shell legs and different composite operators have been discussed recently [6, 7], we will restrict ourselves in the present article to the perturbative expansion of the Sudakov form factor discussed in [4, 5].

We start by introducing the operator

$$\mathcal{O} = \mathrm{Tr}(\phi_{12}\phi_{12})\,, \tag{1.1}$$

where the scalar fields ϕ_{AB} are in the representation $\mathbf{6}$ of $SU(4)$, and $\phi_{AB} = \phi^a_{AB} T_a$, with T_a being the generators of $SU(N)$ in the fundamental representation. The operator $\mathcal{O}$ is a colour singlet and has zero anomalous dimension. In terms of $\mathcal{O}$ the form factor is given by

$$\mathcal{F}_{\mathcal{S}} = \langle \phi^a_{34}(p_1)\phi^b_{34}(p_2)\,\mathcal{O}\rangle \equiv \mathrm{Tr}(T^a T^b) F_S\,. \tag{1.2}$$

The states $\phi^a_{34}(p_1)$ and $\phi^b_{34}(p_2)$ are in the adjoint representation, and the outgoing momenta p_1 and p_2 are massless and on-shell, i.e. $p_1^2 = p_2^2 = 0$, and $q^2 \equiv (p_1+p_2)^2$. In order to regularise IR divergences associated with the on-shell legs we work in dimensional regularisation with $D = 4-2\varepsilon$. In order to facilitate the presentation of the results in sections 3 and 4 we introduce two more quantities, the first one being the dimensionless variable $x = \mu^2/(-q^2 - i\eta)$, with infinitesimal $\eta > 0$. The second quantity is the 't Hooft coupling $a = (g^2 N)/(8\pi^2)\,(4\pi)^{\varepsilon}\, e^{-\varepsilon\gamma_E}$, where g is the gauge coupling of $\mathcal{N}=4$ SYM, N is the number of colours, and $\gamma_E \approx 0.5772$ is the Euler-Mascheroni constant. The loop-expansion of the form factor now assumes the following form,

$$F_S = 1 + a x^{\varepsilon} F_S^{(1)} + a^2 x^{2\varepsilon} F_S^{(2)} + a^3 x^{3\varepsilon} F_S^{(3)} + \mathcal{O}(a^4)\,. \tag{1.3}$$

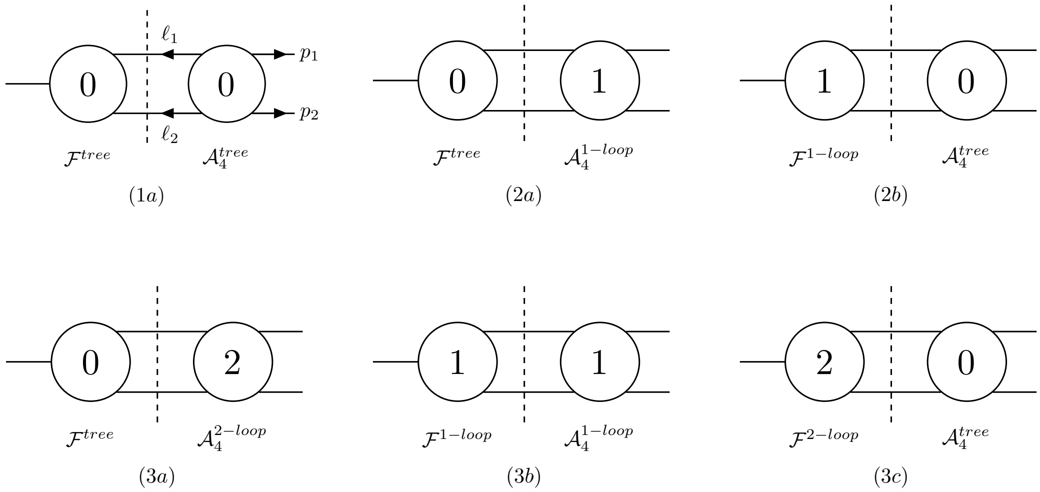

Figure 1: Two-particle cuts up to three loops. The numbers inside the circles indicate the respective number of loops in the form factors and four-particle scattering amplitudes.

