

## Towards Higgs+jet production at NNLO

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**Thomas Gehrmann\***

*Institut für Theoretische Physik, Universität Zürich, CH-8057 Zürich, Switzerland*

*E-mail: [thomas.gehrmann@uzh.ch](mailto:thomas.gehrmann@uzh.ch)*

We report on recent progress made towards the calculation of NNLO corrections to Higgs+jet production at the LHC. The calculation of the two-loop parton-level amplitudes relevant to this process is described. The numerical implementation of this calculation requires a subtraction method to handle infrared divergencies at NNLO. For this purpose, the antenna subtraction method is extended to hadron collider observables, and validated on jet production at NNLO.

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\*Speaker.

## 1. Introduction

The LHC experiments ATLAS and CMS have recently reported [1, 2] the observation of a new boson at a mass of 125 GeV, with properties consistent (within still large error margins) with the Higgs boson of the Standard Model. The precise determination of the new boson's properties will be a major challenge to particle physics in the near future. In this context, numerous production processes and decay channels will be investigated in detail, thereby requiring accurate theory predictions for the interpretation of experimental observations.

Experimental searches for final states containing Higgs boson signatures apply cuts to improve the significance of a potential signal over Standard Model background processes. To implement these cuts in the theoretical description, fully exclusive calculations, which keep track of the kinematical information of all final state particles (Higgs decay products and QCD radiation) are mandatory. In the heavy top quark limit, the NNLO corrections to Higgs production via gluon fusion have been computed fully exclusively, including the Higgs decay to two photons or two weak gauge bosons by two independent groups [3, 4]. These calculations are in the form of flexible parton-level event generators, which can properly account for the final state restrictions used in the experimental studies.

An important final state discriminator is the number of jets observed in addition to the potential Higgs boson decay products. In many searches, it is expected that the  $H + 0j$  and  $H + 1j$  samples contribute roughly equally to the sensitivity. In the above-mentioned NNLO calculations, the  $H + 1j$  final states are included to NLO [5], and the  $H + 2j$  final states to LO. NLO corrections to  $H + 2j$ -production have been derived recently [6].

In the heavy top quark limit, a full NNLO QCD calculation of  $H + 1j$  production requires the computation of the matrix elements of three contributions:

- (a) the tree level  $H \rightarrow 5$  partons amplitudes,
- (b) the one-loop corrections to the  $H \rightarrow 4$  partons amplitudes,
- (c) the two-loop corrections to the  $H \rightarrow ggg$  and  $H \rightarrow q\bar{q}g$  matrix elements.

The tree-level contributions of type (a) can be computed with standard tree-level methods, and compact expressions can be obtained by using MHV-techniques [7]. The one-loop terms of type (b) were derived in an analytic form in [8], and form part of the NLO corrections to  $H + 2j$  final states. The  $H \rightarrow ggg$  and  $H \rightarrow gq\bar{q}$  matrix elements were previously known to one loop [9], and we report here on our recently derived two-loop results [10].

The three different contributions must be combined into a parton-level event generator program. All three are separately infrared-divergent, and only their sum is finite and physically meaningful. To combine the contributions, an infrared subtraction method is required, for example the antenna subtraction method discussed below. A resulting parton-level event generator will allow an NNLO description of both  $H + 1j$  production and of the Higgs boson transverse momentum distribution. In the following, we describe the NNLO antenna subtraction method, which has been extended to jet observables at hadron colliders recently.

## 2. Helicity amplitudes for $H \rightarrow 3$ partons

The general form of the renormalized helicity amplitude  $|\mathcal{M}_{ggg}^{\lambda_1\lambda_2\lambda_3}\rangle$  for the decay,  $H(p_4) \rightarrow g_1(p_1, \lambda_1) + g_2(p_2, \lambda_2) + g_3(p_3, \lambda_3)$  can be written as,

$$|\mathcal{M}_{ggg}^{\lambda_1\lambda_2\lambda_3}\rangle = S_{\mu\nu\rho}(g_1; g_2; g_3) \varepsilon_{1,\lambda_1}^\mu(p_1) \varepsilon_{2,\lambda_2}^\nu(p_2) \varepsilon_{3,\lambda_3}^\rho(p_3), \quad (2.1)$$