The superscripts denote the loop-order, and we normalised the tree-level contribution to unity.

Up to the three-loop level, the L-loop form factor $F_S^{(L)}$ is strictly proportional to N^L, i.e. there is only the leading-in-colour contribution. This changes at four loops since the quartic Casimir $(d_{abcd})^2$ can appear. Whether or not the latter will actually be present at four loops is another very interesting related question, and has to do with the colour dependence of infrared divergences in gauge theories, see e.g. [8] and references therein.

2. Derivation of the form factor from unitarity cuts

We will use the method of unitarity cuts [9, 10] to derive an expression for the Sudakov form factor in $\mathcal{N}=4$ SYM in terms of scalar loop integrals. We will apply two-particle cuts, as well as generalised cuts. The two-particle cuts are displayed schematically in Fig. 1. At a given loop order $L \geq 1$ one has to consider all contributions from cuts of the m-loop form factor with the $(L-1-m)$-loop four-particle scattering amplitude, with $m = 0, \ldots, L-1$. The respective values are shown inside the circles in Fig. 1.

Let us derive the one-loop result explicitly. We follow the notations for unitarity cuts of ref. [11]. We have to compute the two-particle cut (1a) shown in Fig. 1. It is given by

$$\mathscr{F}_{\mathscr{S}}^{1-loop}\Big|_{\mathrm{cut(1a)}} = \int \sum_{P_1,P_2} \frac{d^D k}{(2\pi)^D}\,\frac{i}{\ell_2^2}\,\mathscr{F}_S^{tree}(-\ell_1,-\ell_2)\,\frac{i}{\ell_1^2}\,\mathscr{A}_4^{tree}(\ell_2,\ell_1,p_1,p_2)\Big|_{\ell_1^2=\ell_2^2=0}, \tag{2.1}$$

where ℓ_1 and ℓ_2 are the momenta of the cut legs, and the sum runs over all possible particles across the cut. The four-particle tree ampliutde $\mathscr{A}_4^{tree}(\ell_2,\ell_1,p_1,p_2)$ is given by

$$\mathscr{A}_4^{tree} = g^2\mu^{2\varepsilon} \sum_{\sigma\in S_4/Z_4} \mathrm{Tr}(T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}T^{a_{\sigma(3)}}T^{a_{\sigma(4)}})A_{4;1;1}^{tree}(\sigma(1),\sigma(2),\sigma(3),\sigma(4)), \tag{2.2}$$

with the 'partial amplitudes' $A^{tree}_{4;1,1}(\phi_{12}(1),\phi_{12}(2),\phi_{34}(3),\phi_{34}(4)) = -i\,s_{12}/s_{23}$. The tree-level form factor is simply given by

$$\mathscr{F}^{tree}_{\mathscr{S}}(-\ell_1,-\ell_2) = \mathrm{Tr}(T^aT^b)\,. \tag{2.3}$$

With our choice of external states, only scalars can appear as intermediate particles, and we do not need the spinor helicity formalism. With this, Eq. (2.1) becomes

$$\begin{aligned}\mathscr{F}^{1-loop}_{\mathscr{S}}\Big|_{\mathrm{cut\,(1a)}} &= -2g^2\mu^{2\varepsilon}Nq^2\,\mathrm{Tr}(T^aT^b)\int\frac{d^Dk}{i(2\pi)^D}\frac{1}{k^2(k+p_1)^2(k-p_2)^2}\Big|_{\mathrm{cut\,(1a)}}\\ &= -2g^2\mu^{2\varepsilon}Nq^2\,\mathrm{Tr}(T^aT^b)\,D_1\Big|_{\mathrm{cut\,(1a)}}\,,\end{aligned} \tag{2.4}$$

where we have identified the cut of the one-loop form factor with the cut of the one-loop triangle integral D_1, see Fig. 2. It turns out that this result is exact, i.e. that we can remove the "cut (1a)" in Eq. (2.4) and get

$$F^{1-loop}_S = g^2N\mu^{2\varepsilon}(-q^2)2D_1\,. \tag{2.5}$$

At two loops, following analogous steps, the result for the form factor is given by [4],

$$F^{2-loop}_S = g^4N^2\mu^{4\varepsilon}(-q^2)^2\left[4E_1+E_2\right], \tag{2.6}$$

where the diagrams E_1 and E_2 are also shown in Fig. 2. The unitarity cut (2b) of Fig. 1 detects only the presence of the planar integral E_1. The unitarity cut (2a) of Fig. 1 reveals – besides E_1 – the non-planar integral E_2. The appearance of the latter stems from the fact that we have to use the full one-loop four-point amplitude

$$\begin{aligned}\mathscr{A}^{1-loop}_4 &= g^4\mu^{4\varepsilon}\sum_{\sigma\in S_4/Z_4}N\,\mathrm{Tr}(T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}T^{a_{\sigma(3)}}T^{a_{\sigma(4)}})A^{1-loop}_{4;1,1}(\sigma(1),\sigma(2),\sigma(3),\sigma(4))\\ &+g^4\mu^{4\varepsilon}\sum_{\sigma\in S_4/Z_2^3}\mathrm{Tr}(T^{a_{\sigma(1)}}T^{a_{\sigma(2)}})\mathrm{Tr}(T^{a_{\sigma(3)}}T^{a_{\sigma(4)}})A^{1-loop}_{4;1,3}(\sigma(1),\sigma(2),\sigma(3),\sigma(4))\,,\end{aligned} \tag{2.7}$$

which in addition to single trace terms also contains double trace terms. The latter are subleading in the number of colours N. However, the colour algebra gives rise to another factor of N for those terms, so that they contribute to the form factor at the leading colour, just like the single trace terms.

Finally, at three loops the two-particle cuts are given by cuts (3a) – (3c) of Fig. 1. One finds for their total contribution

$$F^{3-loop}_S\Big|_{2-\mathrm{part.\ cut}} = g^6\mu^{6\varepsilon}N^3(-q^2)^2\left[8(-q^2)F_1-2F_2+4F_3+4F_4-4F_5-4F_6-4F_8\right]\Big|_{2-\mathrm{part.\ cut}}. \tag{2.8}$$

The integrals F_i are given in Fig. 3. It is remarkable that the coefficients of all integrals are small integer numbers. In order to detect also integrals not having any two-particle cuts we study generalised cuts, where we cut all or all but one propagator. This serves as a cross-check on the results already obtained above and detects further integrals such as F_9. The total result at three loops then assumes the form

$$F^{3-loop}_S = g^6\mu^{6\varepsilon}N^3(-q^2)^2\left[8(-q^2)F_1-2F_2+4F_3+4F_4-4F_5-4F_6-4F_8+2F_9\right]. \tag{2.9}$$

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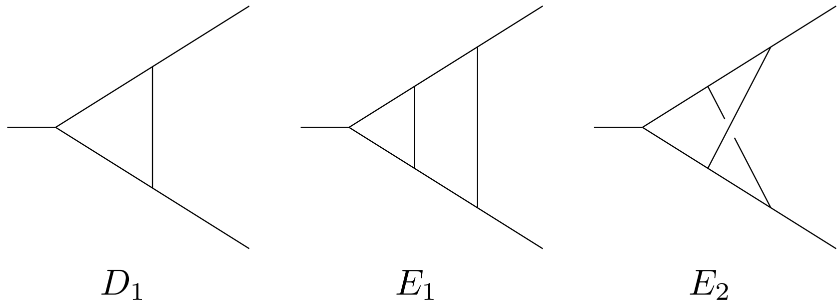

Figure 2: Diagrams that contribute to the one-loop and two-loop form factor in $\mathcal{N}=4$ SYM. All internal lines are massless.