where the  $\lambda_i = \pm$  denote helicity. Similarly, the amplitude for the decay  $|\mathcal{M}_{q\bar{q}g}^{\lambda_1\lambda_2\lambda_3}\rangle$  for the decay,  $H(p_4) \rightarrow q(p_1, \lambda_1) + \bar{q}(p_2, \lambda_2) + g(p_3, \lambda_3)$  can be written as,

$$|\mathcal{M}_{q\bar{q}g}^{\lambda_1\lambda_2\lambda_3}\rangle = T_\rho(q^{\lambda_1}; \bar{q}^{\lambda_2}; g) \varepsilon_{3,\lambda_3}^\rho(p_3). \quad (2.2)$$

By using parity, charge conjugation and Bose symmetry, one finds that only two independent helicity amplitudes for  $H \rightarrow ggg$  and only one for  $H \rightarrow q\bar{q}g$  remain. These can be expressed as:

$$\begin{aligned} |\mathcal{M}_{ggg}^{+++}\rangle &= \alpha \frac{1}{\sqrt{2}} \frac{M_H^4}{\langle p_1 p_2 \rangle \langle p_2 p_3 \rangle \langle p_3 p_1 \rangle}, \\ |\mathcal{M}_{ggg}^{++-}\rangle &= \beta \frac{1}{\sqrt{2}} \frac{[p_1 p_2]^3}{[p_2 p_3][p_1 p_3]}, \\ |\mathcal{M}_{q\bar{q}g}^{-++}\rangle &= \gamma \frac{1}{\sqrt{2}} \frac{[p_2 p_3]^2}{[p_1 p_2]}. \end{aligned} \quad (2.3)$$

The helicity coefficients  $\Omega = \alpha, \beta, \gamma$  can be computed as perturbative series by applying appropriate  $D$ -dimensional projections to the Feynman diagrams contributing to the process under consideration. They are vectors in colour space and have perturbative expansions,

$$\Omega = \lambda \sqrt{4\pi\alpha_s} T_\Omega \left[ \Omega^{(0)} + \left(\frac{\alpha_s}{2\pi}\right) \Omega^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Omega^{(2)} + \mathcal{O}(\alpha_s^3) \right], \quad (2.4)$$

for  $\Omega = \alpha, \beta, \gamma$ . The colour factor is  $T_\alpha = T_\beta = f^{a_1 a_2 a_3}$  and  $T_\gamma = T_{i_1 j_2}^{a_3}$ . At leading order,

$$\alpha^{(0)} = \beta^{(0)} = \gamma^{(0)} = 1. \quad (2.5)$$

For  $H \rightarrow ggg$ , there are four diagrams at tree-level, 60 diagrams at one loop and 1306 diagrams at two loops, while for  $H \rightarrow q\bar{q}g$ , we have one diagram at tree-level, 15 diagrams at one loop and 228 diagrams at two loops. We computed the two-loop diagrams [10] and reduced the two-loop integrals appearing in them using integration-by-parts identities [11, 12] with the REDUZE program [13] to a set of master integrals. The master integrals relevant to this process are two-loop four-point functions with one off-shell leg, they were derived previously [14] in the context of the two-loop corrections to  $\gamma^* \rightarrow q\bar{q}g$ .

After ultraviolet renormalization, the amplitudes contain infrared singularities that will be analytically canceled by those occurring in radiative processes of the same order. Catani [15] has shown how to organize the infrared pole structure of the one- and two-loop contributions renormalized in the  $\overline{\text{MS}}$  scheme in terms of the tree and renormalized one-loop amplitudes. The finite two-loop remainder is then obtained by subtracting the predicted infrared structure (expanded through

to  $\mathcal{O}(\epsilon^0)$ ) from the renormalized helicity coefficient. We further decompose the finite remainder according to the colour casimirs as follows,

$$\Omega^{(2),finite} = \left( N^2 A_\Omega^{(2)} + N^0 B_\Omega^{(2)} + \frac{1}{N^2} C_\Omega^{(2)} + \frac{N_F}{N} D_\Omega^{(2)} + NN_F E_\Omega^{(2)} + N_F^2 F_\Omega^{(2)} \right). \quad (2.6)$$

Analytic expressions for all one- and two-loop coefficients are given in [10] in terms of one- and two-dimensional harmonic polylogarithms [16].