3. Final result for the form factor up to three loops

Using unitarity cut methods described in the previous section we obtain the following result for the $\mathcal{N}=4$ SYM form factor up to three loops [5].

$$\begin{aligned}
F_S = \; & 1 + g^2 N \mu^{2\varepsilon} \cdot (-q^2) \cdot 2 D_1 + g^4 N^2 \mu^{4\varepsilon} \cdot (-q^2)^2 \cdot [4E_1 + E_2] \\
& + g^6 N^3 \mu^{6\varepsilon} \cdot (-q^2)^2 \cdot \left[8\,(-q^2) F_1 - 2F_2 + 4F_3 + 4F_4 - 4F_5 - 4F_6 - 4F_8 + 2F_9\right] \\
& + \mathcal{O}(g^8) \,.
\end{aligned} \tag{3.1}$$

All diagrams are shown in Figs. 2 and 3. It is remarkable that the form factor up to three loops is given by a small number of scalar loop integrals, each having a small integer coefficient. Working in dimensional regularisation with $D = 4 - 2\varepsilon$, the Laurent-series expansions of all diagrams are known from the calculation of the QCD quark and gluon form factor [12–18]. They yield for the Sudakov form factor in $\mathcal{N}=4$ SYM

$$\begin{aligned}
F_S^{(1)} = \; & -\frac{1}{\varepsilon^2} + \frac{\pi^2}{12} + \frac{7\zeta_3}{3}\,\varepsilon + \frac{47\pi^4}{1440}\,\varepsilon^2 + \varepsilon^3\left(\frac{31\zeta_5}{5} - \frac{7\pi^2\zeta_3}{36}\right) + \varepsilon^4\left(\frac{949\pi^6}{120960} - \frac{49\zeta_3^2}{18}\right) \\
& + \varepsilon^5\left(-\frac{329\pi^4\zeta_3}{4320} - \frac{31\pi^2\zeta_5}{60} + \frac{127\zeta_7}{7}\right) + \varepsilon^6\left(\frac{49\pi^2\zeta_3^2}{216} - \frac{217\zeta_3\zeta_5}{15} + \frac{18593\pi^8}{9676800}\right) \\
& + \mathcal{O}(\varepsilon^7) \,,
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
F_S^{(2)} = \; & +\frac{1}{2\varepsilon^4} - \frac{\pi^2}{24\varepsilon^2} - \frac{25\zeta_3}{12\varepsilon} - \frac{7\pi^4}{240} + \varepsilon\left(\frac{23\pi^2\zeta_3}{72} + \frac{71\zeta_5}{20}\right) + \varepsilon^2\left(\frac{901\zeta_3^2}{36} + \frac{257\pi^6}{6720}\right) \\
& + \varepsilon^3\left(\frac{1291\pi^4\zeta_3}{1440} - \frac{313\pi^2\zeta_5}{120} + \frac{3169\zeta_7}{14}\right) \\
& + \varepsilon^4\left(-66\zeta_{5,3} + \frac{845\zeta_3\zeta_5}{6} - \frac{1547\pi^2\zeta_3^2}{216} + \frac{50419\pi^8}{518400}\right) + \mathcal{O}(\varepsilon^5) \,,
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
F_S^{(3)} = \; & -\frac{1}{6\varepsilon^6} + \frac{11\zeta_3}{12\varepsilon^3} + \frac{247\pi^4}{25920\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{85\pi^2\zeta_3}{432} - \frac{439\zeta_5}{60}\right) \\
& - \frac{883\zeta_3^2}{36} - \frac{22523\pi^6}{466560} + \varepsilon\left(-\frac{47803\pi^4\zeta_3}{51840} + \frac{2449\pi^2\zeta_5}{432} - \frac{385579\zeta_7}{1008}\right) \\
& + \varepsilon^2\left(\frac{1549}{45}\zeta_{5,3} - \frac{22499\zeta_3\zeta_5}{30} + \frac{496\pi^2\zeta_3^2}{27} - \frac{1183759981\pi^8}{7838208000}\right) + \mathcal{O}(\varepsilon^3) \,.
\end{aligned} \tag{3.4}$$

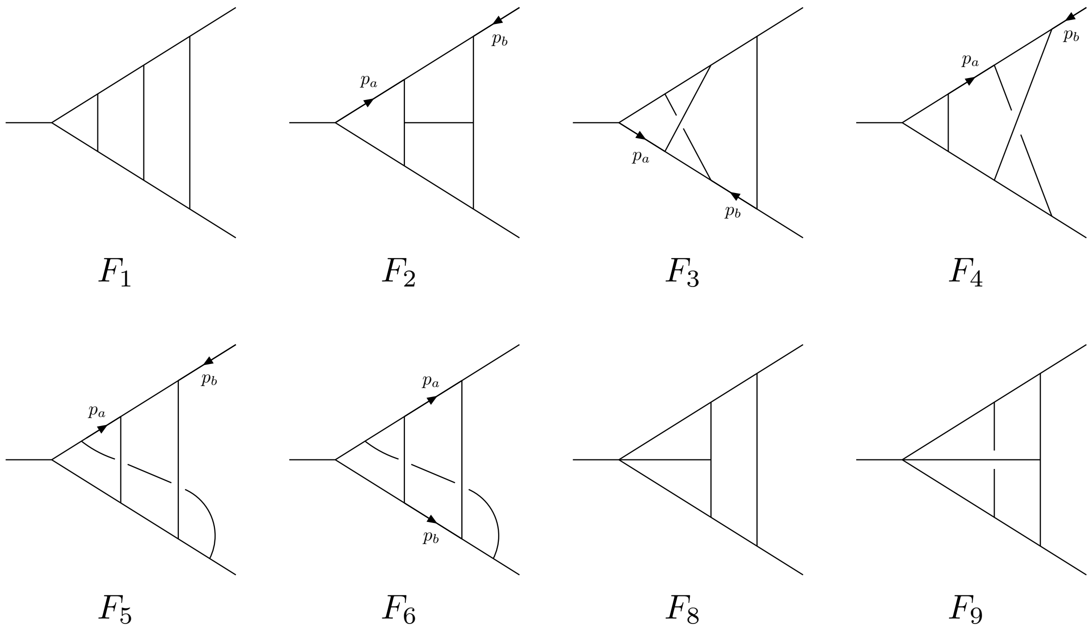

Figure 3: Diagrams that contribute to the three-loop form factor in $\mathcal{N}=4$ SYM. All internal lines are massless. p_a and p_b on arrow lines denote an irreducible scalar product $(p_a+p_b)^2$ in the numerator.

The coefficients of the ε-expansions are of increasing transcendentality (or weight) in the Riemann ζ-function[1]. One recognizes that each coefficient in the above formulas has homogeneous weight; a property that does not only hold true for the final result, but for each of the diagrams in Eq. (3.1) contributing to it. We also remark that in order to obtain all finite pieces of the logarithm of the form factor (see section 4) we need the ε-expansion through terms of transcendental weight six. We emphasize that our expressions contain two more orders in ε and therefore contain already all information required for exponentiation at four loops.

Let us elaborate here on yet another very interesting observation, namely the leading transcendentality principle [19]. To this end, let us specify the QCD quark and gluon form factor – which do *not* have the homogeneous-weight property – to a supersymmetric Yang-Mills theory with a bosonic and fermionic degree of freedom in the same colour representation. This is achieved by setting $C_A = C_F = 2\,T_F$ and $n_f = 1$ in the QCD result [14]. We find that with this adjustment the leading (i.e. highest) transcendentality pieces of the quark and gluon form factor become equal, and moreover coincide with the Sudakov form factor in $\mathcal{N}=4$ SYM presented here. This equality holds true at one, two, and three loops and in all coefficients up to transcendental weight eight, and it serves as an important check of our result.