Shortly after the release of our results, Brandhuber, Travaglini and Yang validated the leading transcendentality contribution to the  $H \rightarrow ggg$  amplitude from a calculation in  $N = 4$  SYM theory [17]. By applying the calculus of symbols to our results, Duhr could re-express the  $H \rightarrow ggg$  amplitude in terms of ordinary polylogarithms, thereby obtaining a very compact and elegant expression for them [18].

### 3. Antenna subtraction formalism for hadronic collisions

The  $H + 2$ -parton and  $H + 3$ -parton matrix elements contribute to  $H + 1$ -jet observables at NNLO if the extra partons are unresolved or are clustered to form an  $H + 1$ -jet final state. Consequently, these extra partons are unconstrained in the soft and collinear regions, and yield infrared divergences. To determine the contribution to NNLO jet observables from these configurations, one has to find subtraction terms which coincide with the full matrix element in the unresolved limits and are still sufficiently simple to be integrated analytically in order to cancel their infrared pole structure with the virtual contributions.

A systematic method to derive these subtraction terms to NNLO is provided by the antenna subtraction technique. This method was initially derived for  $e^+e^-$  annihilation [19] in the context of the calculation of NNLO corrections to three-jet production and related event shapes [20]. The antenna subtraction formalism constructs the subtraction terms from antenna functions. Each antenna function encapsulates all singular limits due to the emission of one or two unresolved partons between two colour-connected hard radiator partons. This construction exploits the universal factorisation of matrix elements and phase space in all unresolved limits.

While both radiator partons are in the final state for  $e^+e^-$ -initiated processes, one encounters two new types of antenna functions [21] in hadronic collisions: initial-final antenna functions with one radiator parton in the initial state, and initial-initial antenna functions with both radiator partons in the initial state. The initial-final and initial-initial antenna functions appearing in the NNLO subtraction terms are obtained from crossing the final-final antennae. Their integration has to be performed over the appropriate phase space. In the case of the initial-final antennae, this has been accomplished in [22]. The initial-initial one-loop antenna functions were integrated in [23], and the integration of the initial-initial tree-level double real antenna functions has been completed most recently in [24, 25].

A first application of antenna subtraction for hadron collider processes is the calculation of two-jet production at NNLO, where the unintegrated subtraction terms for purely gluonic processes have already been derived and tested for the double real radiation at tree-level [26] and the

single real radiation at one-loop [27]. In these, the all-gluon NNLO antenna functions appear in all configurations, and their integrated forms constitute the integrated subtraction terms for the two-jet production process. They contribute to the cancellation of all infrared poles when combined with mass factorization terms and with the virtual two-loop matrix elements [28]. This pole cancellation has been verified [28, 29] and can be regarded as a strong check on integrated antenna functions, and on the applicability of the antenna subtraction method to hadron collider processes.

#### 4. Conclusions and outlook

In this talk, we reported on recent progress towards the calculation of NNLO corrections to Higgs+jet production at the LHC. We described the analytical calculation of the two-loop helicity amplitudes contributing to this process, based on a reduction to known master integrals. The implementation of all subprocesses contributing at this order into a parton-level event generator requires a subtraction method to handle infrared singular radiation. We described the extension of the NNLO antenna subtraction method to hadron collider observables by introducing antenna functions for initial state radiation.

These new analytical results and technical developments will allow the construction of an NNLO parton-level event generator for Higgs+jet production, as well as for other jet final states at hadron colliders. Work on these is currently ongoing.