4. Logarithm of the form factor

The logarithm of the form factor is given by

$$\ln(F_S) \;=\; \ln\Big(1 + a\,x^{\varepsilon}\,F_S^{(1)} + a^2\,x^{2\varepsilon}\,F_S^{(2)} + a^3\,x^{3\varepsilon}\,F_S^{(3)} + \mathscr{O}(a^4)\Big)$$

[1] One assigns to π^i the weight i and to ζ_k the weight k. Their product has weight $i+k$.

$$= a x^{\varepsilon} F_S^{(1)} + a^2 x^{2\varepsilon} \left[F_S^{(2)} - \frac{1}{2}\left(F_S^{(1)}\right)^2 \right] + a^3 x^{3\varepsilon} \left[F_S^{(3)} - F_S^{(1)} F_S^{(2)} + \frac{1}{3}\left(F_S^{(1)}\right)^3 \right] + \mathcal{O}(a^4)\,. \tag{4.1}$$

Plugging in the results from Eqs. (3.2) – (3.4) we verify the cancellation of all poles higher than $1/\varepsilon^2$, as expected from exponentiation of infrared divergences. The logarithm of the form factor therefore has the generic structure [20]

$$\ln(F_S) = \sum_{L=1}^{\infty} a^L x^{L\varepsilon} \left[-\frac{\gamma^{(L)}}{4(L\varepsilon)^2} - \frac{\mathcal{G}_0^{(L)}}{2L\varepsilon} \right] + \mathcal{O}(\varepsilon^0)\,, \tag{4.2}$$

and we confirm up to $L=3$ the L-loop cusp $\gamma^{(L)}$ and collinear $\mathcal{G}_0^{(L)}$ anomalous dimensions [21]

$$\gamma(a) = \sum_{L=1}^{\infty} a^L \gamma^{(L)} = 4a - 4\zeta_2 a^2 + 22\zeta_4 a^3 + \mathcal{O}(a^4)\,, \tag{4.3}$$

$$\mathcal{G}_0(a) = \sum_{L=1}^{\infty} a^L \mathcal{G}_0^{(L)} = -\zeta_3 a^2 + \left(4\zeta_5 + \frac{10}{3}\zeta_2\zeta_3\right) a^3 + \mathcal{O}(a^4)\,. \tag{4.4}$$

5. Ultraviolet divergences in higher dimensions

The Sudakov form factor is ultraviolet (UV) finite in $D=4$ dimensions. One can now study the form factor as a function of the number D of space-time dimensions and investigate at which D it first develops UV divergences. This particular D is called "critical dimension" and depends on the number of loops. Hence we denote it by $D_c(L)$. The knowledge of D_c at a given loop order is useful since it can allow for a cross-check of computations, or constrain the types of loop integrals that can appear (or, even more important, that *cannot* appear). There is a bound on D_c based on power counting for supergraphs and the background field method which reads [22, 23],

$$D_c(L) \geq 4 + \frac{2(\mathcal{N}-1)}{L} = 4 + \frac{4}{L}\,, \qquad L > 1\,. \tag{5.1}$$

The formula is valid for $L>1$ only. For $D<D_c$ the theory is UV finite. We plugged in $\mathcal{N}=3$ in (5.1) since here $\mathcal{N}$ denotes on the number of supersymmetries that can be realized off-shell.

We will now investigate whether the lower bound (5.1) for D_c is saturated, or if the formula gives a bound that is too conservative. There is no statement from Eq. (5.1) for the one-loop case, but one can easily see from Fig. 2 that $D_c(L=1)=6$. From the same Figure, one can see that also at two-loops we have $D_c(L=2)=6$, which follows from naïve power counting. Hence at two-loops the bound (5.1) is indeed saturated. At three loops, Eq. (5.1) becomes $D_c \geq 16/3$. We will now investigate if we have $D_c(L=3)=16/3$ or if the form factor at three loops is better behaved in the UV than expected from (5.1). To this end we take the UV limit of the three-loop term of Eq. (3.1) by giving all propagators (and also all numerators) a common mass m and by nullifying the external momenta. This is possible since there are no sub-divergences in $D=16/3$. In this limit we get [5]