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#### References

- [1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716** (2012) 1 [arXiv:1207.7214].
- [2] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716** (2012) 30 [arXiv:1207.7235].
- [3] C. Anastasiou, K. Melnikov, F. Petriello, Nucl. Phys. **B724** (2005) 197 [hep-ph/0501130];  
C. Anastasiou, G. Dissertori, F. Stockli, JHEP **0709** (2007) 018 [arXiv:0707.2373].
- [4] M. Grazzini, JHEP **0802** (2008) 043 [arXiv:0801.3232].
- [5] D. de Florian, M. Grazzini, Z. Kunszt, Phys. Rev. Lett. **82** (1999) 5209 [hep-ph/9902483];  
V. Ravindran, J. Smith, W. L. Van Neerven, Nucl. Phys. **B634** (2002) 247 [hep-ph/0201114].
- [6] J. M. Campbell, R. K. Ellis, G. Zanderighi, JHEP **0610**, 028 (2006). [hep-ph/0608194];  
J. M. Campbell, R. K. Ellis, C. Williams, Phys. Rev. **D81** (2010) 074023. [arXiv:1001.4495].
- [7] L. J. Dixon, E. W. N. Glover, V. V. Khoze, JHEP **0412** (2004) 015 [hep-th/0411092];  
S. D. Badger, E. W. N. Glover and V. V. Khoze, JHEP **0503** (2005) 023 [hep-th/0412275].

- [8] S. D. Badger and E. W. N. Glover, Nucl. Phys. Proc. Suppl. **160** (2006) 71 [hep-ph/0607139].  
L. J. Dixon, Y. Sofianatos, JHEP **0908** (2009) 058 [arXiv:0906.0008];  
S. Badger, E. W. N. Glover, P. Mastrolia, C. Williams, JHEP **1001** (2010) 036 [arXiv:0909.4475];  
S. Badger, J. M. Campbell, R. K. Ellis, C. Williams, JHEP **0912** (2009) 035 [arXiv:0910.4481].
- [9] C. R. Schmidt, Phys. Lett. B **413** (1997) 391 [hep-ph/9707448].
- [10] T. Gehrmann, M. Jaquier, E. W. N. Glover and A. Koukoutsakis, JHEP **1202** (2012) 056 [arXiv:1112.3554].
- [11] F.V. Tkachov, Phys. Lett. **100B** (1981) 65;  
K.G. Chetyrkin and F.V. Tkachov, Nucl. Phys. **B192** (1981) 159.
- [12] S. Laporta, Int. J. Mod. Phys. A **15** (2000) 5087 [hep-ph/0102033].
- [13] C. Studerus, Comput. Phys. Commun. **181** (2010) 1293. [arXiv:0912.2546].
- [14] T. Gehrmann and E. Remiddi, Nucl. Phys. **B601** (2001) 248 [hep-ph/0008287]; **B601** (2001) 287 [hep-ph/0101124].
- [15] S. Catani, Phys. Lett. **B427** (1998) 161 [hep-ph/9802439].
- [16] E. Remiddi and J.A.M. Vermaseren, Int. J. Mod. Phys. **A15** (2000) 725 [hep-ph/9905237].
- [17] A. Brandhuber, G. Travaglini and G. Yang, JHEP **1205** (2012) 082 [arXiv:1201.4170].
- [18] C. Duhr, JHEP **1208** (2012) 043 [arXiv:1203.0454].
- [19] A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, JHEP **0509** (2005) 056 [hep-ph/0505111].
- [20] A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover and G. Heinrich, JHEP **0711** (2007) 058 [arXiv:0710.0346]; Phys. Rev. Lett. **100** (2008) 172001 [arXiv:0802.0813];  
S. Weinzierl, Phys. Rev. Lett. **101** (2008) 162001 [arXiv:0807.3241]; JHEP **0907** (2009) 009 [arXiv:0904.1145].
- [21] A. Daleo, T. Gehrmann and D. Maître, JHEP **0704** (2007) 016 [hep-ph/0612257].
- [22] A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann and G. Luisoni, JHEP **1001** (2010) 118 [arXiv:0912.0374].
- [23] T. Gehrmann and P. F. Monni, JHEP **1112** (2011) 049 [arXiv:1107.4037].
- [24] R. Boughezal, A. Gehrmann-De Ridder and M. Ritzmann, JHEP **1102** (2011) 098 [arXiv:1011.6631].
- [25] A. Gehrmann-De Ridder, T. Gehrmann and M. Ritzmann, arXiv:1207.5779.
- [26] E. W. N. Glover and J. Pires, JHEP **1006** (2010) 096 [arXiv:1003.2824].
- [27] A. Gehrmann-De Ridder, E. W. N. Glover and J. Pires, JHEP **1202** (2012) 141 [arXiv:1112.3613].
- [28] A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, J. Pires, in preparation.
- [29] J. Pires, these proceedings.