$$F_S^{3-loop} \propto (-q^2)\,[8F_1 + 2F_3^* + 2F_4^*] - 2F_2 + 4F_5^* - 2F_9\,. \tag{5.2}$$

where the asterisk on F_3 and F_4 indicates the respective integral with unit numerator. F_5^* is obtained from F_5 by replacing in the numerator $(p_a^{F_5}+p_b^{F_5})^2 \longrightarrow (p_a^{F_6}+p_b^{F_6}-p_a^{F_5}+p_b^{F_5})^2$. The first three integrals are finite by naïve power counting, and the last three integrals become equal in the aforementioned UV limit, and cancel due to their pre-factors. This renders the three-loop form factor finite in $D = 16/3$ dimensions. It is therefore better behaved in the UV than suggested by Eq. (5.1).

The next value of D where the form factor can – and indeed does – develop UV divergences is $D_c(L=3) = 6$. We have therefore found $D_c(L) = 6$ for $L = 1, 2, 3$. We now take a closer look at the UV properties of the form factor in six dimensions. Specifying $D = 6-2\varepsilon$ and taking the aforementioned UV limit we find that the leading UV pole at L loops is $1/\varepsilon^L$. Moreover, the leading pole is always produced by the L-loop planar ladder diagram. All other diagrams start at most at a subleading pole in ε. When considering $\log(F_S)$ in the UV limit all higher poles cancel and there are only simple $1/\varepsilon$ poles up to three loops.

An equation similar to (5.1) holds also for scattering amplitudes in the UV limit. In this case one even finds the stronger bound $D_c(L) \geq 4+6/L$, which is saturated at two and three loops [3]. At one loop one finds $D_c(L=1) = 8$ for the four-particle scattering amplitude. So despite the fact that the form factor is better behaved in the UV than expected, four-particle scattering amplitudes are even better behaved in the UV than the form factor. One reason for this is the fact that amplitudes, at least in the planar limit, are dual conformal invariant, whereas form factors are not. Another reason is the fact that in $D = 6$ the operator $\mathcal{O}$ in (1.1) has the counterterm $g^2\Box\,\mathrm{tr}\,(\phi^2)$, and other operators having the same quantum numbers; and operator mixing can occur at one loop.

6. Conclusion

We presented the results for the Sudakov form factor in $\mathcal{N}=4$ super Yang-Mills theory up to the three-loop level. We employed the unitarity-based method to derive the answer in terms of both, planar and non-planar loop integrals. At each loop order, the form factor is expressed as a linear combination of only a handful scalar integrals, with small integer coefficients. We evaluated the form factor in dimensional regularisation to $\mathcal{O}(\varepsilon^{8-2L})$ (L is the number of loops) and found that the expansion coefficients of each integral exhibit homogeneous transcendentality in the Riemann ζ-function. Moreover, we verified the exponentiation of infrared divergences, and reproduced the correct values of the cusp and collinear anomalous dimensions.

In addition, we observed that the heuristic leading transcendentality principle that relates anomalous dimensions in QCD with those in $\mathcal{N}=4$ SYM also holds for the form factor. We verified this principle to three loops, and through to terms of transcendentality eight.

Finally, we studied the UV behaviour of the form factor in higher dimensions, and found that the critical dimension ist given by $D_c(L) = 6$ up to three loops. This means that the three-loop result is better behaved in the UV than suggested by Eq. (5.1). In particular, it is finite in $D = 16/3$ dimensions.

An interesting further direction of the present calculation would be its extension to four loops, since it would allow to get insight into the non-planar colour structure. Whether the anomalous dimension associated with the quartic Casimir $(d_{abcd})^2$ vanishes is a hot topic and has to do with the general question of colour dependence of infrared divergences in gauge theories [8].

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